Lesson 9   Slope of a Linear   May 10, 2024
Function and Properties Continued

Recall. Definition

slope $\longrightarrow$ rate of change   of $f(x)$

(y)

(visually, slope = steepness of a hill)

$$\boxed{m = \frac{rise}{run}}$$

slope $\longrightarrow$ how fast sthg is changing

$\longrightarrow$ if that change is an increase **(+)**

or an decrease **(–)**

slope

$\downarrow$

$$m = \frac{\Delta y}{\Delta x}$$

$$a = \frac{rise}{run}$$

slope

$\dfrac{\sharp}{hr}$

Slope and Reading Graph

Electrician A's Salary as a Function of Weekly Hours Worked ①
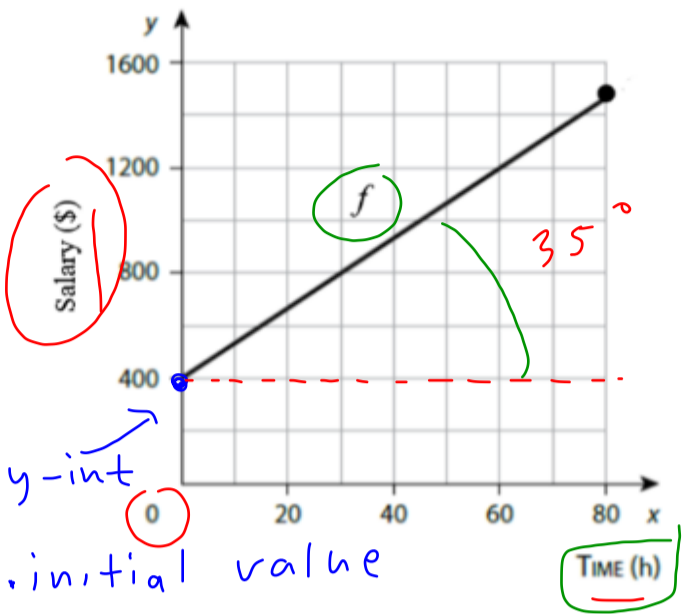
(graph: Salary ($) vs Time (h), line labeled f, angle 35°, y-int at 400, dashed line at 400)

y-int
• initial value — TIME (h)

Electrician B's Salary as a Function of Weekly Hours Worked

(graph: Salary ($) vs Time (h), line labeled g, angle 5°)

TIME (h)

(protractor image)
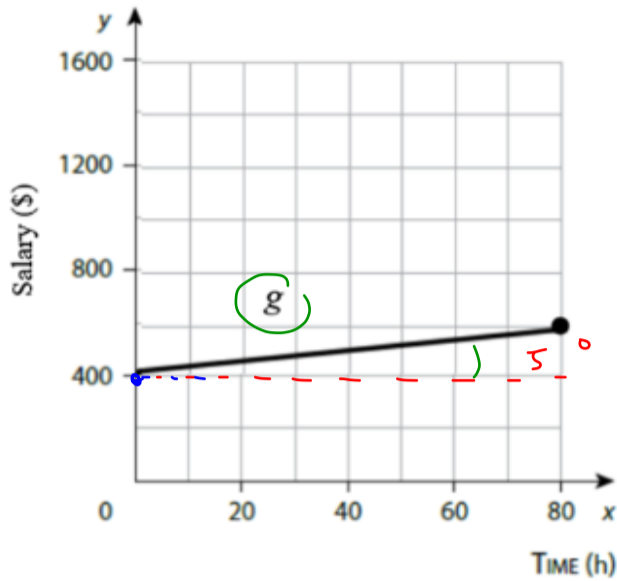
a) Which line is steeper? That is, which function is changing (increasing) faster?
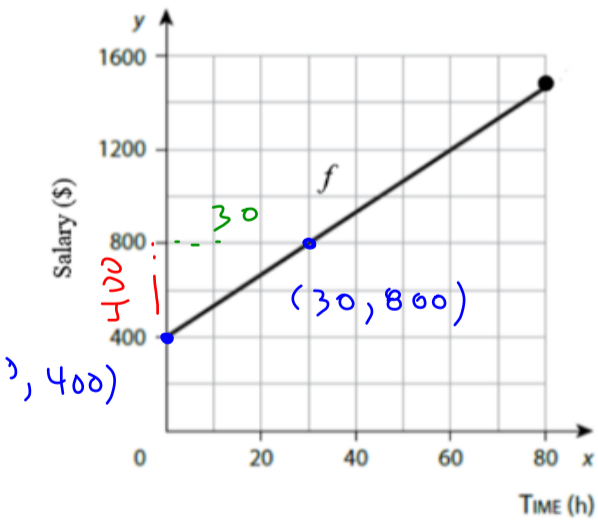
f   steep (adj) = making a large angle w̄ the horizon

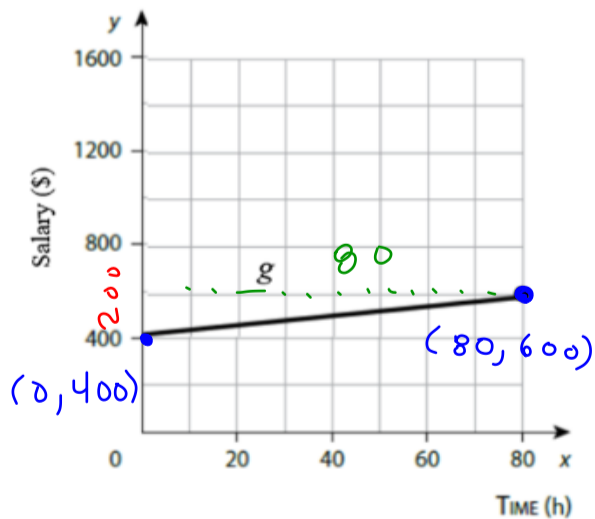### b) Are the two slopes positive or negative?

if f is increasing, then a is +

if f is decreasing, then a is —

c) Verify your answers by calculating the slope of each line using the formula rise over run.

Electrician A's Salary as a Function of Weekly Hours Worked

(graph with point (30, 800), line f, 400, (0, 400))

$$a = \frac{rise}{run}$$

$$a = \frac{400}{30}$$

$$a = 13.30 \ \$/hr$$

Electrician B's Salary as a Function of Weekly Hours Worked

(graph with line g, 80, 200, (80, 600), (0, 400))

$$a = \frac{200}{80}$$

$$a = 2.50 \ \$/hr$$

step i: find 2 points on grid lines

step ii: count how units rise (up → +) (down → —)

rise = 400

count how many units you run (right +)

run = 30

Part B:

Electrician A's Salary as a Function of Weekly Hours Worked

a) During the entire observation period, over which time interval does electrician A's salary increase?   [0, 80] hr

b) During the entire domain, what is the increasing interval of function g?   [0, 80] hr

c) During the entire observation period, (for how long) does electrician A's salary increase? 80hr

d) During the entire domain, (for how long) does function g increase? for 80hr

e) When does electrician A's salary decrease?   Ø

f) What is the decreasing interval of function g?   Ø

g) Over which time interval is electrician A's salary negative?   Ø

h) What is the negative interval of function g?   Ø

i) What is the maximum salary of electrician A? $1500

j) What is the maximum of function g? $600

k) What is the range of electrician A's salary?   [400, 1500] $

l) What is the range of function g?   [400, 600] $

m) What is the domain of function f?   [0, 80] hr

n) What is the domain of function g?   [0, 80] hr

o) What is Electrician A's initial salary? $400

p) What is the initial value of function g? $400
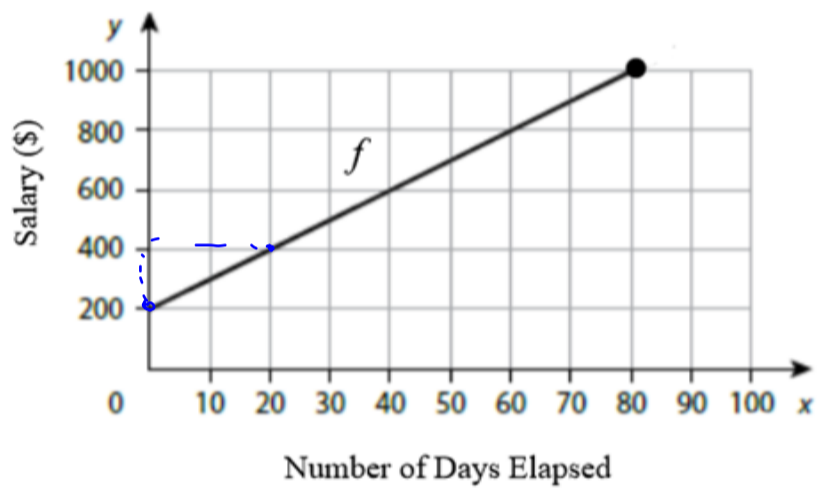
when x = 0
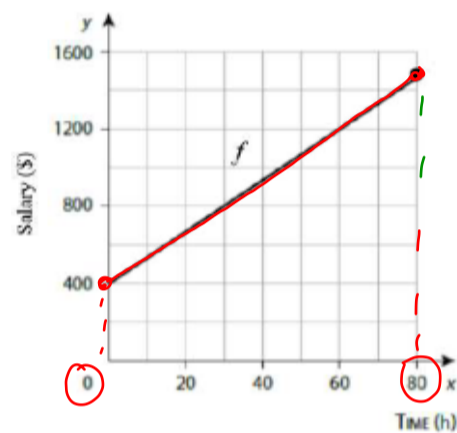
Electrician B's Salary as a Function of Weekly Hours Worked
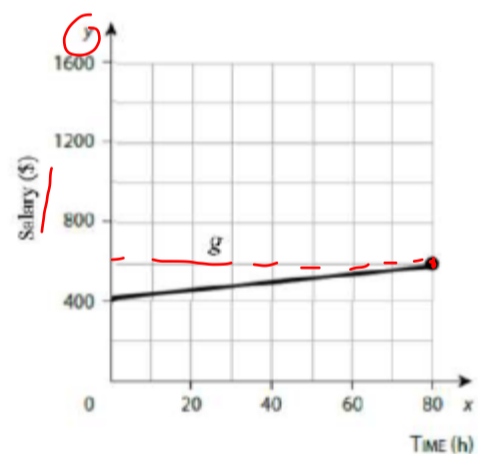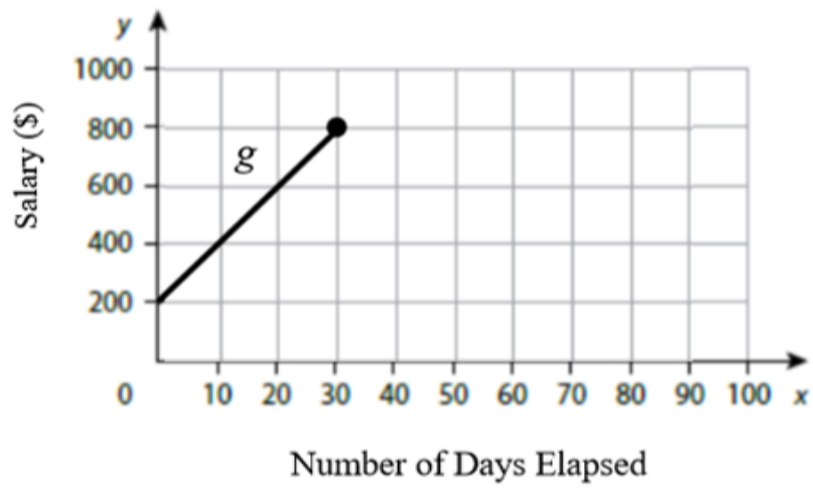
You do
Q2 part A
and part B

for how long ≠ when
   "
duration
$d = |x_2 - x_1|$

Isabelle's Salary Since the Start of the Year

Salary ($) vs Number of Days Elapsed — line labeled f

Harry's Salary Since the Start of the Year

Salary ($) vs Number of Days Elapsed — line labeled g

a) Harry

b) +

c) $\dfrac{200}{20}$

$a = \dfrac{200}{20}$
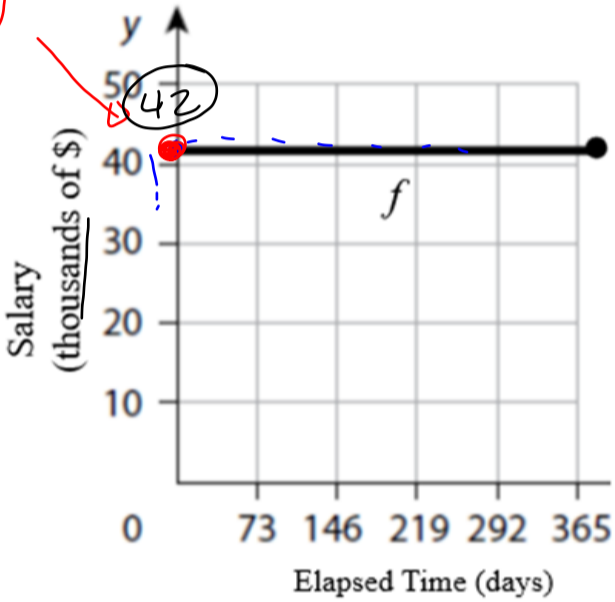
$a = 10 \ \$/day$

$a = 20 \ \$/day$
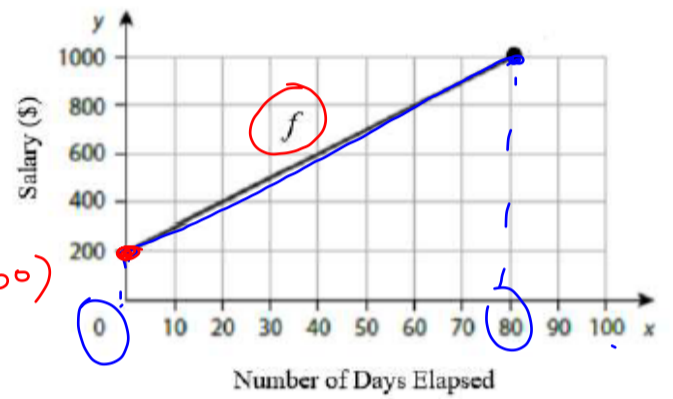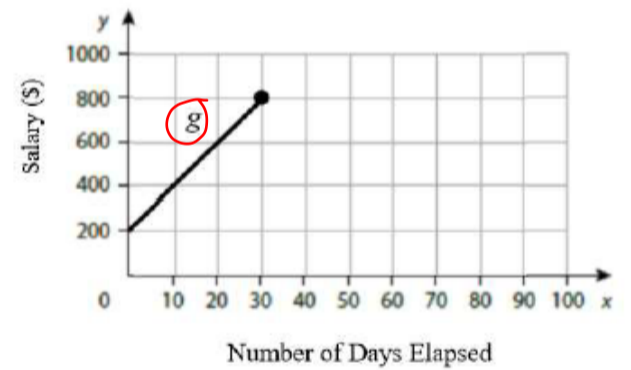
Part B:

a) Over what time interval does Isabelle earn salary? $[0, 80]$ day

b) What is the increasing interval of function g? $[0, 30]$ day

c) For how many days does Isabelle's salary increase? 80 day

d) For how long does function g increase? 30 day

e) What is Isabelle's minimum salary? $200

f) What is the minimum value of function g? $200

g) Over what time interval does Isabelle's salary decrease? ∅

h) What is the decreasing interval of function g? ∅

i) What is the range of Isabelle's salary? $[200, 1000]$ $

j) What is the range of function g? $[200, 800]$ $

k) What is the domain of function f? $[0, 80]$ days

l) What is the domain of function g? $(0, 30]$ day

m) What is Isabelle's initial salary? $200

n) What is the y-intercept of function g? $200

$(0, 200)$

Isabelle's Salary Since the Start of the Year

Salary ($) vs Number of Days Elapsed — function f

Harry's Salary Since the Start of the Year

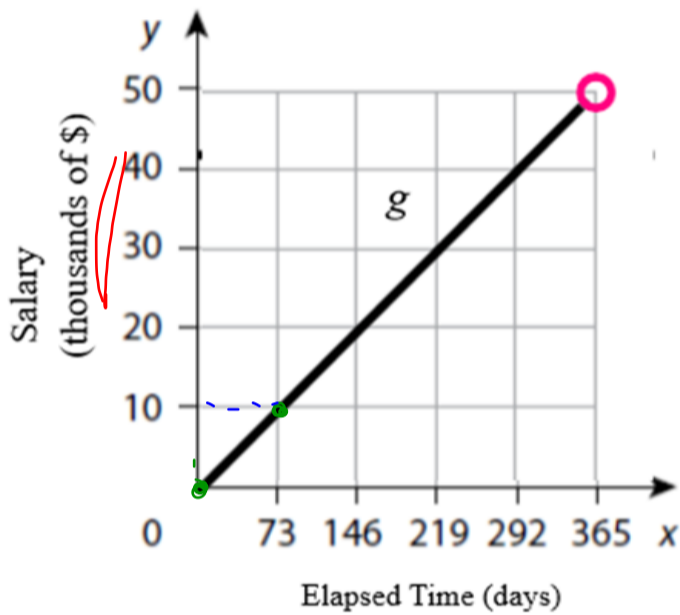Salary ($) vs Number of Days Elapsed — function g

Think — Pair — Share

A Regular Teacher's Yearly Starting Salary

y-int / initial value (b)

A Substitute Teacher's Yearly Salary

What's the story the graph is telling?

a) Which line is steeper? That is, which function is changing (increasing) faster?

? !

$$a = \frac{0}{365}$$

$$a = 0 \; \$/day$$

$$y = ax + b$$

$$y = 0x + 42000$$

$$\boxed{y = 42000}$$

The constant Function

$$\boxed{y = b}$$
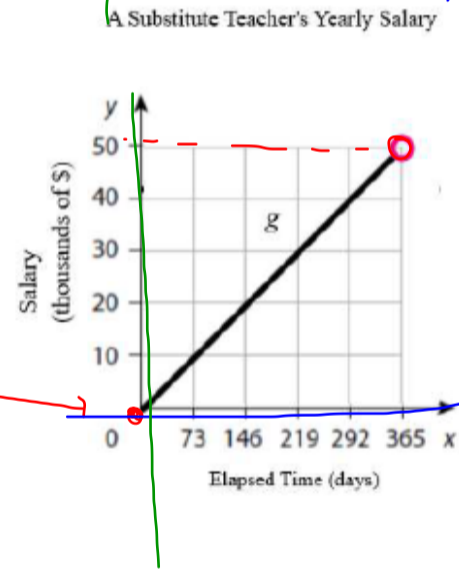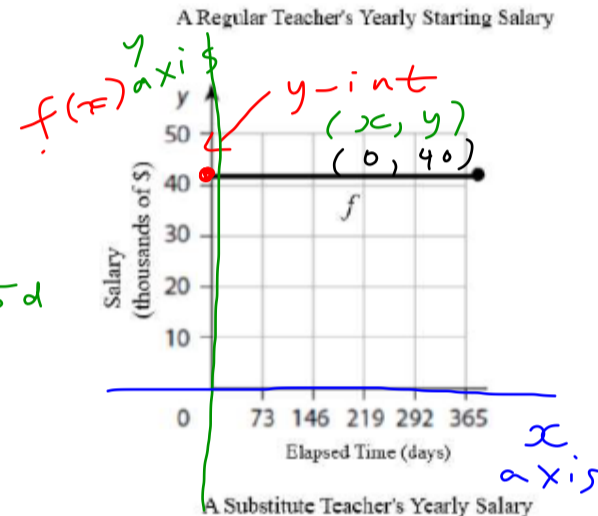
$$a = \frac{10000\$}{73 \; day}$$

$$a = 136 \; \$/day$$

$$a = 0.14 \; \$/day$$

[handwritten: Pg 3]

Part B:

a) During the entire observation period, over which time interval does a regular teacher's salary increase? *[handwritten: ∅]*

b) During the entire domain, what is the increasing interval of function g? *[handwritten: [0, 365[day]*

c) During the entire observation period, for how long is a regular teacher's salary positive? *[handwritten: for 365 d]*

d) During the entire domain, for how long is function g positive? *[handwritten: for 365 d.]*

e) When does a regular teacher's salary decrease? *[handwritten: ∅]*

f) What is the decreasing interval of function g? *[handwritten: ∅]*

g) Over which time interval is a regular teacher's salary negative? *[handwritten: ∅]*

h) What is the negative interval of function g? *[handwritten: ∅]*

i) What is the maximum salary of a regular teacher? *[handwritten: 42 K $]*

j) What is the maximum of function g? *[handwritten: 50 K ?]*

k) What is the range of a regular teacher's salary? *[handwritten: {42}]*

l) What is the range of function g? *[handwritten: [0, 50[]*

m) What is the domain of function f? *[handwritten: [0, 365]]*

n) What is the domain of function g? *[handwritten: [0, 365[day]*

o) What is function f's y-intercept? *[handwritten: 40]*

p) What is the initial value of function g? *[handwritten: 0]*

[handwritten note: 50 & Range]

A Regular Teacher's Yearly Starting Salary

[handwritten annotations: f(F) axis, y-int (x, y) (0, 40), x.axis]

A Substitute Teacher's Yearly Salary

[handwritten annotations: y-int, x-int]
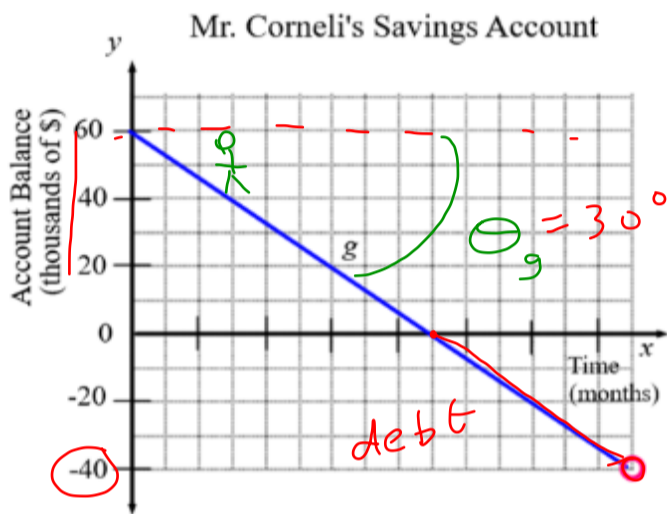
Think – Pair – Share
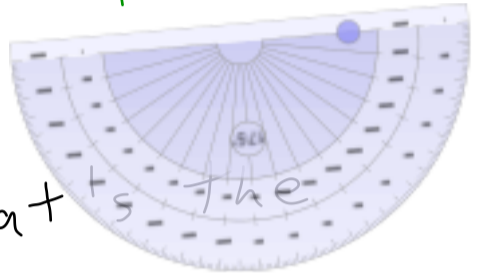
What is the story behind the graph?

Ms. Short's Savings Account

Account Balance (thousands of $)

$\theta_f = 60°$

f

Time (months)

Mr. Corneli's Savings Account

Account Balance (thousands of $)

$\theta_g = 30°$

g

debt
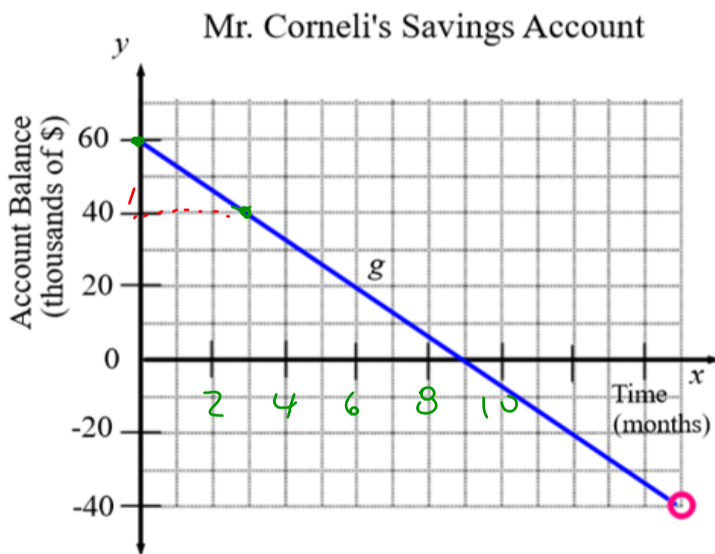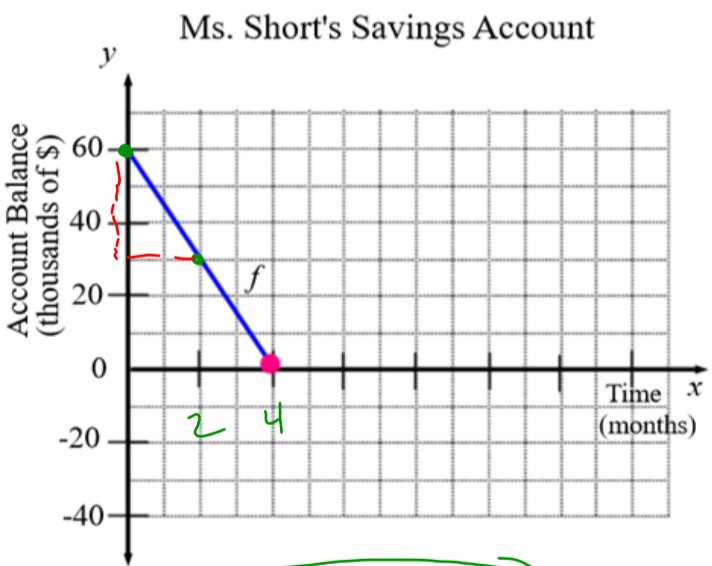
Time (months)

Is this a happy or a sad story?
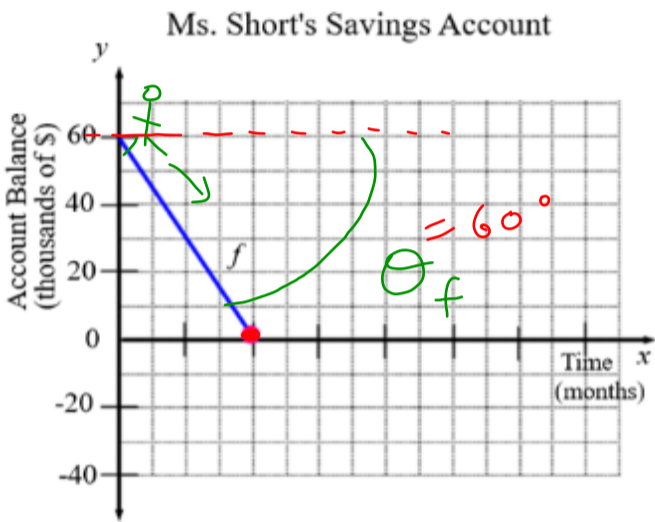
a) Which line is steeper? That is, which function is changing (decreasing) faster?   f)
b) Are the two slopes positive or negative?   (−)
c) Verify your answers by calculating the slope of each line using the formula rise over run.

Ms. Short's Savings Account

Account Balance (thousands of $)

f

2 4

Time (months)

Mr. Corneli's Savings Account

Account Balance (thousands of $)

g

2 4 6 8 10

Time (months)

You do Part B on Pg 3 and pg 4

$$a = \frac{rise}{run}$$ ✓

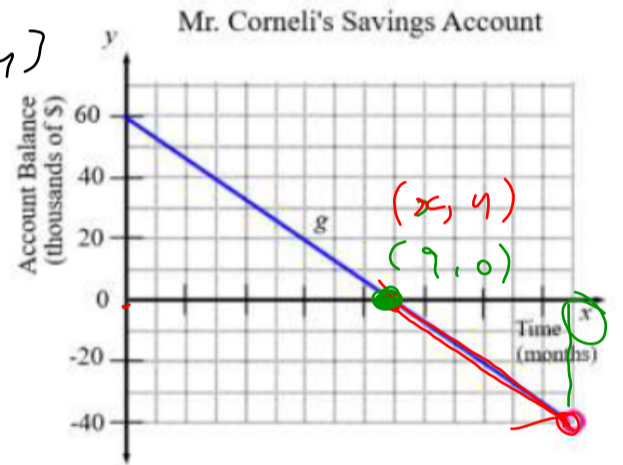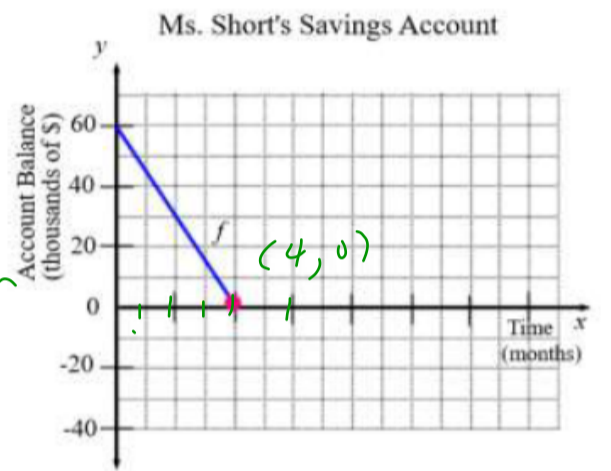$$a = \frac{-30}{2}$$ ← minus cuz down
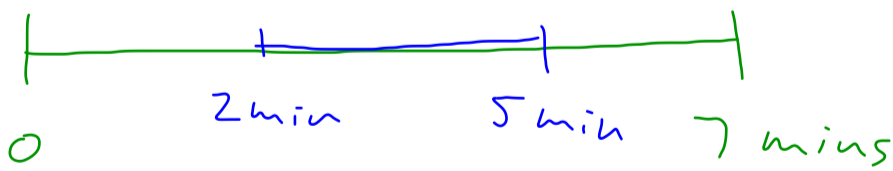
$$a = -15 \ \$/month$$

$$a = \frac{-20}{3}$$

$$a = -6.7 \ \$/month$$

Part B:

Pg 4

a) Over what time interval does Ms. Short save money? ∅
b) What is the increasing interval of function g? ∅
c) What is Ms. Short's minimum account balance? 0 $
d) What is the minimum value of function g? −40 $
e) Over what time interval does Ms. Short's account balance decrease? [0, 4] month
f) What is the decreasing interval of function g? [0, 15[
g) Over what time interval is Ms. Short's account balance negative? ∅
h) What is the negative interval of function g? [9, 15[month
i) For how long is Ms. Short's account balance negative? for 0 month
j) For how long is function g negative? $d = |x_2 - x_1| = 15 - 9 = 6$ months.
k) What is the range of Ms. Short's account balance? [0, 60] [min y, max y]
l) What is the range of function g? [60, −40[x   ]−40, 60]
m) What is the domain of function f? [0, 4] months
n) What is the domain of function g? [0, 15[
o) What is Ms. Short's initial account balance? 60 $
p) What is the y-intercept of function g? 60 $
q) What are the zeros of function f? 4 months
r) What are the x-intercepts of function g? 9 months
s) At what moment does Ms. Short lose all her money? @ the 4th month
t) At what moment does Mr. Corneli go into debt? @ the 9th month

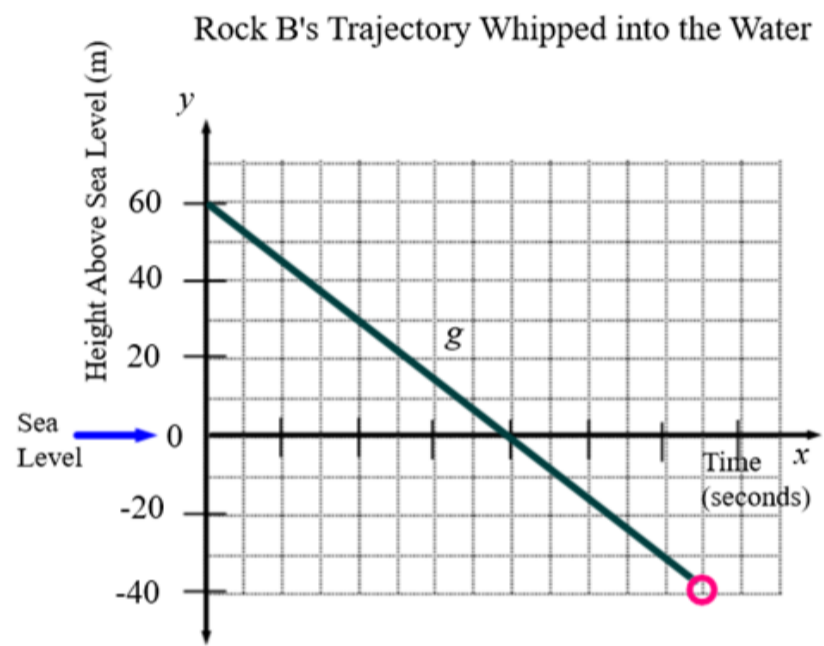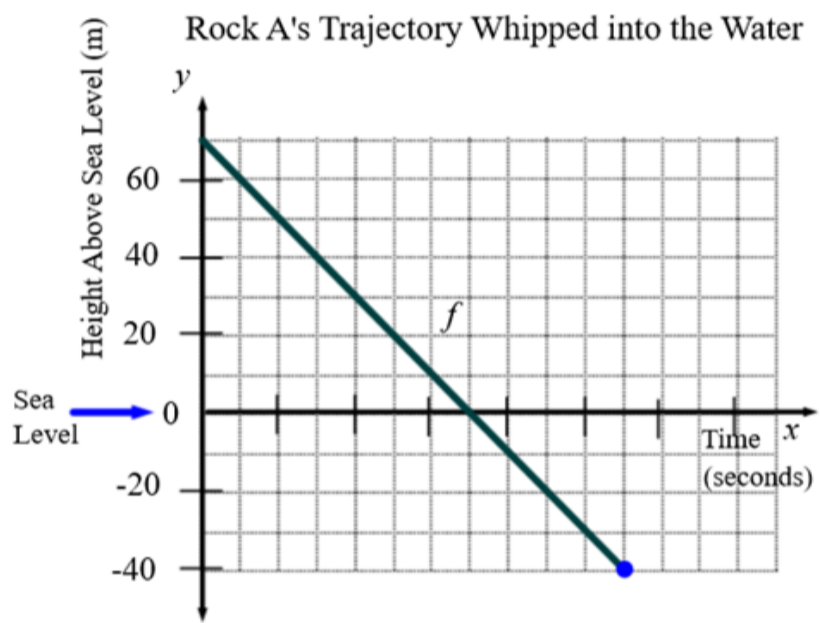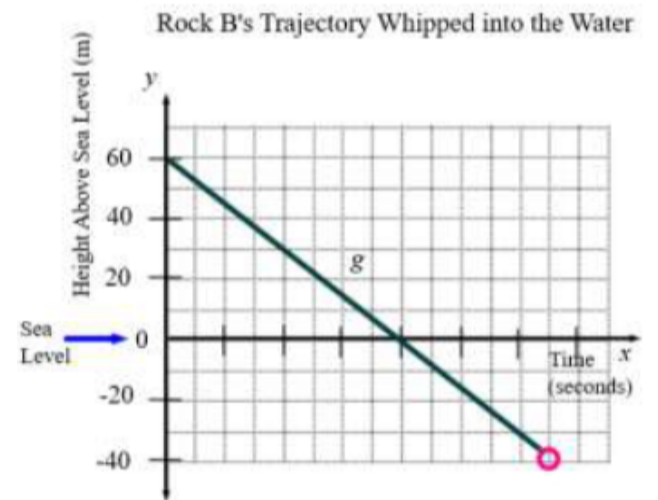Ms. Short's Savings Account

f   (4, 0)

Mr. Corneli's Savings Account

g   (x, y)
(9, 0)

[min x, max x]

Rihanna

2min   5min   7 mins

Drake [2, 5) mins
x_1   x_2

$d = |x_2 - x_1|$

Rock A's Trajectory Whipped into the Water

Rock B's Trajectory Whipped into the Water
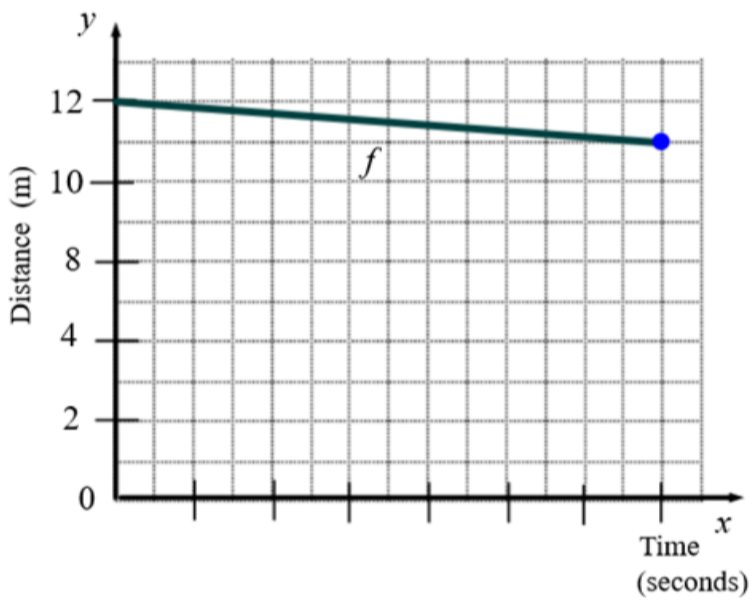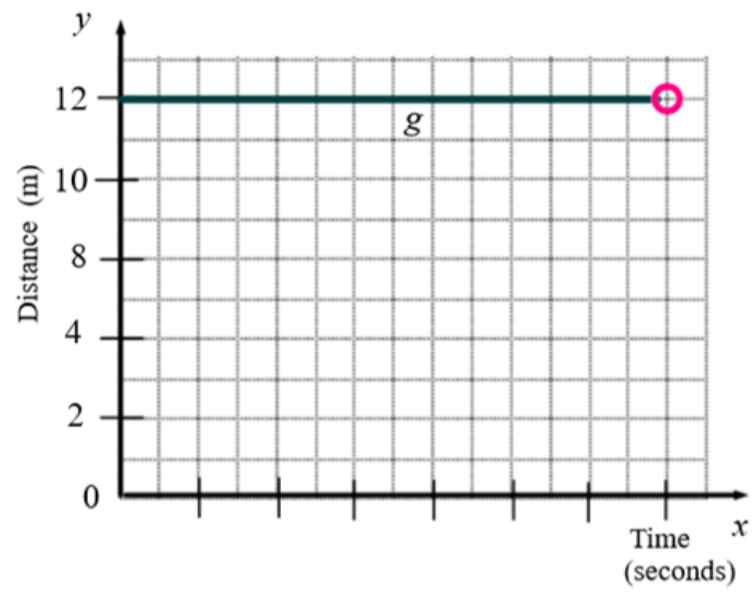
Part B:

a) During the entire observation period, over which time interval does rock A's altitude increase?
b) During the entire domain, what is the increasing interval of function g?
c) During the entire observation period, for how long is rock A's altitude positive?
d) During the entire domain, for how long is function g positive?
e) When does rock A's height decrease?
f) What is the decreasing interval of function g?
g) Over which time interval is rock A's height negative?
h) What is the negative interval of function g?
i) During the entire observation period, for how long is rock A underwater?
j) During the entire domain, for how long is function g negative?
k) What is rock A's maximum altitude?
l) What is the maximum of function g?
m) What is the range of rock A's height?
n) What is the range of function g?
o) What is the domain of function f?
p) What is the domain of function g?
q) What is rock A's initial height?
r) What is the initial value of function g?
s) What are the x-intercepts of function f?
t) What are the zeros of function g?
u) At what moment does rock A go into the water?
v) At what moment does rock B reach sea level?

Rock A's Trajectory Whipped into the Water

Rock B's Trajectory Whipped into the Water

Distance Between Ms. Short and Yufan

f

Distance (m) — y axis (0, 2, 4, 8, 10, 12)

Time (seconds) — x axis
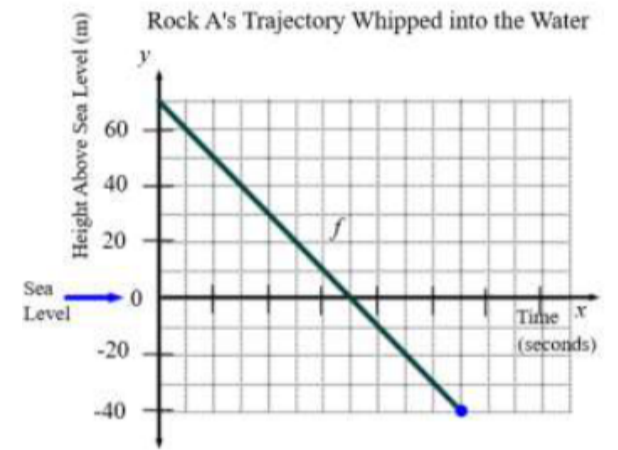
Distance Between Ms. Short and Deelan

g

Distance (m) — y axis (0, 2, 4, 8, 10, 12)

Time (seconds) — x axis
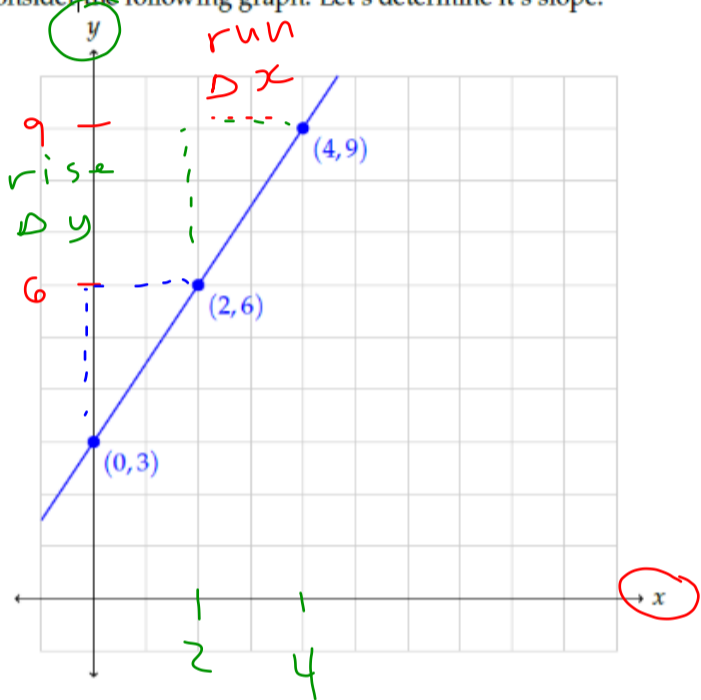
2 Formulas for Slope

Consider the following graph. Let's determine it's slope.

$$a = \frac{\text{rise}}{\text{run}}$$

$$a = \frac{\Delta y}{\Delta x}$$

$\leftarrow$ "change in" y

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

Can we come up with a formula for calculating the slope of a line?

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$P_1 (2, 6)$
$\quad x_1, y_1$

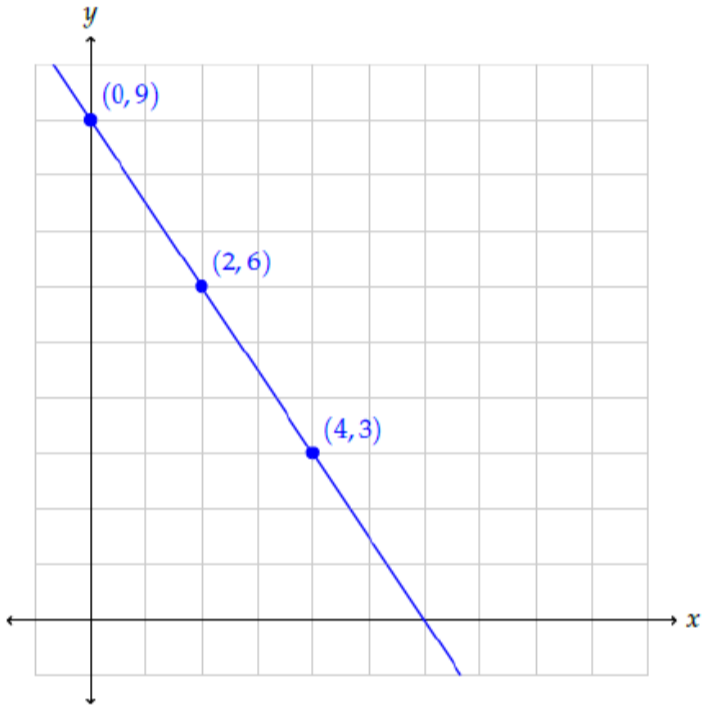$P_2 (4, 9)$
$\quad x_2, y_2$

$$a = \frac{(9 - 6)}{(4 - 2)}$$

$$a = \frac{\text{rise}}{\text{run}}$$

$$a = \frac{3}{2} \quad \leftarrow \text{better}$$

$$a = 1.5$$

1.1.1 Practice

Determine the slope of the following linear function. First find it graphically, then use the formula for the slope we just discovered!
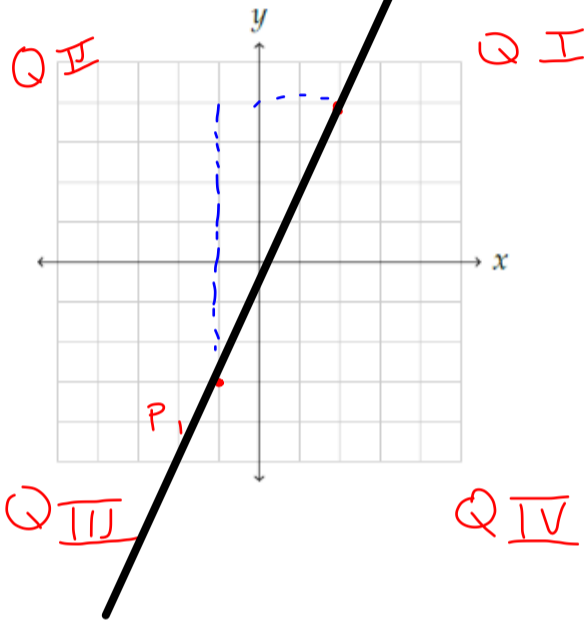

You do

$a = \dfrac{3}{2}$ ✗

$a = \dfrac{3}{-2}$ ✓

$a = \dfrac{-3}{2}$

$a = -\dfrac{3}{2}$

nota bene: if f is decreasing, then a is $(-)$

1.2 Example

Determine the slope of the line passing through the following two points: $(-1, -3)$ and $(2, 4)$

Use graph to verify:

$P_1 \quad x_1, y_1 \qquad x_2 \quad y_2$

label

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$a = \frac{4 - (-3)}{2 - (-1)}$$

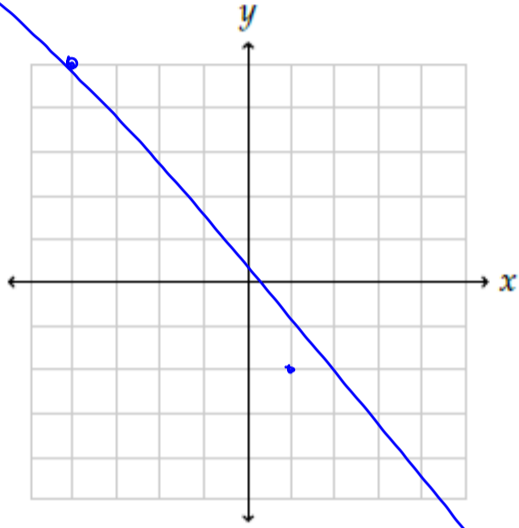$$a = \frac{4 + 3}{2 + 1}$$

$$a = \frac{7}{3}$$

— verify by plotting points.

You do

Practice
1.2.1   a) and
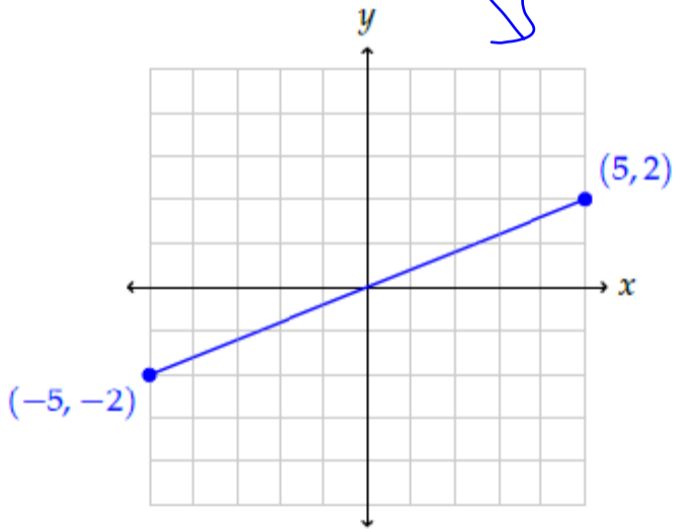   b)

Practice plotting points.

1.2.1 Practice

(a) Determine the slope of the line passing through the following two points: $(-4, 5)$ and $(1, -2)$

$$a = \frac{-7}{5}$$

(b) Determine the slope of the following linear function:

$(5, 2)$

$(-5, -2)$

$$a = \frac{-4}{-10}$$

$$a = \frac{2}{5}$$

if f increasing,
then a is $(+)$

1.3   Example: Finding the Slope of a Line Through Two Points Presented in a Table

Determine the slope of the line passing through the following points:

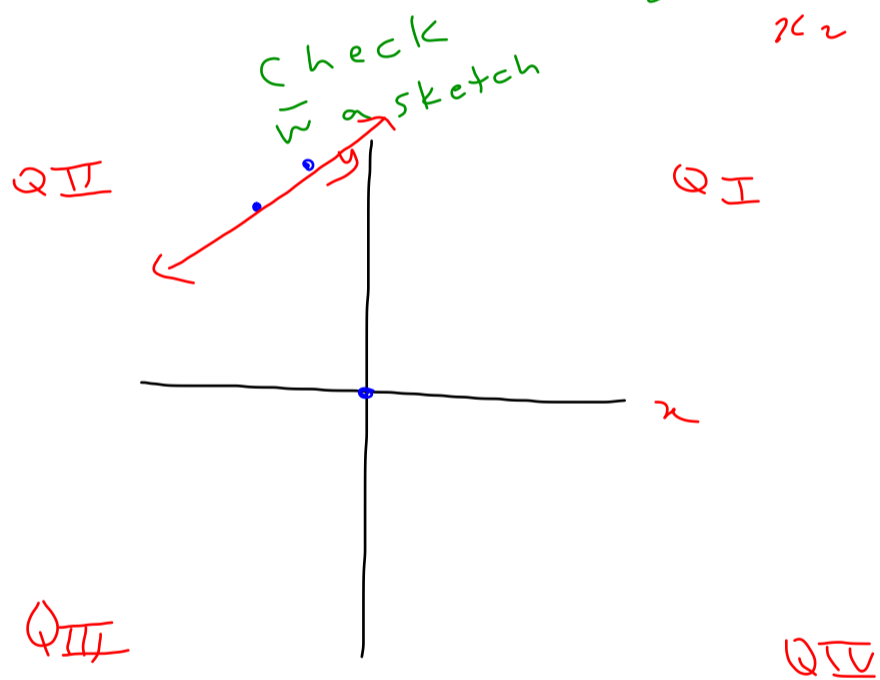| x	f(x)
-3	12
-2	17

$$P_1 (-3, 12)$$
$$\quad x_1 \quad y_1$$

$$P_2 (-2, 17)$$
$$\quad x_2 \quad y_2$$

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$
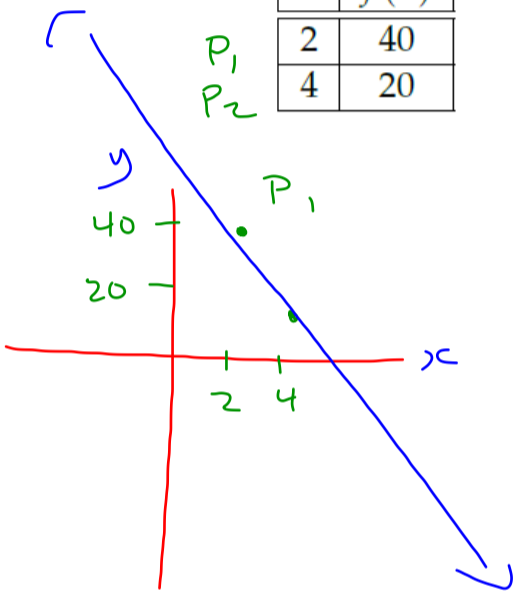
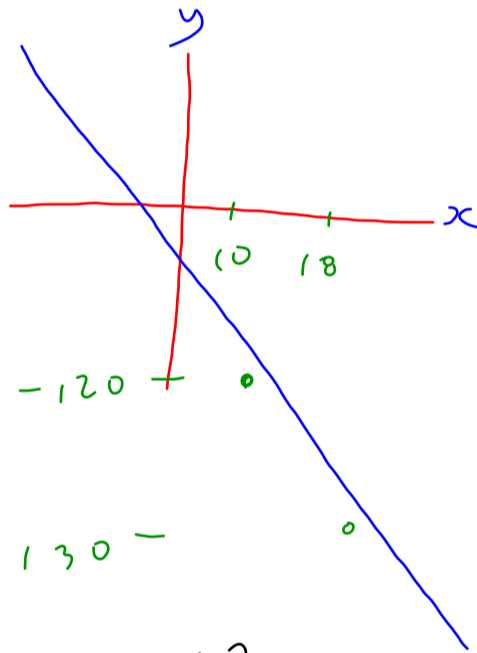$$a = \frac{17 - 12}{-2 - (-3)}$$

$$a = \frac{+5}{1}$$

$$a = +5$$

Check
w/ a sketch

QII

QI

QIII

QIV

x

1.3.1 Practice

Determine the slope of the line passing through the following points: *and verify w̄ a sketch.*

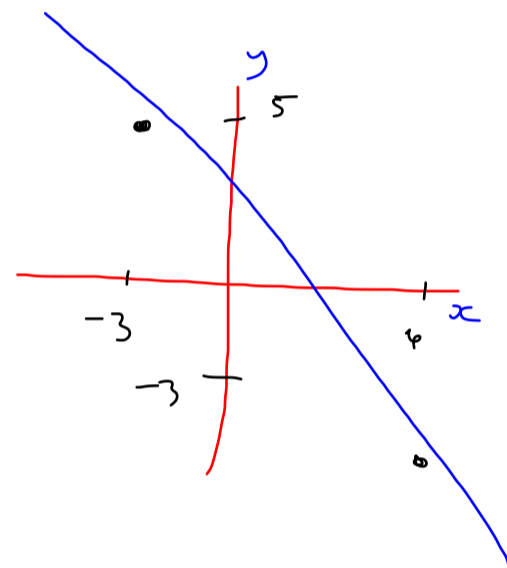| x	$f(x)$
2	40
4	20

P_1
P_2

x	y
10	-120
18	-130

x	$f(x)$
-3	5
6	-3

$a = -10$

$a = -\dfrac{10}{8}$

$a = -\dfrac{5}{4}$

$a = -\dfrac{8}{9}$

1.4 Example: Finding the Slope of a Line – Horizontal Table

Determine the slope of the line passing through the following points:

x	-5	-3
$f(x)$	-2	-5

P_1 P_2

$$P_1 \ (-5, -2)$$
$$x_1 \quad y_1$$

$$P_2 \ (-3, -5)$$
$$x_2 \quad y_2$$

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$a = \frac{-5 - (-2)}{-3 - (-5)}$$

$$a = \frac{-5 + 2}{-3 + 5}$$

$$a = \frac{-3}{2}$$

You do

1.4.1

and

Exit Ticket

HMWK:

pg 152, #3.24

pg 129, #3.7

pg 130, #3.8

1.4.1 Practice

Determine the slope of the line passing through the following points:

x	-2	3
$f(x)$	-1	-3