Lesson 9 b): Finding the Rule of a Linear Function → Equation May 13, 2024

→ $y = ax + b$

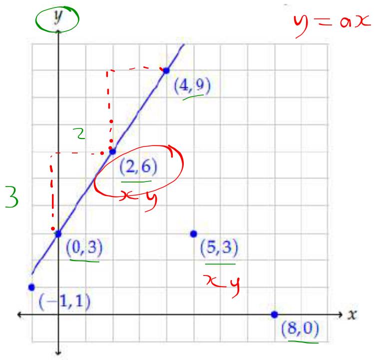
· slope $a = \frac{rise}{run}$ (steepness)

y-intercept (0, b) (initial value)

### 2.1 Discovering the Steps to Finding the Rule of any Linear Function

First, let's consider the linear function $y = \frac{3}{2}x + 3$ (or $f(x) = \frac{3}{2}x + 3$)

$y = ax + b$ y-int (0, b) (0, 3)




Notice the points (0,3), (2,6), (4,9) are **on** (or part of) the line. The points (−1,1), (5,3), (8,0) are **not on** (part of) the line

(a) Plug (sub) in each point on the line into the algebraic rule (equation) of the line. What do you notice?

(b) Plug in each point that is *not* on the line into the equation. What do you notice?

a) (0, 3) x y "sub in" "plug in"

$y = \frac{3}{2}x + 3$

$3 = \frac{3}{2} \cdot (0) + 3$

$3 = 0 + 3$

$3 = 3$ ✓ True

Why?

b) (5, 3) x y sub in

$y = \frac{3}{2}x + 3$

$3 = \frac{3}{2}(5) + 3$

$3 = 10.5$ ✗ False

(2, 6) $y = \frac{3}{2}x + 3$

$6 = 6$ ✓ True

(8, 0) x y $y = \frac{3}{2}x + 3$

$0 = 15$ False

∴ is not on line

∴ (8, 0) is NOT a solution

$x + 1 = 10$

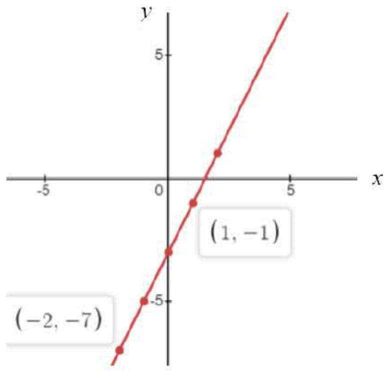
$7 + 1 = 10$

$8 = 10$ False

7 is not a solution

# How to Find the Rule of a Linear Function

1.1 Example: Find the rule of the following linear function:




– memory aid –

$$\boxed{y = ax + b}$$

WANT: a (1 unk.)

TOOL: 1 eq.

step i Find a w equation + 2 points.

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$P_1$ $(-2, -7)$ ($x_1$, $y_1$)

$P_2$ $(1, -1)$ ($x_2$, $y_2$)

$$a = \frac{-1 - (-7)}{1 - (-2)}$$

$$a = \frac{-1 + 7}{1 + 3}$$

$$a = \frac{6}{4} \quad \text{or} \quad a = \frac{3}{2}$$

step ii : sub value of a into:

$$y = ax + b$$

$$y = \frac{3}{2}x + b$$

step iii . Find value of b by subbing in a point $(x, y)$ into

$$y = \frac{3}{2}x + b \qquad (1, -1)$$

x y

$$-1 = \frac{3}{2}(1) + b$$

. sub
. simplify
. solve
w o.o.

$$-1 = \frac{3}{2} + b$$

$$-\frac{3}{2} \qquad -\frac{3}{2}$$

$$b = -\frac{5}{2} \quad \text{or} \quad b = -2.5$$

step iv state the rule by subbing the value of a and b and check w graph

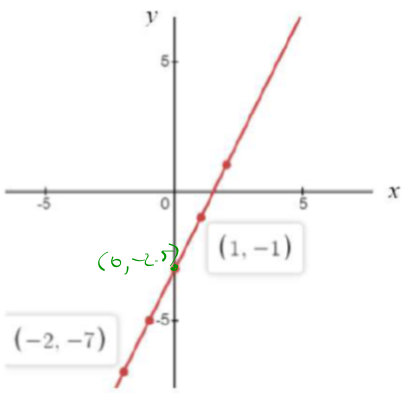
$a = 3/2$ $b = -2.5$

$$y = ax + b$$

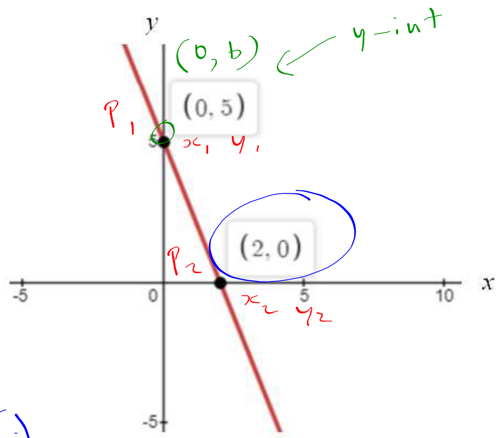
$$y = \frac{3}{2}x - 2.5$$

slope

y-int



2.1 Example: Find the rule of the following linear function:




i) $a = \dfrac{y_2 - y_1}{x_2 - x_1}$

$a = \dfrac{0 - 5}{2 - 0}$

$a = \dfrac{-5}{2}$

$a = -2.5$ ✓ since f is decreasing

ii) $y = ax + b$

$y = -2.5x + b$

iii) find b:

$y = -2.5x + b$

sub (2, 0)
x y

- sub
- simplify
- solve

$0 = -2.5(2) + b$

$0 = -\cancel{5} + b$
$+5 \quad \cancel{+5}$

$b = 5$

You do pg 3
Practice 2.2

iv) $a = -2.5 \quad b = 5$

$y = ax + b$

$$\boxed{y = -2.5x + 5}$$

or

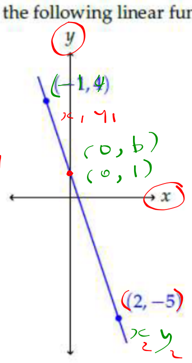
$$\boxed{f(x) = -2.5x + 5}$$

2.2 Practice:

Find the rule of the following linear functions:

$y=-3x+1$

$y=ax+b$




$y=ax+b$

$a=\frac{y_2-y_1}{x_2-x_1}$

be careful

use only points

$y=ax+b$

$y=-3x+b$

$-5=-3(2)+b$

$-5=-6+b$

$+6 \quad +6$

$\boxed{b=1}$

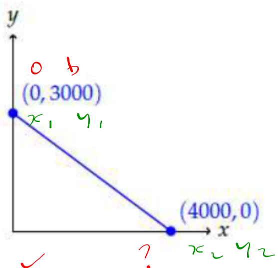
$a=\frac{-5-4}{2-(-1)}$

$a=\frac{-9}{3}$

$\boxed{a=-3}$

$y=ax+b$

$y=-3x+1$




$y=ax+b$

$a=\frac{y_2-y_1}{x_2-x_1}$

$a=\frac{0-3000}{4000-0}$

$\boxed{a=-\frac{3}{4}}$ and $\boxed{b=3000}$

$y=ax+b$

$y=\frac{-3}{4}x+3000$

Find the Rule from a Table of Values

3.1 Example: Verify the following functions are linear and then find their rule.

a)

| | $x$ | $f(x)$ |
|---|---|---|
| | -3 | 5 $y_1$ |
| $P_1$ | 6 | -3 $y_2$ $y_1$ |
| $P_2$ | 15 | -11 $y_2$ |

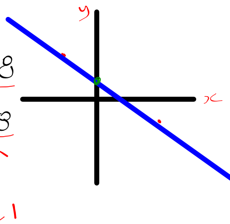
$\Delta y = y_2 - y_1$

$-3 - 5 = \underline{-8}$

$-11 - (-3) = \underline{-8}$

constant!

$\therefore f$ is linear!




a table of points $(x, y)$ ✓✓

(NOT slope!)

step i: verify the $f$ is linear (w̄ a sketch of points or verify $\Delta y$ is constant)

$y = ax + b$

step ii → same as before

$a = \dfrac{y_2 - y_1}{x_2 - x_1}$ $P_1\ (\underset{x_1}{-3}, \underset{y_1}{5})$ $P_2\ (\underset{x_2}{6}, \underset{y_2}{-3})$

$a = \dfrac{-3-5}{6-(-3)}$

$a = \dfrac{-8}{9}$

$y = ax + b$ $(\underset{x}{-3}, \underset{y}{5})$

$y = -\dfrac{8}{9}x + b$

$5 = \dfrac{-8}{9}\cdot(-3) + b$

$5 = \dfrac{8}{3} + b$

$-\dfrac{8}{3}$ $-\cancel{\dfrac{8}{3}}$

$b = \dfrac{7}{3}$

$y = \dfrac{-8}{9}x + \dfrac{7}{3}$

b)

| $x$ | -6 | -4 | -2 |
|---|---|---|---|
| $f(x)$ | -10 | -5 | 0 |

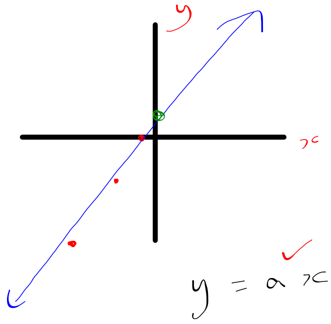
| $x$ | $y$ |
|---|---|
| $-6$ | $-10$ $y_1$ |
| $-4$ | $-5$ $y_2$ $y_1$ |
| $-2$ | $0$ $y_2$ |

$\Delta y = y_2 - y_1$

$-5 - (-10) = 5$

$0 - (-5) = 5$

constant! ∴ linear




$y = ax + b$ ?  ✓

$a = \dfrac{y_2 - y_1}{x_2 - x_1}$

$(-6, -10)$
$x_1 \quad y_1$

$(-4, -5)$
$x_2 \quad y_2$

$a = \dfrac{-5 - (-10)}{-4 - (-6)}$

$a = \dfrac{5}{2}$

$y = ax + b$

$y = \dfrac{5}{2}x + b$

$(-4, -5)$
$x \quad y$

$-5 = \dfrac{5}{\cancel{2}}(\cancel{-4}) + b$

$-5 = -10 + b$
$+10 \qquad +10$

$b = 5$

$y = ax + b$

$y = \dfrac{5}{2}x + 5$

You do:
- pg 5 #3, 2 a) and b)
- start pg 6.

3.2 Practice: Verify the following functions are linear and then find their rule:

a)

| $x$ | $f(x)$ |
|---|---|
| -3 | -4 |
| 0 | -2 |
| 3 | 0 |

$y = \frac{2}{3}x - 2$

b)

| $x$ | 2 | 4 | 6 |
|---|---|---|---|
| $f(x)$ | 40 | 20 | 0 |

$a = \frac{-20}{2}$

$\boxed{a = -10}$

$y = ax + b$

$(0, 6)$ ✗

$(6, 0)$ ✓

x y

$y = -10x + b$

$0 = -10(6) + b$

$0 = -60 + b$

$+60 \qquad +60$

$\boxed{b = 60}$

$\boxed{y = -10x + 60}$

or

$\boxed{f(x) = -10x + 60}$

# Finding the Rule Using the Properties

slope = a

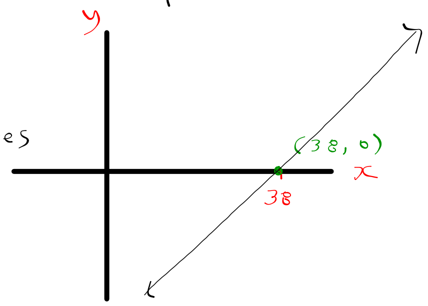
not a coordinate in a point (x, y)

4.1 Example: Find the rule of linear function $f(x)$ given that the rate of change is $\boxed{2}$ and that the $x$-intercept is 38.

$f(x) = ax + b$

Step i: Do a sketch and plot point.

a point on x-axis w/ coordinates $(x, 0)$




$y = ax + b$

$a = 2$

$y = 2x + b$

sub in a point

$(38, 0)$
$x \quad y$

$0 = 2(38) + b$

$0 = 76 + b$
$-76 \quad -76$

$b = -76$

$$\boxed{y = 2x - 76}$$

4.1.2 Example: Find the rule of linear function $g(x)$ given that $g(-1) = 4$ and that the negative interval of $g(x)$ *is* $[1, \infty[$

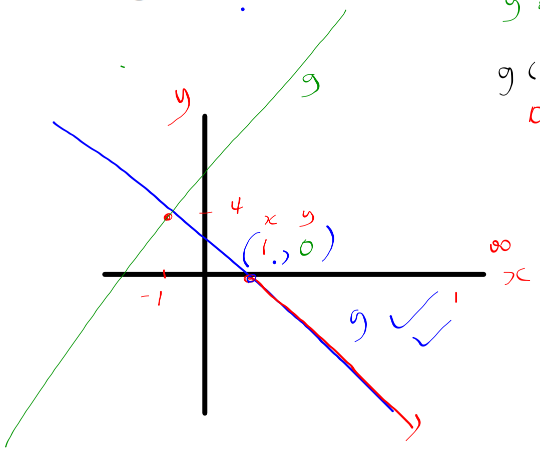
g of x

$g(x) = y \rightarrow \therefore (-1, 4)\ P_1$

$x_1 \quad y_1$

$(1, 0)\ P_2$

$x_2 \quad y_2$




$g(x) = ax + b$

$0 = a(1) + b$

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$a = \frac{0 - 4}{1 - (-1)}$$

$$a = \frac{-4}{2}$$

$$\boxed{a = -2}$$

$y = ax + b$

$y = -2x + b$

$x \quad y$

$(1, 0)$

$0 = -2(1) + b$

$0 = -2 + b$

$+2 \qquad +2$

$\boxed{b = 2}$

$y = ax + b$

$\boxed{g(x) = -2x + 2}$

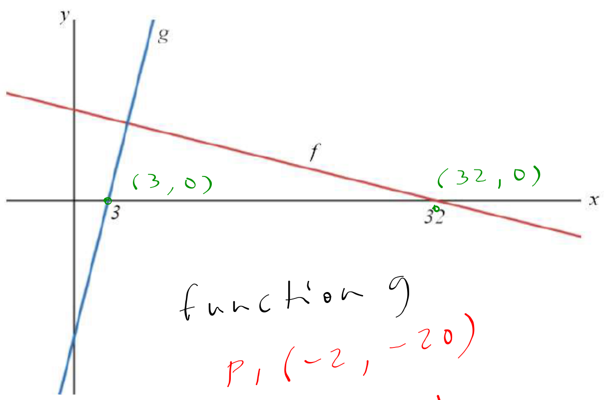
you do
pg 7

4.2 Practice: Linear functions $f$ and $g$ are represented in the Cartesian plane below, where the rate of change of $f$ is $-\frac{1}{4}$ and the $x$-intercept of $f$ is 32. Find the two different rules of $f$ and $g$ given that $g(-2) = -20$ and that the positive interval of $g$ is $[3, \infty[$

↳ equation

take notes:

$g(x) = y \to \therefore (-2, -20)$




function $g$

$P_1(-2, -20)$

$P_2(3, 0)$

$g(x) = 4x + (-12)$

$g(x) = 4x - 12$ ✓

function $f$ : $x$ $y$

$a = -\frac{1}{4}$

$P(32, 0)$

$y = ax + b$ $\quad a = -\frac{1}{4}$

$y = -\frac{1}{4}x + b$

$0 = -\frac{1}{4}(32) + b$

$0 = -8 + b$

$+8 \quad +8$

$b = 8$

$\therefore y = ax + b$

$\boxed{f(x) = -\frac{1}{4}x + 8}$

Find the Rule from a Word Question

$y = ax + b$

### 3 Linear Functions and Short Word Problems

Tips:

Example: Jimmy is travelling from *Lala Land* to *Imaginary Place* at a constant speed. When he began, he was 601km from *Imaginary Place*. After 3 hours, he was 370km from his destination. When will he be 300km from his destination?

$x_1 = 0\,hr.$ $= y_1$ $x_2$ $\parallel$ $y_2$

(i) Understand and define variables

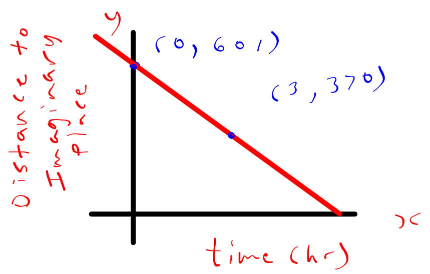
WANT: $x$ when $y = 300\,km$

$x$ = time (hrs)

$y$ = distance (km) to imaginary place

(ii). Determine if #'s are coordinates or slope (what is change) $a = \frac{\Delta y}{\Delta x}$ $\frac{km}{hr}$ look @ units

(iii) Draw a sketch of graph




$x_1$, $y_1$: $(0, 601)$

$x_2$, $y_2$: $(3, 370)$

$y = ax + b$

$a = \frac{y_2 - y_1}{x_2 - x_1}$

$a = \frac{370 - 601}{3 - 0}$

$\boxed{a = -77}$ km/hr

$b$ = y-intercept

$(0, b)$

$(0, 601)$

$\boxed{b = 601}$

WANT: $x$ when $y = 300\,km$

TOOL: 1 eq → $\boxed{y = -77x + 601}$

$y = -77x + 601$

sub $y = 300$

$300 = -77x + 601$

$-601 \qquad -601$

· simplify? · solve! w o.o.

B E D M A S

$\frac{-301}{-77} = \frac{-77x}{-77}$

$x = 3.9\,hr$

$\therefore$ it will take Jimmy $3.9\,hr$ to be 300 km from his destination.

3hr + 54 min

$60 \times \left(\frac{0.9\,hr}{1\,hr}\right) = \left(\frac{x \text{ mins}}{60 \text{ min}}\right) \times 60$

$x = 54\,min$

60min = 1hr

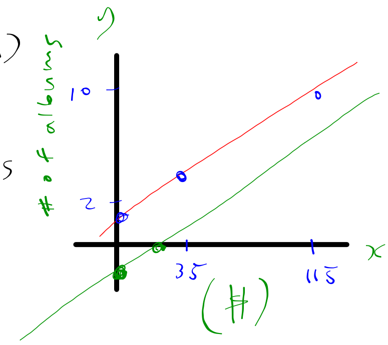
### 3.1 Example

Costco

In long forgotten ancient times, a company named *Columbia House* would provide music albums at a reduced price, and required members to pay a monthly membership fee. Assuming the price per album was the same, we need to figure out how many albums $= y$ we would be able to purchase with $265 ($x$) in a given month. We know that Arelius, an avid music fan, paid $115 in a month for 10 music albums. Jeremiah bought 2 albums that same month and spent $35. — the want

$x$ = money spent ($)

$y$ = # of albums bought




You do 3.1 and 3.3

(115, 10)

(35, 2)

$$a = \frac{y_2 - y_1}{x_2 - x_1} \qquad (\underset{x_1}{115}, \underset{y_1}{10}) \quad (\underset{x_2}{35}, \underset{y_2}{2})$$

$$a = \frac{2 - 10}{35 - 115}$$

$$a = 0.1 \ \frac{\text{ablums}}{(\$)}$$

$y = ax + b$ $\qquad (\underset{x}{115}, \underset{y}{10})$

$y = 0.1x + b$

$10 = 0.1(115) + b$

$10 = 11.5 + b$

$-11.5 \qquad -\cancel{11}.5$

$b = -1.5$

$\therefore$ $\boxed{y = 0.1x - 1.5}$

WANT: $y$ when $x = 265\$$

TOOL:

$y = 0.1(265) - 1.5$

$y = 25$ ablums

### 3.3 Example

Slim Jim, a used car salesman, is paid a certain fixed rate for every car he sells in addition to a fixed amount of \$2700 regardless of how many cars he sells. When Jim sold 20 cars, he earned a total income of \$6700. If Jim sells just 12 cars, what will be his total income?

ans: $\begin{cases} y = 200x + 2700 \\ y = 5100 \$ \end{cases}$

Word Questions where slope is Given

rate of change of $f(x)$ $a = \frac{\Delta y}{\Delta x}$ } units

### 3.4 Example

a) Roberto is a plumber who is saving money for the purchase of a high-tech tool. The cost of this tool is \$2500. He charges his customers \$45 per hour plus an additional \$15 for travel expenses. How many hours will it take Roberto to save for this tool?

not slope   slope   not slope

"$y_1$  $x_1 = ?$  $x_1 = 0$

e.x.
\$ per hr
km per hr
\$ a day

$y$   $x$

step i: state slope's units: $\frac{\$}{hr}$ $\leftarrow \Delta y$ / $\leftarrow \Delta x$ $= a$

step ii: Define $x$ and $y$ variable

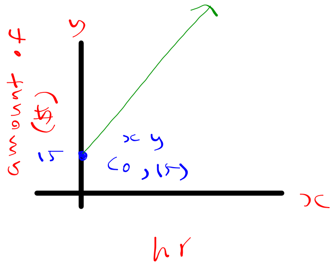
$x$ = hr of work

$y$ = \$ total amount of (\$)

$a = \frac{y_2 - y_1}{x_2 - x_1}$

WANT: $x$ when $y = 2500$

step iii sketch




$$\boxed{y = 45x + 15}$$

HMWK:

pg 139 #3.13-3.14
pg 138 #3.11
pg 154 #3.27
pg 155 #3.29-3.30
pg 140 #3.15-3.16

$$2500 = 45x + \cancel{15}$$
$$-15 \qquad\qquad -\cancel{15}$$

$$\frac{2485}{45} = \frac{\cancel{45} \cdot x}{\cancel{45}}$$

$$x = 55.2 \text{ hr}$$

$\therefore$ it will take Bob 55.2 hr to save up for the tool.

You do: pg 12 part a)

b) If Roberto increases his hourly rate by \$10, determine the rule of the function that models this new salary situation. Draw and label a sketch of his old and new salary situation.

## 3.5 Example

a) Kashana is saving money for a one-week educational trip to Eastern Europe. The cost of the trip is \$1200. She already has \$300 saved and plans to add \$50 per month to her saving. How long will it take Kashana to save for the trip?

b) If Kashana increases her monthly saving by \$25, determine the rule of the function that models this new saving situation.