Lesson 7 Determining if a Relation is a Function

May 8th, 2024

Graphing Linear Functions

e.x.

$y = 2x + 1$ ← an equation w/ 2 unknowns

← not solvable cuz there's an infinite amount of solutions

x	y
0	1
1	3
2	5
3	7
4	9
5	11

solution?

$(0, 1)$

x " , y "

$y = 2x + 1$

$1 = 2(0) + 1$

$1 = 1$ ✓

solution

$(5, 11)$

x " , y "

$y = 2x + 1$

$11 = 2(5) + 1$

$11 = 11$ ✓

→ there's a pattern!

→ there's a $\boxed{\text{relation}}$ between x + y

↳ $y = 2x + 1$

Squares and Rectangles

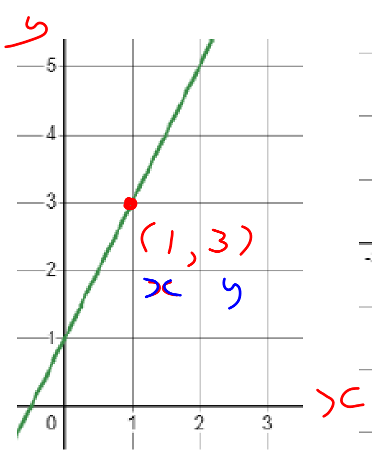
Functions and Relations

A relation is a "link" between two variables (usually x and y). Functions are special ~~special~~ kinds of relations. Indeed, functions are sometimes called: functional relations

Q: What is so special about functions?

For all the independant values → (the x's → the input) there exists one and only one dependant value → (the y's → the output)

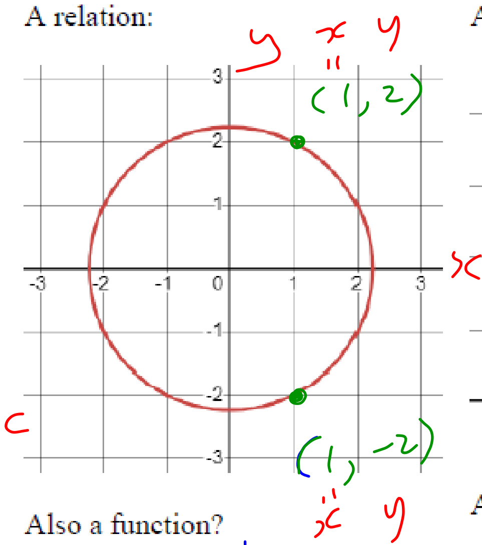
A relation:



Also a function?

YES!

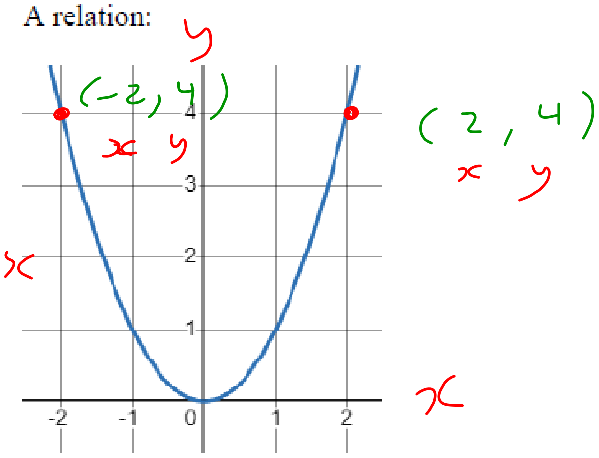
A relation:



Also a function?

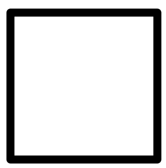
NO!

A relation:



Also a function?

YES!



are all squares rectangles?

YES!

i Are all functions relations?

no, no, yeah, yes, yes

ii Are all relations functions?

no, no, no, yes, no

NO

are all rectangles squares?

2.2 Determining whether a Relation is a Function from a Table of Values

Recall the definition of a function:

Functions and Relations

A *function* is a relation between an independent variable (input - x) and a dependent variable (output - y) such that:

- for any given input value (x) there is **one and only one** output value (y)

If a relation is *not* a function, then we simply call it is a *relation*. If a relation is a functional relation, then we simply call it a *function*.

Now consider the following relations. Determine whether they are functions or not.

x	y
0	10
1	9
2	8
3	7
4	6
5	5

a function ✓

x	y
0	10
4	9
5	8
3	7
4	6
6	5

NOT ~~a~~ function!

x	y
0	10
2	10
4	8
3	7
5	10
7	5

a function ✓

repeats of x → not okay

repeats of y → okay ✓

2.3 Determining whether a Relation is a Function from a Diagram

Recall the definition of a function:

Functions and Relations

A *function* is a relation between an independent variable (input - x) and a dependent variable (output - y) such that:

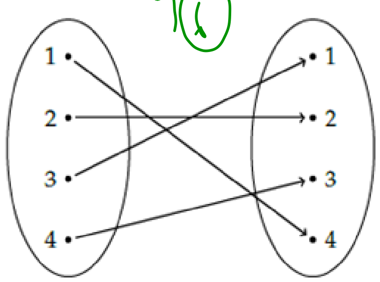
- for any given input value (x) there is **one and only one** output value (y)

If a relation is *not* a function, then we simply call it a *relation*. If a relation is a functional relation, then we simply call it a *function*.

Consider the following relations and determine which are functions and which are not.

yes f

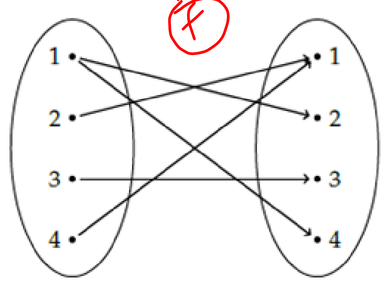
x y



x	y
1	4
2	2
3	1
4	3

Not a f

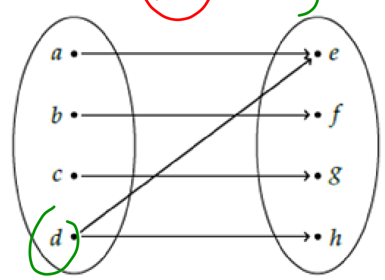
x y



x	y
1	2
1	4

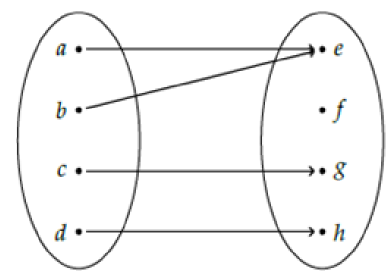
Not a f

x y



x	y
a	e
d	e
d	h

Yes a



x	y
a	e
b	e

You do pg 2 and 3 Practice 2.2.1 and 2.3.1

pg 2

2.2.1 Practice

Consider the following relations. Determine whether they are functions or not.

i

x	y
0	-3
10	-3
20	-3
30	-3
40	-3
50	-3

yes

ii

x	y
-1	10
2	9
5	8
8	7
11	6
14	5

yes

iii

x	y
a	10
b	10
c	8
a	7
e	10
f	5

no

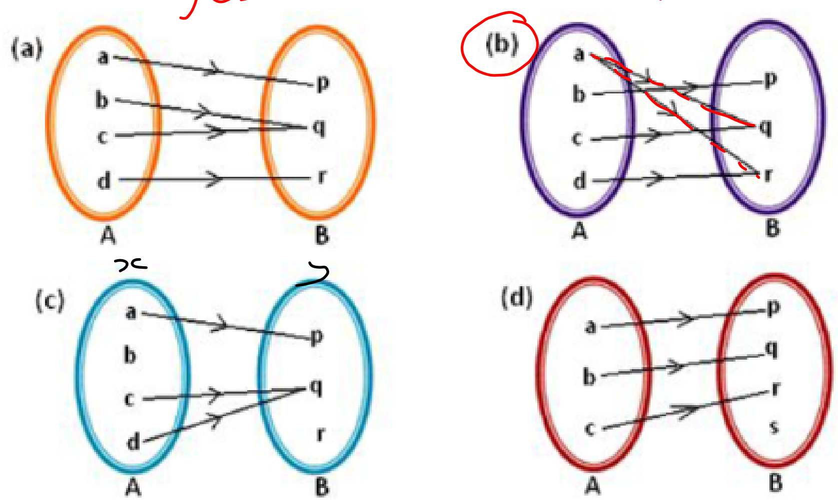
2.3.1 Practice

Consider the following relations. Determine which are functions and which are not:

yes no

(a, q)
(a, r)

pg 3



yes yes!

x	y
a	p
c	q
d	q

2.4 Determining whether a Relation is a Function from a Set of Points (Ordered Pairs)

(x, y)

Recall the definition of a function:

> **Functions and Relations**
>
> A *function* is a relation between an independent variable (input - x) and a dependent variable (output - y) such that:
>
> - for any given input value (x) there is **one and only one** output value (y)
>
> If a relation is *not* a function, then we simply call it is a *relation*. If a relation is a functional relation, then we simply call it a *function*.

Consider the following relations. Determine which are functions and which are not:

1. $\{(1,1),(2,3),(4,5),(6,7)\}$ → yes!
   x y x y x y x y
2. $\{(1,1),(2,1),(3,1),(4,1)\}$ → yes!

x	y
1	1
2	1
3	1
4	1

3. $\{(1,1),(1,3),(4,7),(5,8)\}$
   x y x y
   → no!

$[\ ,\ ]$ square brackets for intervals

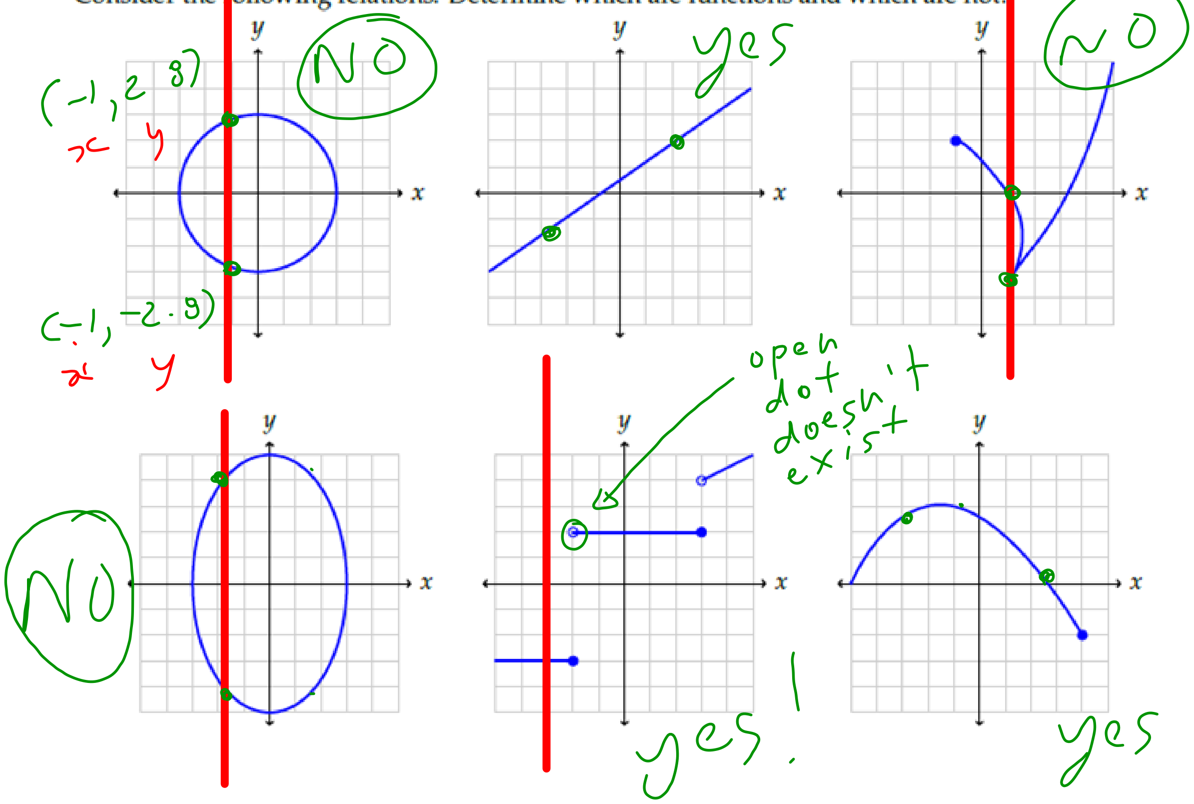
$\{\ ,\ \}$ curly for sets

$(\ ,\ )$ → for points.

2.5 Determining whether a Relation is a Function from a Graph

→ to check: ?
vertical line must pass only once thru the relation

Consider the following relations. Determine which are functions and which are not.



You do pg 4 #2.4.1
AND from textbook pg 54 "In a graph" part

pg 4

2.4.1 Practice

Consider the following relations. Determine which are functions and which are not:

1. $\{(-1,1),(2,-3),(-4,5),(6,-7)\}$ yes
2. $\{(0,0),(2,1),(3,0),(4,1)\}$ Yes
3. $\{(-5,1),(-6,3),(-5,7),(5,8)\}$ No

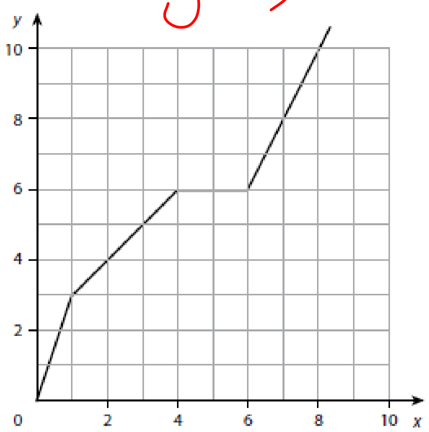
x y x y

In a graph, it is relatively easy to see whether you are dealing with a function or a relation.

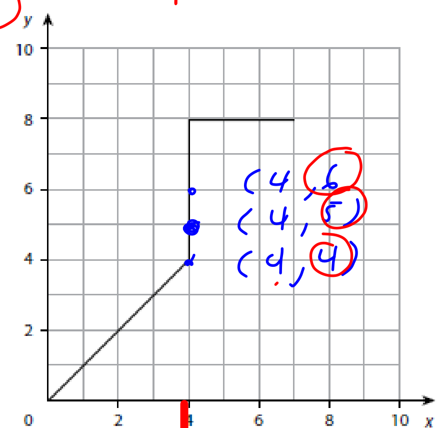
Which of the following graphs do you think illustrate a function? ________________

Graphs 2.1

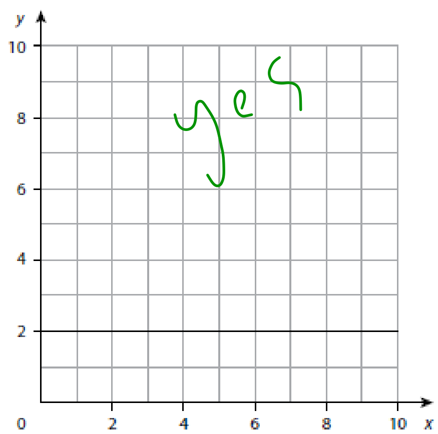
1) yes



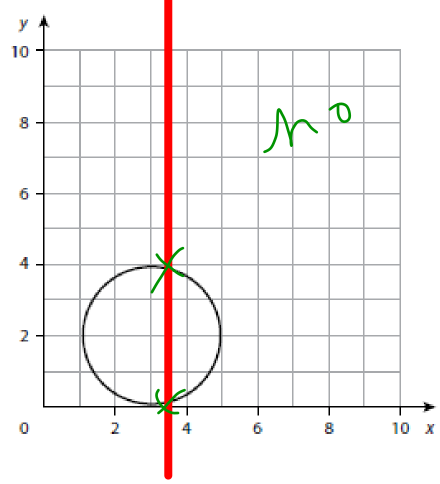
2) No



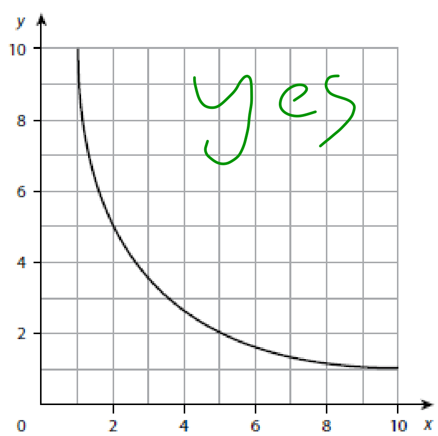
3)



4)



5)

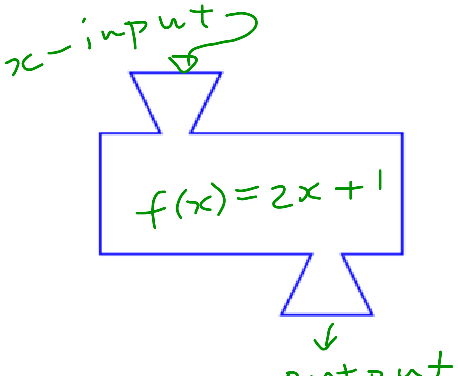


pg 5: Graphing linear Functions

3 Function Notation

Let's now add a new piece of vocabulary in our mathematical language: *function notation*. We will see why this is actually quite useful and makes certain things easier.

$y = 2x + 1$



y depends on x

$f(x) = 2x + 1$

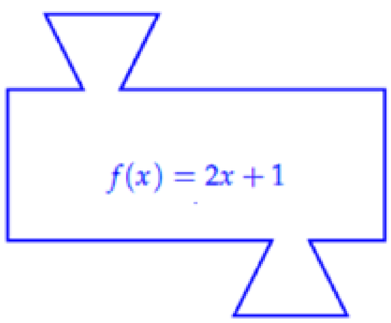
y is a function of x

this reads, "f of x"

y
f(x)

3.1 Example

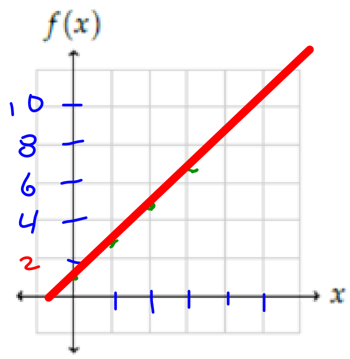
Complete the associated table of values for the following function and then plot the points on a Cartesian Graph and complete the graph of the function:



TOV

x	$f(x)$
0	1
1	3
2	5
3	7
4	9
5	11

+2
+2
+2



plot points
coordinates
. ordered pair
(x, y)

draw line or curve through points

find $f(0) \rightarrow$ find y when $x = 0$

$f(x) = 2x + 1$

$f(0) = 2(0) + 1$

$f(0) = 1 \rightarrow \therefore (0, 1)$

x y

$y = 2x + 1$

$y = 2(0) + 1$

$y = 1$

You do pg 6 #3 2 a) + b)

and from textbook

pg 37 #1.23

pg 38-39 #1.24-1.25

find $f(1) \rightarrow$ find y when $x = 1$

$f(x) = 2x + 1$

$f(1) = 2(1) + 1$

$f(1) = 3$ (1, 3)
(x, y)

find $f(2)$

$f(x) = 2x + 1$

$f(2) = 2(2) + 1$ (2, 5)

$f(2) = 5$

3.2 Practice

(a) Complete the associated table of values for the following function and then plot the points on a Cartesian Graph and complete the graph of the function:

slope y-int

$f(x) = mx + b$

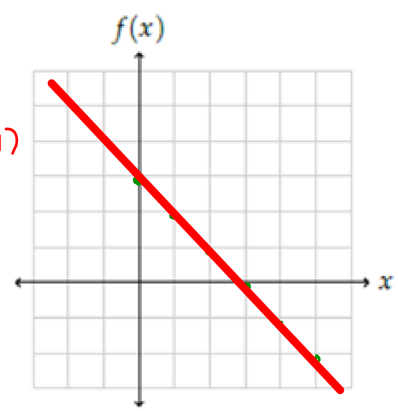
$f(x) = -x + 3$

m

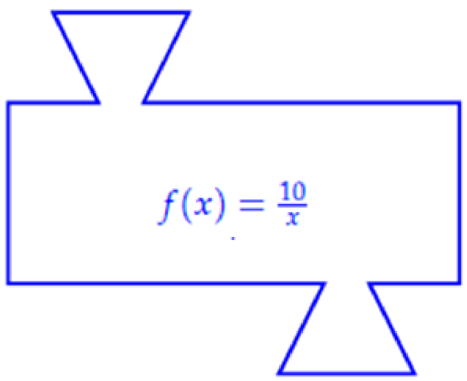
b

x	$f(x)$
0	3
1	2
2	1
3	0
4	−1
5	−2

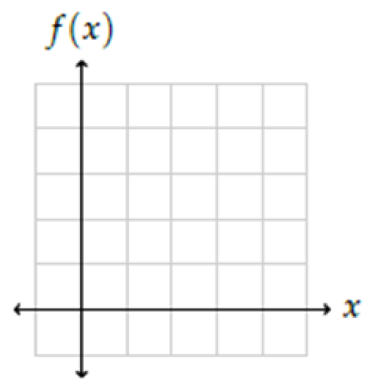
+(−1)
−1
−1
−1



(b) Complete the associated table of values for the following function and then plot the points on a Cartesian Graph and complete the graph of the function:



x	$f(x)$
1	
2	
3	
4	
5	



Evaluating and Solving

3.3 Example: Determining Values in Function Notation

Now let's discover some reasons why function notation is algebraically useful:

Consider the following function:

$$y = 3x - 2$$

1. What is the value of y when $x = 2$?
2. What is the value of x when $y = 7$?

Consider the following function:

$$f(x) = 3x - 2$$

$f(x)$

1. What is $f(2)$?
2. What is the value of x if $f(x) = 7$

"f of x"

#1

want: y

$=$

1 unk

tool: 1eq

$f(x) = 3x - 2$

info $x = 2$

#1.

$f(x) = 3x - 2$

$f(x) = 3(2) - 2$

$f(x) = 4$

#2

want: x

tool: 1eq

$f(x) = 3x - 2$

info:

$y = 7$

$f(x) = 7$

#2 $f(x) = 3x - 2$

$7 = 3x - \cancel{2}$
$+2 \qquad +2$

$\frac{9}{3} = \frac{\cancel{3}x}{\cancel{3}}$

$x = 3$

You do

#3.4 and #3.5 pg 7 and 8

3.4 Example: Determining Values in Function Notation

Now let's discover some reasons why function notation is algebraically useful:

Consider the following function:

$$y = x^2 - 2x + 1$$

same

1. What is the value of y when $x = 2$?
2. What is the value of y when $x = -3$?

same

Consider the following function:

$$f(x) = x^2 - 2x + 1$$

1. What is $f(2)$?
2. What is $f(-3)$?

y depends on x

y is a function of x

$f(x) = x^2 - 2x + 1$

$f(2) = 2^2 - 2(2) + 1$

$f(2) = 1$

$f(-3) = (-3)^2 - 2(-3) + 1$

$f(-3) = 16$

$(-3)^2 = 9$

$-3^2 =$

$-1 \cdot 9 = -9$

3.5 Practice

Given the following function:

$$f(x) = \frac{100}{x}$$

i (a) What is $f(25)$?

ii (b) What is $f(-2)$?

iii (c) What is the value of x when $f(x) = 1000$

iv (d) What is the value of x when $f(x) = -2$?

#1

$$f(x) = \frac{100}{x}$$

$$f(25) = \frac{100}{25}$$

$$f(25) = 4$$

$(25, 4)$

x, y

#

$$f(x) = \frac{100}{x}$$

$$f(-2) = \frac{100}{(-2)}$$

$$f(-2) = -50$$

$(-2, -50)$

#3 sub $f(x) = 1000$

$$f(x) = \frac{100}{x}$$

solve!

memory aid

$$\frac{1000}{1} = \frac{100}{x}$$

$$\frac{1000x}{1000} = \frac{100}{1000}$$

$$x = 0.1$$

#4 $x = ?$

sub $f(x) = -2$

$$f(x) = \frac{100}{x}$$

$$\frac{-2}{1} = \frac{100}{x}$$

$$\frac{-2x}{-2} = \frac{100}{-2}$$

Reading Graphs!

1 Graphing Linear Relations to Model Real Life Situations

Let's try to understand a linear function by seeing how it can *model* a real-life situation.

v. important for marks on exam!

1.1 Example

Create a model for the following situation:

Step i: Define your variable (x, y) w/ words
↳ sthg that varies
sthg that changes
label x/y axis

> Alex opens up a new savings account with nothing saved initially. Alex sets up a savings plan and begins saving \$110 per month. (n.b. Because of low interest rates, the bank does not offer Alex any interest on his deposits).

Step ii: fill out chart by picking values of x!

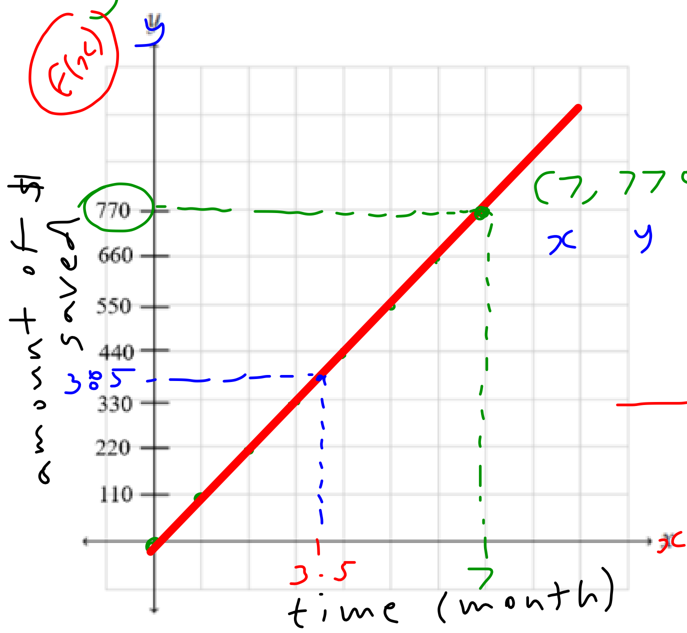
Dependent Variable: (y)
\$ saved

Independent Variable: (x)
time (month)

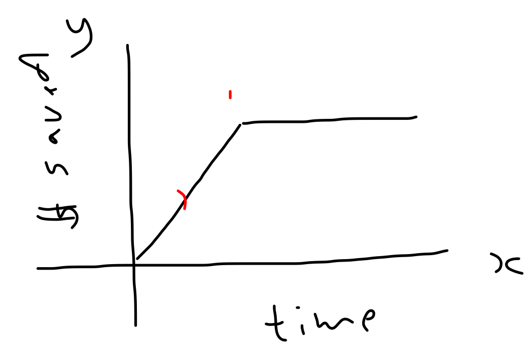
x	$f(x)$
0	0
1	110
2	220
3	330
4	440
5	550

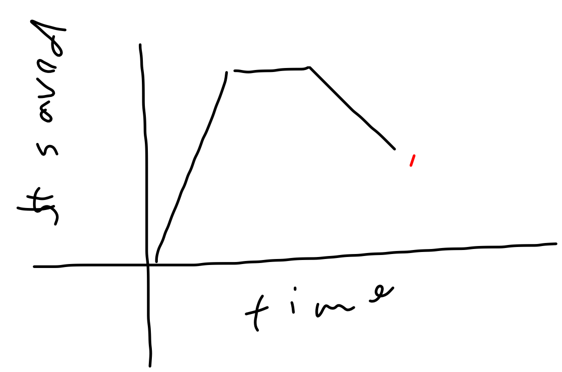
+110
+110
+110

graphical model:



→ the story of the graph a success!





Question 1: Use the graph and extrapolate to determine how much Alex will have saved at 7 months.

\$ 770

Question 2: Use the graph and interpolate to determine how much Alex will have saved at 3.5 months.

\$ 385

Question 3: What is the rate of change (slope) of the linear function that models this situation?

? a y — amount of \$

$a = \frac{\Delta y}{\Delta x}$ $a = 110$ \$/month

$a = \frac{110\ \$}{\text{month}}$

Question 4: What are the units of the rate of change?

$\frac{\$}{\text{month}}$ ← y
← x

You do pg 2 Practice 1.2 and Question 1-4

1.2 Practice

Create a model for the following situation:

Francesca loves amusement parks. During a recent vacation, Francesca went to *LoLoLand*, which doesn't charge admission, but charges \$1.50 for each ride.

per   y   x

step 1

$a = \frac{\Delta y}{\Delta x}$

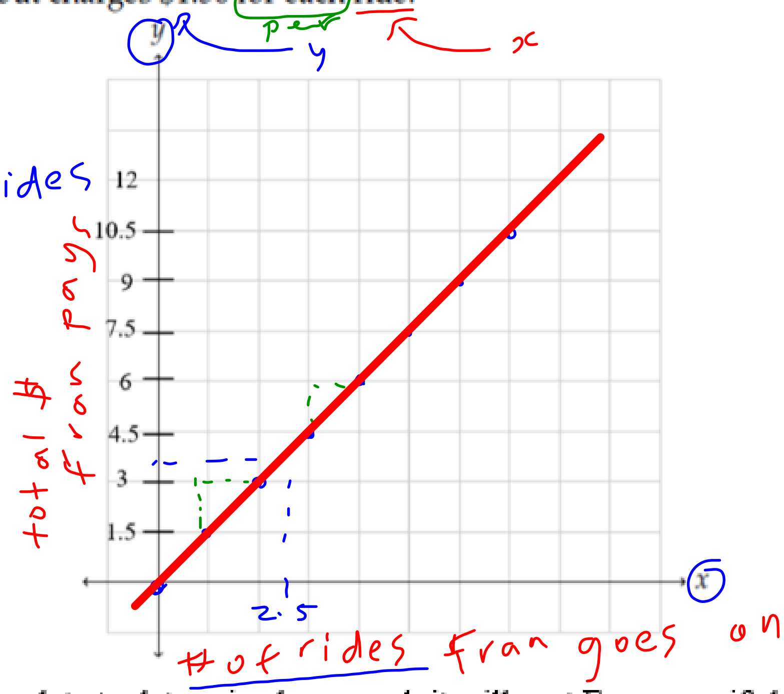
$a = \Delta y / \Delta x$

per

Dependent Variable: \$

Independent Variable: # of rides

x	$f(x)$



Question 1: Use the graph and extrapolate to determine how much it will cost Francesca if she plans on going on 8 rides.

\$12

Question 2: Use the graph and interpolate to determine how much it will cost Francesca if she plans on going on "2.5 rides".

= x

\$3.75

Question 3: What is the rate of change (slope) of the linear function that models this situation?

\$1.50/per ride

Question 4: What are the units of the rate of change?

$\frac{\$}{\text{ride}}$

1.3 Practice

$ ← y

ride ← x

Create a model for the following situation:

Dae loves amusement parks. During a recent vacation, Dae went to *Fantasy Circuit*, which charges \$10 for admission and \$1 for each ride.

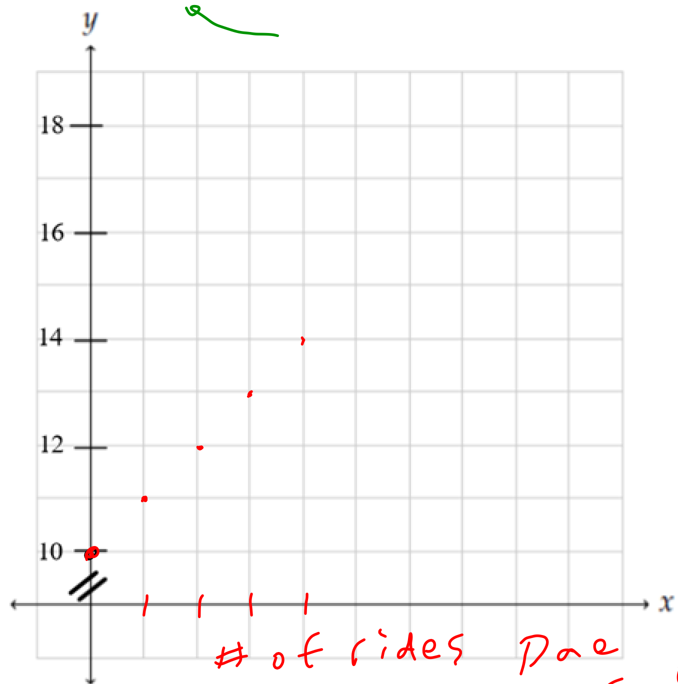
You do.

Dependent Variable:

Independent Variable:

x	$f(x)$

total $ spent



\# of rides Dae goes on

Question 1: Use the graph to determine how much it will cost Dae to go on 7 rides.

$17

Question 2: Use the graph to determine how much it will cost Dae to go on 6.5 rides.

$16.5

Question 3: What is the rate of change (slope) of the linear function that models this situation?

1 $/ride

Question 4: What are the units of the rate of change?

$\frac{\$}{\text{ride}}$

1.4 Example

Create a model for the following situation:

$ ← y

month ← x

$m = \frac{\Delta y}{\Delta x}$

step: x y

define in words.

Feng opens up a new savings account and initially deposits \$350. Feng sets up a savings plan and begins saving \$100 per month. (n.b. Because of low interest rates, the bank does not offer Feng any interest on his deposits).

Dependent Variable:

Independent Variable:

x	$f(x)$
0	350
1	450
2	550
3	650
4	750
5	850

initial value (y-intercept)

+100

Question 1: Use the graph to determine how much Feng will have saved at 6 months.

\$950

Question 2: Use the graph to determine how much Fend will have saved at 4.5 months.

\$800

Question 3: What is the rate of change (slope) of the linear function that models this situation?

100 \$/month

$f(0) = 350$

$f(1) = 350 + 100$

$f(2) = 350 + 100 + 100 = 350 + 2(100)$

$f(3) = 350 + 100 + 100 + 100 = 350 + 3(100)$

Question 4: What are the units of the rate of change?

\$/month

$f(4) = 350 + 100 + 100 + 100 + 100 = 350 + 4(100)$

$f(x) = 350 + \underbrace{100 + 100 + \cdots + 100}_{x\text{-times}} = 350 + x(100)$

$f(x) = 100x + 350$

slope

y-int.

2 An Introduction to Linear Functions

2.1 The Rule of a Linear Function

Let's go back and check the rules (equations) for the linear functions we used in Section 1 and list them here. Can we find a pattern?

$f(x) = a x + b$

a = slope (coefficient of x)

$f(x) = 110 x + 0$

coefficient

constant

b = y-intercept (constant)

$f(x) = 1.5 x + 0$

$f(x) = 100 x + 350$

$f(x) = 1 \cdot x + 10$

2.2.1 Practice

For each of the following functions, determine the *slope* and *initial value* (y-intercept) of the function:

$f(x) = a x + b$

1. $f(x) = 3x + 1$

$f(x)$ $a x + b$

2. $y = -3x - 4$

$f(x) = a x + b$

3. $f(x) = \frac{1}{3}x + 10$

$f(x) = a x + b$

4. $f(x) = -\frac{x}{4} - 1$

$y = -\frac{1x}{4} - 1$

$y = -\frac{1}{4}x - 1$

$f(x) = a x + b$

5. $y = x + 4$

$f(x)$ $a x + b$

6. $y = -x + 4$

$a x + b$

7. $f(x) = 9x$ $+ 0$

8. $y = -13x$

9. $f(x) = 9$

10. $y = -13$

11. $f(x) = 0$

HMWK:

pg 116, #3.4

pg 128, #3.5

pg 130, #3.8

pg 101 and pg 68 #2.7

pg 37 #1.23

pg 38-39 #1.24-1.25

	a	b:
1.	3	1
2.	-3	-4
3	$\frac{1}{3}$	10
4.	$-\frac{1}{4}$	-1
5.	1	4
6.	-1	4
7.	9	0
8.	-13	0
9.	0	9
10.	0	-13
11.	0	0