Unit 4: Proofs w Vectors

vectors in component form

if $\vec{u} = (a, b)$
$\vec{v} = (c, d)$

prove $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

take the L.S. and rewrite until you get right side.

- rewrite in comp form
- perform operation
- use properites of scalars
- do operation in reverse

vectors in norm/angle form

if $\vec{u} = \|\vec{u}\|, \theta_{\vec{u}}$
$\vec{v} = \|\vec{v}\|, \theta_{\vec{v}}$

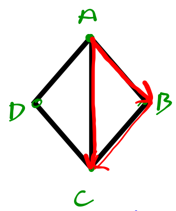
$\vec{u} \cdot \vec{v} = 0$

take the L.S. and rewrite until you get right side.

- rewrite using Chasles Law

$\vec{AC} = \vec{AB} + \vec{BC}$

$\vec{AC} = \vec{DC} + \vec{BC}$



* starting point of a vector doesn't matter.

* $\vec{AB} = -\vec{BA}$

* if $\vec{u} \perp \vec{v}$ then $\vec{u} \cdot \vec{v} = 0$

* $\vec{u} \cdot \vec{u} = \|\vec{u}\|^2$

$\vec{u} \cdot \vec{u} = \|\vec{u}\| \cdot \|\vec{u}\| \cos\theta$
$= \|\vec{u}\| \cdot \|\vec{u}\| \cos 0$
$= \|\vec{u}\|^2$

Prove $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
(The conclusion to be proved)

Hypothesis: (what you know is true)

$\vec{u} = (a, b)$

$\vec{v} = (c, d)$

Statements	Justification
L.S.	
$\vec{u} + \vec{v}$	hypo
$(a,b) + (c,d)$	Vector addition
$(a+c, b+d)$	properities of scalars (number) $3+2=2+3$
$(c+a, d+b)$	vector addition in reverse
$(c,d) + (a,b)$	
$\vec{v} + \vec{u}$ R.S.	

Prove $k(\vec{u}+\vec{v}) = k\vec{u} + k\vec{v}$

distrib w vectors

Hypothesis
$\vec{u} = (a, b)$
$\vec{v} = (c, d)$

Statements	Justification
L.S.	
$k(\vec{u}+\vec{v})$	hypo
$k((a,b)+(c,d))$	vector addition ✓
$k(a+c, b+d)$	multiplication of a vector w a scalar ✓
$(k(a+c), k(b+d))$	distribution between scalars.
$(k\cdot a + k\cdot c, k\cdot b + k\cdot d)$	
$(k\cdot a, k\cdot b) + (kc, k\cdot d)$	vector addition in reverse
$k(a,b) + k(c,d)$	multiplication in reverse
$k\vec{u} + k\vec{v}$ R.S. ∎	

Prove $k_1 \vec{u} \cdot k_2 \vec{v} = k_1 k_2 \vec{u} \cdot \vec{v}$

Hypo

$\vec{u} = (a, b)$

$\vec{v} = (c, d)$

Statements	Justification.

Prove that $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

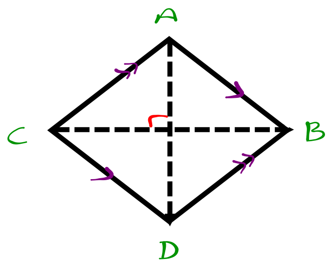
Hypo

$\vec{u} = (a, b)$

$\vec{v} = (c, d)$

$\vec{w} = (e, f)$

- Prove that the diagonals of a rhombus are perpendicular



Hypothesis: (what you know is true)

$\vec{AB} = \vec{CD}$

$\vec{AC} = \vec{BD}$

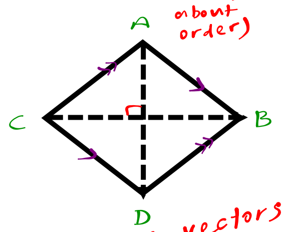
$\|\vec{AB}\| = \|\vec{CD}\| = \|\vec{AC}\| = \|\vec{BD}\|$

Conclusion (to be proved): $\vec{AD} \cdot \vec{CB} = 0$

Statements	Justification
L.S.	
$\vec{AD} \cdot \vec{CB}$	
$(\vec{AC} + \vec{CD}) \cdot (\vec{CA} + \vec{AB})$	Chasles' Law
$\vec{AC} \cdot \vec{CA} + \vec{AC} \cdot \vec{AB} + \vec{CD} \cdot \vec{CA} + \vec{CD} \cdot \vec{AB}$	distribution
$\vec{AC} \cdot \vec{CA} + '' + '' + \vec{AB} \cdot \vec{AB}$	hypo
$-\vec{CA} \cdot \vec{CA}$ $\quad \|\vec{AB}\|^2$	
$-\|\vec{CA}\|^2 + '' + '' + \|\vec{AB}\|^2$	
$\cancel{\|\vec{AB}\|^2 - \|\vec{CA}\|^2} + '' + ''$	
$\vec{AC} \cdot \vec{AB} + \vec{CD} \cdot \vec{CA}$	hypo
$\vec{AC} \cdot \vec{AB} + \vec{AB} \cdot \vec{CA}$	
$\vec{AC} \cdot \vec{AB} + \vec{CA} \cdot \vec{AB}$	
$\vec{AC} \cdot \vec{AB} - \vec{AC} \cdot \vec{AB}$	
0	

i. check if any dot product pairs equal zero ($\vec{u} \perp \vec{v}$)

ii check if you can rewrite the pairs w same letters (don't worry about order)

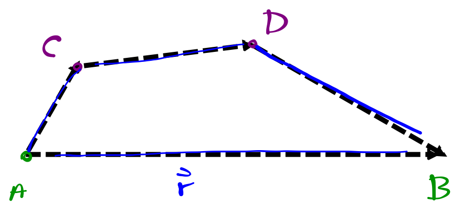


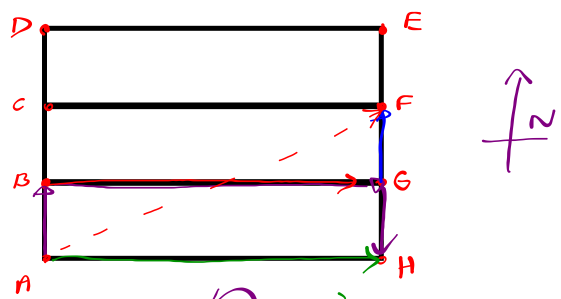
iii get same vectors by flipping letters and putting a negative sign.

iv. check if you can make remaining pairs the same.

Chasles' Relation

(a way to rewrite vectors)





$\vec{AB} = \underbrace{\vec{AC} + \vec{CD}} + \underbrace{\vec{DB}}$

- Rewrite $\vec{CF}$ in as many ways as possible

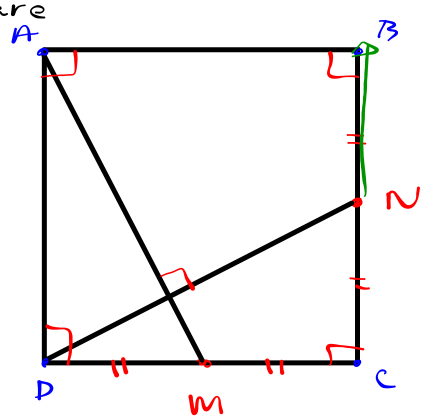
$\cdot \vec{CF}$

$\vec{AH} = \vec{AB} + \vec{BG} + \vec{GH}$ T / F

$\vec{AH} = \vec{AB} + \vec{CF} + \vec{GH}$

$\vec{AH} = \vec{AB} + \vec{DE} + \vec{FE}$ F

Show that line segments from vertices to midpoints in a square are $\perp$



Hypo:

$\vec{AB} = \vec{DC}$

$\vec{AD} = \vec{BC}$

$\vec{DM} = \vec{MC}$

$\vec{CN} = \vec{NB}$

$\|\vec{DM}\| = \|\vec{MC}\| = \|\vec{CN}\| = \|\vec{NB}\|$

Conclusion to be proved: $\vec{AM} \cdot \vec{DN} = 0$

Statement	Justification

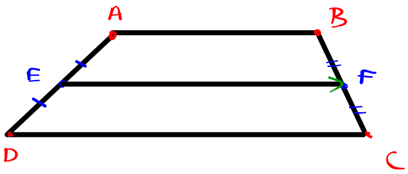
hypothesis:

$\overrightarrow{DE} = \overrightarrow{EA}$

$\overrightarrow{CF} = \overrightarrow{FB}$

Prove:

$\overrightarrow{EF} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{DC})$



proof
norm
angle

homework

p 4.26 #1

p 4.37 #4

p 4.19 Ex 10

proof
component
form

p 4.12 – 4.16

Chaslos #1-3
p 4.33
4.37

Review
Activity