Unit 3: Linear Combination ($\vec{w}$) of Two Vectors and Scalar Product between Two Vectors

Definition: Linear Combination ($\vec{w}$): the addition of 2 vectors that have been multiplied by 2 scalars.

$$\vec{w} = k_1 \vec{u} + k_2 \vec{v}$$

- Vector basis: the L.C. of 2 non-collinear vectors.
  non-parallel

- Orthonormal basis: the L.C. of two unit vector (norm = 1 unit) that are orthogonal $\perp$

L.C. of 2 vectors in component form

$\vec{w}$

$\vec{u} = (a, b)$
$\vec{v} = (c, d)$

$\vec{u} = (\vec{u}_x, \vec{u}_y)$

$\vec{v} = (\vec{v}_x, \vec{v}_y)$

$\vec{w} = k_1\vec{u} + k_2\vec{v}$

$k_1\vec{u} = (k_1 \cdot \vec{u}_x, k_1 \cdot \vec{u}_y)$

$k_2\vec{v} = (k_2 \cdot \vec{v}_x, k_2 \cdot \vec{v}_y)$

$\vec{w} = (\underbrace{k_1\vec{u}_x + k_2\vec{v}_x}_{=\vec{w}_x}, \underbrace{k_1\vec{u}_y + k_2\vec{v}_y}_{\vec{w}_y})$

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if $\vec{u} = (2, 3)$
$\vec{v} = (-1, 4)$

find $\vec{w} = 3\vec{u} + 4\vec{v}$

step ① find $3\vec{u}$ ✓
$4\vec{v}$ ✓

$3\vec{u} = (3 \cdot 2, 3 \cdot 3)$

$3\vec{u} = (6, 9)$

$4\vec{v} = (4 \cdot (-1), 4 \cdot 4)$

$4\vec{v} = (-4, 16)$

step ②: Find $\vec{w}$
by adding x comp
and y comp.

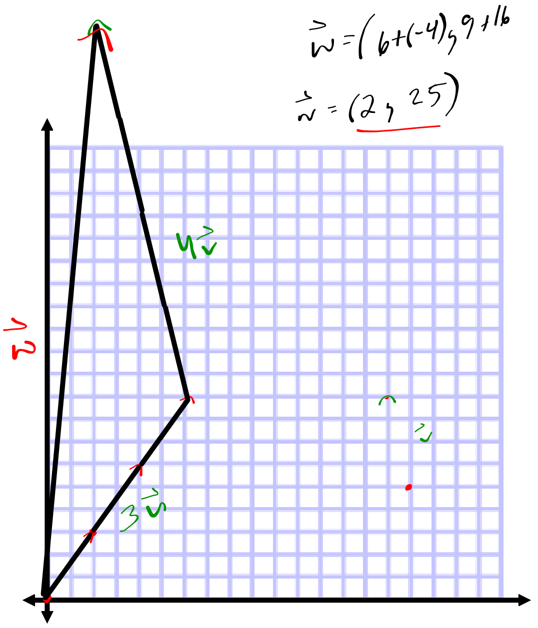
$\vec{w} = 3\vec{u} + 4\vec{v}$

$\vec{x}$

$3\vec{u} = (6, 9)$
$4\vec{v} = (-4, 16)$

$\vec{w} = (6 + (-4), 9 + 16)$

$\vec{w} = (2, 25)$



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if $\vec{u} = (-2, 3)$
$\vec{v} = (1, -4)$

find $\vec{w} = -2\vec{u} + 3\vec{v}$

$-2\vec{u} = (4, -6)$

$3\vec{v} = (3, -12)$

$\vec{w} = (7, -18)$

Finding the scalars in a Linear Combination.

if $\vec{u} = (1, 2)$ $(3\ \ 6)$

$\vec{v} = (3, -1)$ $(6, -2)$

$\vec{w} = k_1\vec{u} + k_2\vec{v}$

$\vec{w} = (9, 4)$

find k_1 and k_2

step ① Rewrite $k_1\vec{u}$, $k_2\vec{v}$

$k_1\vec{u} = (k_1 \cdot 1, k_1 \cdot 2)$

$+ \qquad +$

$k_2\vec{v} = (k_2 \cdot 3, k_2 \cdot -1)$

$\text{"} \qquad \text{"}$

$\vec{w} = (9, 4)$

step ②: add up the x comps of $\vec{u} + \vec{v}$ and put equal to x comp of $\vec{w}$. Do same for ys.

$-2\times$ ① $9 = (k_1 + 3k_2)^{-2}$

② $4 = 2k - k_2$

step ③: use sub/comp/elimination to find k_1 + k_2

i. times ① by -2

$-18 = -2k_1 - 6k_2$ ①

$+\ \ 4 = 2k_1 - k_2$ ②

ii add ① + ② to eliminate k_1

$\frac{-14}{-7} = \frac{\cancel{-7}k_2}{\cancel{-7}}$

$k_2 = 2$

iii sub k_2 into ① to find k_1

$9 = k_1 + 3(2)$

$9 - 6 = k_1 + 6 - 6$

$k_1 = 3$

ANS $\boxed{\vec{w} = 3\vec{u} + 2\vec{v}}$

$\boxed{k_1 = 3 \quad k_2 = 2}$ ANS

find k_1 and k_2 if

$\vec{u} = (3, -3)$

$\vec{v} = (-2, -5)$

$\vec{w} = (2, -16)$

$\boxed{\text{pg } 3.62}$

Linear Combination of two Vectors in Norm/Angle Form

$\vec{w}$

$\vec{u} = \|\vec{u}\|, \theta_{\vec{u}}$

ex. 3.3 p 3.6

$\vec{u} = 4.2$ units, $45°$

$\vec{v} = 3.6$ unit, $326°$

find $\vec{w} = 2\vec{u} + 2\vec{v}$

$(\|\vec{w}\|, \theta_{\vec{w}})$

step i. find $2\vec{u}$, $2\vec{v}$

$2\vec{u} = |2| \cdot 4.2$, $45°$ since k is positive

$= 8.4$ units

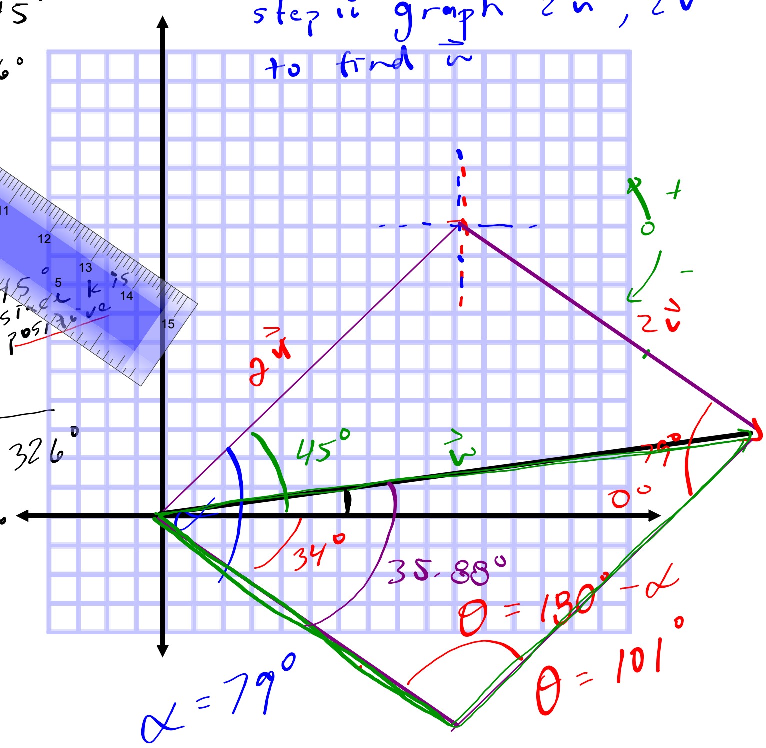
$45°$

$2\vec{v} = 2 \times 3.6$, $326°$

$2\vec{v} = 7.2$ unit, $326°$

$-34°$

step ii graph $2\vec{u}$, $2\vec{v}$ to find $\vec{w}$



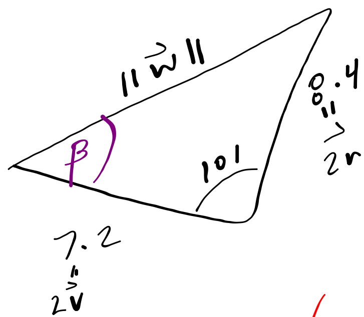
$$\|\vec{w}\| = \sqrt{(7.2)^2 + (8.4)^2 - 2(7.2)(8.4)\cos 101}$$

$\|\vec{w}\| = 12.06$ units.

$\theta_{\vec{w}} = 43.14° - 34$

$\theta_{\vec{w}} = 9.14°$

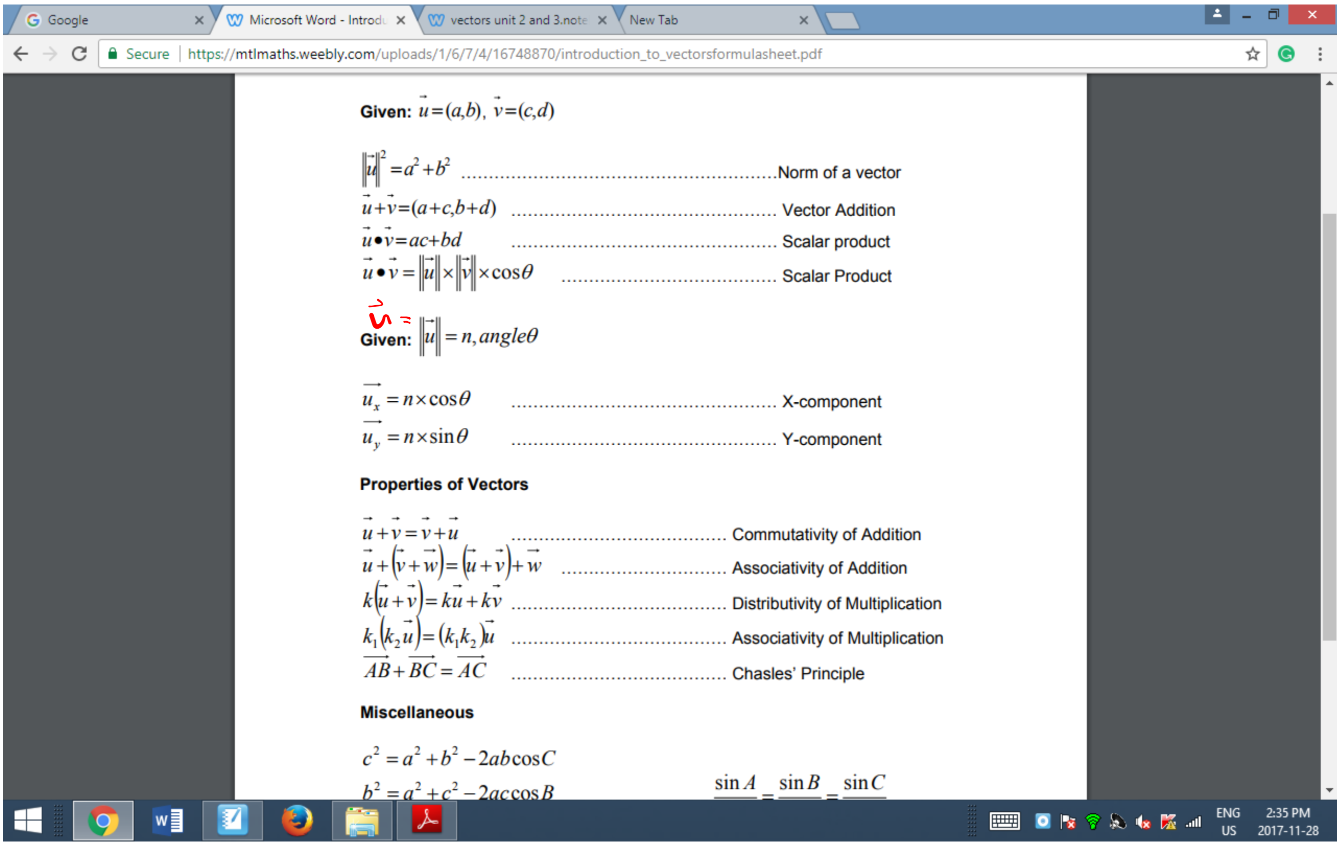
$$\boxed{\vec{w} = 12.06 \text{ units}, 9.14°}$$



$$\frac{8.4 \sin 101}{12.06} = \left(\frac{\sin \beta}{\cancel{8.4}}\right) \cancel{8.4}$$

$$\sin^{-1}\left(\frac{8.4 \sin 101}{12.06}\right) = \cancel{\sin^{-1}} \cancel{\sin} \beta$$

$\beta = 43.14°$



Given: $\vec{u}=(a,b),\ \vec{v}=(c,d)$

$\|\vec{u}\|^2 = a^2+b^2$ ....Norm of a vector

$\vec{u}+\vec{v}=(a+c,b+d)$ .... Vector Addition

$\vec{u}\bullet\vec{v}=ac+bd$ .... Scalar product

$\vec{u}\bullet\vec{v}=\|\vec{u}\|\times\|\vec{v}\|\times\cos\theta$ .... Scalar Product

$\vec{u} =$

Given: $\|\vec{u}\| = n, angle\theta$

$\vec{u_x} = n\times\cos\theta$ .... X-component

$\vec{u_y} = n\times\sin\theta$ .... Y-component

Properties of Vectors

$\vec{u}+\vec{v}=\vec{v}+\vec{u}$ .... Commutativity of Addition

$\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$ .... Associativity of Addition

$k(\vec{u}+\vec{v})=k\vec{u}+k\vec{v}$ .... Distributivity of Multiplication

$k_1(k_2\vec{u})=(k_1k_2)\vec{u}$ .... Associativity of Multiplication

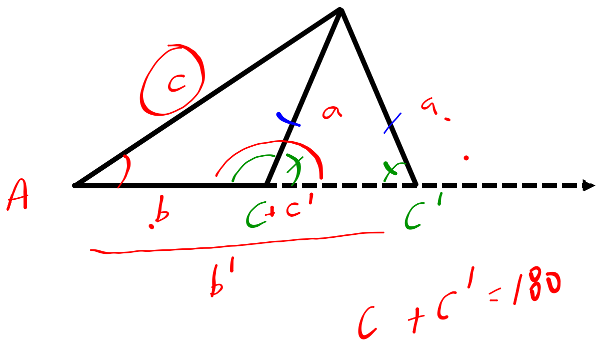
$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ .... Chasles' Principle

Miscellaneous

$c^2 = a^2+b^2-2ab\cos C$

$b^2 = a^2+c^2-2ac\cos B$ $\qquad \sin A = \sin B = \sin C$

The ambiguous case of Sine Law



$$\frac{\sin A}{a} = \left[\frac{\sin B}{b} = \frac{\sin C}{c} \right]$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

3.12

b) $\vec{u} = 8.5, 45^\circ$

$\vec{v} = 8.2, 14^\circ$

find $\vec{w} = \frac{1}{3}\vec{u} + \frac{1}{2}\vec{v}$

20mins or less

4:03

Stop everything

https://mtlmaths.weebly.com/uploads/1/6/7/4/16748870/introduction_to_vectorsformulasheet.pdf

Given: $\vec{u}=(a,b),\ \vec{v}=(c,d)$

$\|\vec{u}\|^2=a^2+b^2$ .... Norm of a vector

$\vec{u}+\vec{v}=(a+c,b+d)$ .... Vector Addition

$\vec{u}\bullet\vec{v}=ac+bd$ .... Scalar product

$\vec{u}\bullet\vec{v}=\|\vec{u}\|\times\|\vec{v}\|\times\cos(\theta_{\vec{u}}-\theta_{\vec{v}})$ .... Scalar Product

Given: $\|\vec{u}\|=n, angle\theta$

$\vec{u_x}=n\times\cos\theta$ .... X-component

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Properties of Vectors

$\vec{u}+\vec{v}=\vec{v}+\vec{u}$ .... Commutativity of Addition

$\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$ .... Associativity of Addition

$k(\vec{u}+\vec{v})=k\vec{u}+k\vec{v}$ .... Distributivity of Multiplication

$k_1(k_2\vec{u})=(k_1k_2)\vec{u}$ .... Associativity of Multiplication

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Miscellaneous

$c^2=a^2+b^2-2ab\cos C$

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$\frac{\sin A}{} = \frac{\sin B}{} = \frac{\sin C}{}$

Scalar (Dot Product) Product of Two Vectors in component form

$\vec{u} = (a, b)$

$\vec{v} = (c, d)$

$\vec{u} \cdot \vec{v} = ac + bd$

ex. $\vec{u} = (3, 3)$ — $\vec{u}_x, \vec{u}_y$

$\vec{v} = (6, 2)$ — $\vec{v}_x, \vec{v}_y$

find $\vec{u} \cdot \vec{v} = \vec{u}_x \cdot \vec{v}_x + \vec{u}_y \cdot \vec{v}_y$

$= 3 \cdot 6 + 3 \cdot 2$

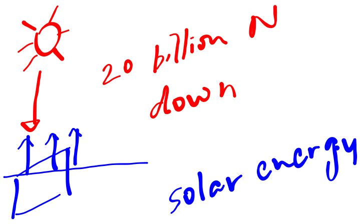
$= 24 \text{ unit}^2$

$\vec{u} \cdot \vec{v} = 24 \text{ unit}^2$

Nota Bene:

if $\vec{u} \perp \vec{v}$

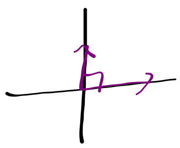
then $\vec{u} \cdot \vec{v} = 0$



if $\vec{u}(0, 3)$

$\vec{v}(4, 0)$

find dot product



if $\vec{s}(-2, 3)$

$\vec{r}(-1, 4)$

find scalar product

$\vec{s} \cdot \vec{r} = -2 \times -1 + 3 \cdot 4$

$\vec{s} \cdot \vec{r} = 14 \text{ unit}^2$

Scalar Product of two vectors in Norm/angle Form

$\vec{u} = \|\vec{u}\|, \theta_{\vec{u}}$

$\vec{v} = \|\vec{v}\|, \theta_{\vec{v}}$

$\vec{u} = 2$ units, 45°

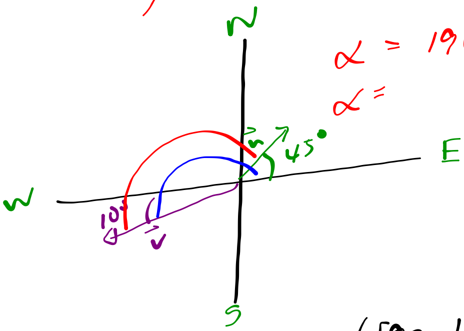
$\vec{v} = 5$ units, W10°S $= 190^\circ$

$\vec{u} \cdot \vec{v} = \|\vec{u}\|\|\vec{v}\| \cos(\theta_{\vec{u}} - \theta_{\vec{v}})$

angle in between two vectors.

$\alpha = 190 - 45^\circ$

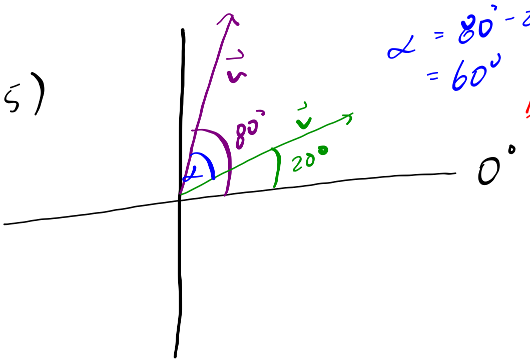
$\alpha =$



$\theta_{\vec{u}} = 80^\circ$

$\theta_{\vec{v}} = 20^\circ$

$\alpha = 80^\circ - 20^\circ$
$= 60^\circ$



$\vec{u} \cdot \vec{v} = 2 \cdot 5 \cdot \cos(190 - 45)$

$\vec{u} \cdot \vec{v} = -8.19$ units^2

$\vec{u} = 4$, W

$\vec{v} = 3$, S20°E

find $\vec{u} \cdot \vec{v}$

$\vec{t} = 3$ units, -30°

$\vec{s} = 4$ units, 60°

find $\vec{t} \cdot \vec{s}$

Properities of Scalar Product

$3 \cdot 2 = 2 \cdot 3$

$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

$3(x+1)$
$3x+3$

$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

$k_1 \vec{u} \cdot k_2 \vec{v} = k_1 k_2 \vec{u} \cdot \vec{v}$

if $\vec{u} = (a, b)$ $\quad \vec{u} \cdot \vec{u} = a \cdot a + b \cdot b$

$\vec{u} = (a, b)$

$\vec{u} \cdot \vec{u} = a^2 + b^2$

$\|\vec{u}\|^2 = a^2 + b^2$

$\vec{u} \cdot \vec{u} = \|\vec{u}\|^2$

cat = p?an
cat = gorter

$\vec{u} = \|\vec{u}\|, \theta_{\vec{u}}$

$\vec{u} \cdot \vec{u} = \|\vec{u}\| \|\vec{u}\| \times \cos(\theta_{\vec{u}} - \theta_{\vec{u}})$

$\vec{u} \cdot \vec{u} = \|\vec{u}\| \cdot \|\vec{u}\| \cdot 1$

$\vec{u} \cdot \vec{u} = \|\vec{u}\|^2$

HMWK
P 3.71
3.82