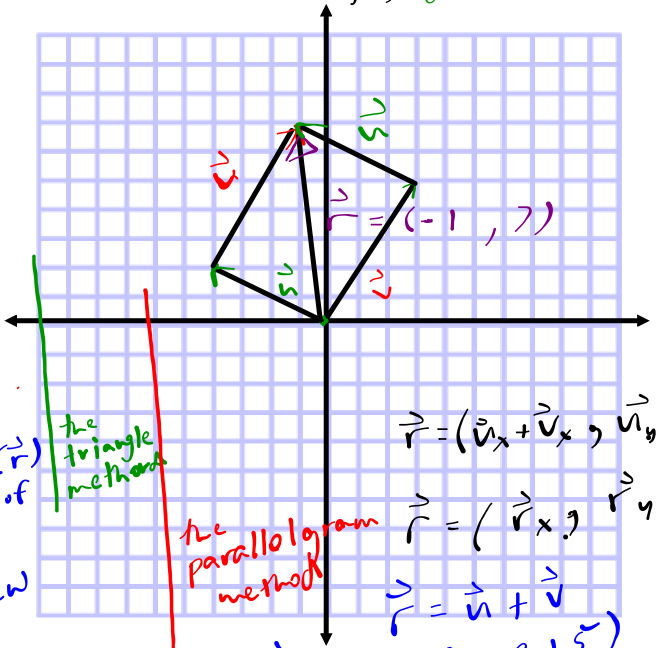
Unit 2: Vector Addition and Multiplication of a Vector by a Scalar

Vector Addition given $\vec{u} = (a, b)$, $\vec{v} = (c, d)$ } Component form.

ex. if $\vec{u} = (-4, 2)$.
$\vec{v} = (3, 5)$.

find $\vec{r} = \vec{u} + \vec{v}$



(resulting vector)
to add vector

step ① Draw 1st vector at the origin

step ② Draw 2nd vector at the head of 1st

step ③ Draw resultant vector ($\vec{r}$) from tail of 1st vector to head of 2nd

step ④ Redo steps ①-② but draw 2nd first.

$\vec{r} = (\vec{u}_x + \vec{v}_x, \vec{u}_y + \vec{v}_y)$

$\vec{r} = (\vec{r}_x, \vec{r}_y)$

$\vec{r} = \vec{u} + \vec{v}$

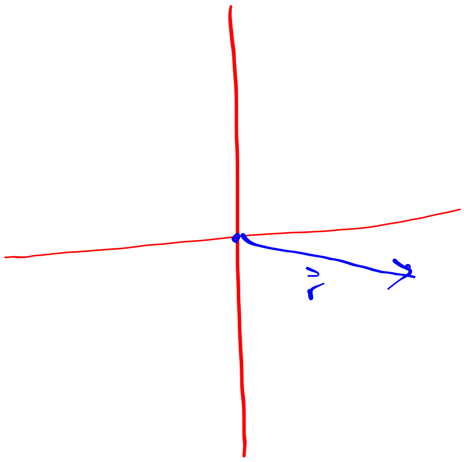
$\vec{r} = (-4 + 3, 2 + 5)$

$\vec{r} = (-1, 7)$

P 2.13

#1 find $\vec{r}$ - using parallogram
graph $\vec{r}$
if $\vec{u} = (5, 1)$
$\vec{v} = (2, -3)$

$\vec{r} = (7, -2)$



Introduction to Vectors

Given: $\vec{u}=(a,b)$, $\vec{v}=(c,d)$

$\|\vec{u}\|^2=a^2+b^2$ .... Norm of a vector

$\vec{u}+\vec{v}=(a+c,b+d)$ .... Vector Addition

$\vec{u}\bullet\vec{v}=ac+bd$ .... Scalar product

$\vec{u}\bullet\vec{v}=\|\vec{u}\|\times\|\vec{v}\|\times\cos\theta$ .... Scalar Product

Given: $\|\vec{u}\|=n, angle\theta$

$\vec{u_x}=n\times\cos\theta$ .... X-component

$\vec{u_y}=n\times\sin\theta$ .... Y-component

Properties of Vectors

$\vec{u}+\vec{v}=\vec{v}+\vec{u}$ .... Commutativity of Addition

$\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$ .... Associativity of Addition

Addition of Vectors in Norm Angle Form

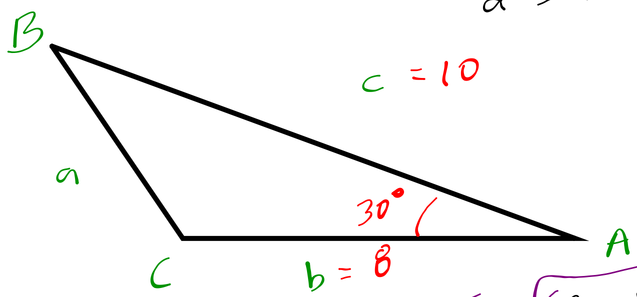
(ex $\vec{u} = 2\text{km}, 30°$)

Recall Necessary Tools

Cosine Law : $c^2 = a^2 + b^2 - 2ab\cos C$

$a^2 = b^2 + c^2 - 2bc\cos A$

find a



$\sqrt{a^2} = \sqrt{8^2 + 10^2 - 2(8)(10)\cos 30}$

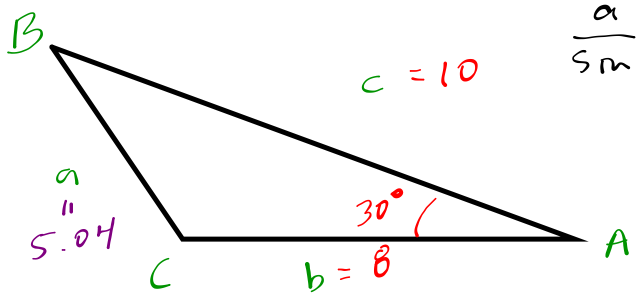
$a = 5.04$ units

Necessary Tool:

Sine Law:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



find B

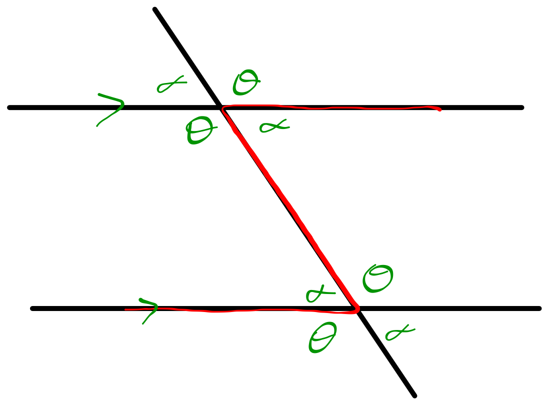
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

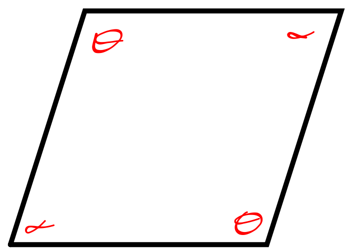
$$\frac{5.04}{\sin 30} = \frac{8}{\sin B}$$

$$\frac{5.04 \sin B}{5.04} = \frac{8 \sin 30}{5.04}$$

$$\sin^{-1} \sin B = \sin^{-1}\left(\frac{8 \sin 30}{5.04}\right)$$

$$B = 52.53°$$



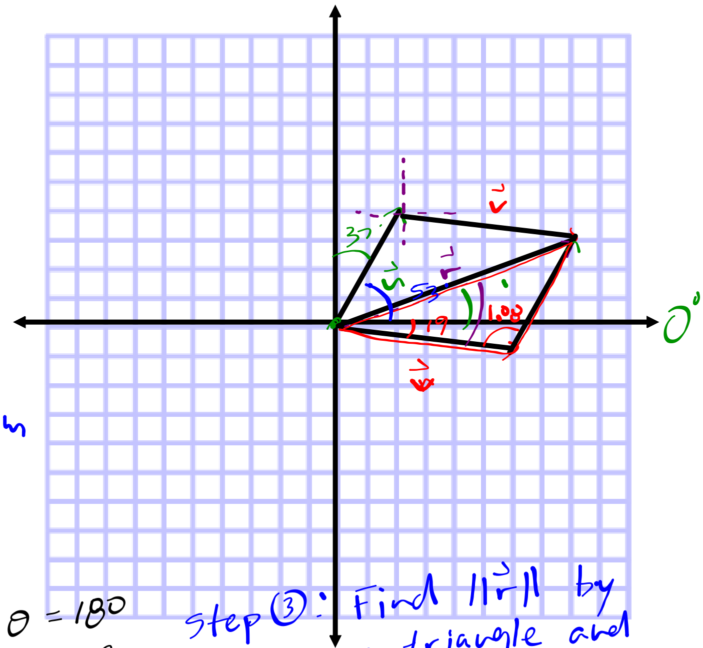


$\alpha + \theta = 180^\circ$

Norm / Angle Form (Addition)

if
$\vec{u}$ = 5 unit, N 37° E (53°)
$\vec{v}$ = 6.3 unit, −19°

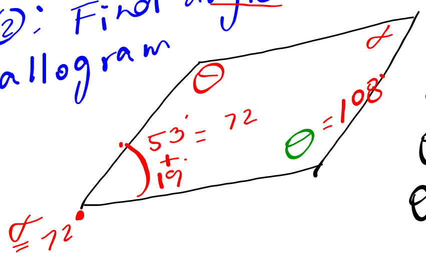
find $\vec{r} = \vec{u} + \vec{v}$
(want $\|\vec{r}\|$ ✓
$\theta_{\vec{r}}$ ✓)



(same steps as before)

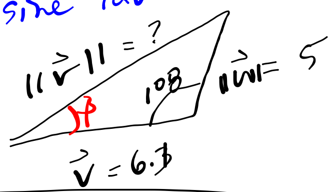
step ① Graph $\vec{r}$ using parallelogram method

step ②: Find angles in parallelogram



$\alpha + \theta = 180$
$\theta = 180 - 72$
$\theta = 108°$

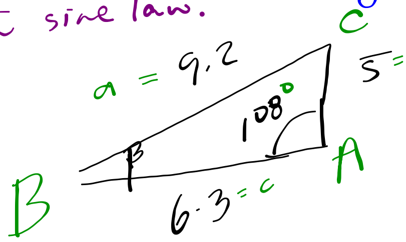
step ③: Find $\|\vec{r}\|$ by focusing on triangle and using cosine law.



$$r = \sqrt{5^2 + (6.3)^2 - 2(5)(6.3)\cos 108}$$

$r = 9.2$ units

step ④ Find $\theta_{\vec{r}}$ by first find β in triangle.
w/ sine law.



$$\frac{\sin A}{9.2} = \frac{\sin \beta}{5} \qquad \frac{\sin A}{a} = \frac{\sin B}{b}$$

$$5\left(\frac{\sin 108}{9.2}\right) = \left(\frac{\sin \beta}{5}\right) 5$$

$$\sin^{-1}\left(\frac{5 \sin 108}{9.2}\right) = \sin^{-1} \sin \beta$$

$\beta = 31.12°$

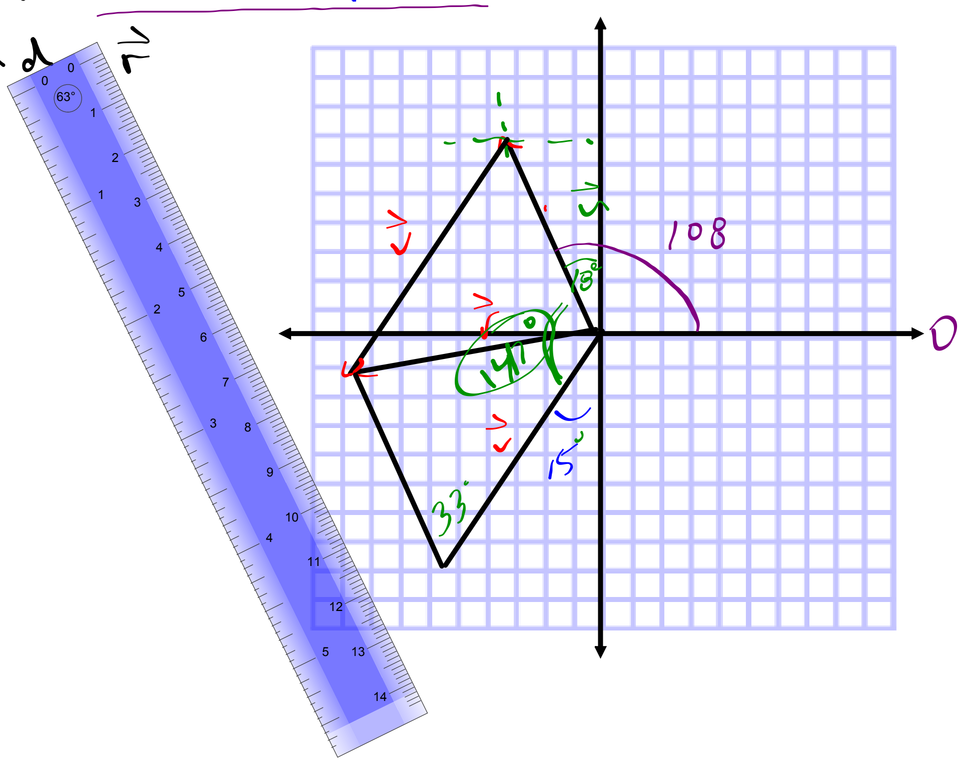
$\theta_{\vec{r}} = 31.12° - 19°$

$\theta_{\vec{r}} = 12.12°$

$$\boxed{\vec{r} = 9.2 \text{ unit}, \ 12.12°}$$

if $\vec{u}$ = 4 units 108°
$\vec{v}$ = 5.5 units S 15° W

find $\vec{r}$



Multiplication of a Vector by a Scalar

when vector is in component form:

$k \cdot \vec{u}$, $k \in \mathbb{R}$ $\vec{u} = (a, b)$

$k \cdot \vec{u} = (k \cdot a, k \cdot b)$

graph $\vec{u} = (-3, 4)$

$\vec{u} = \vec{AB} \neq \vec{BA}$

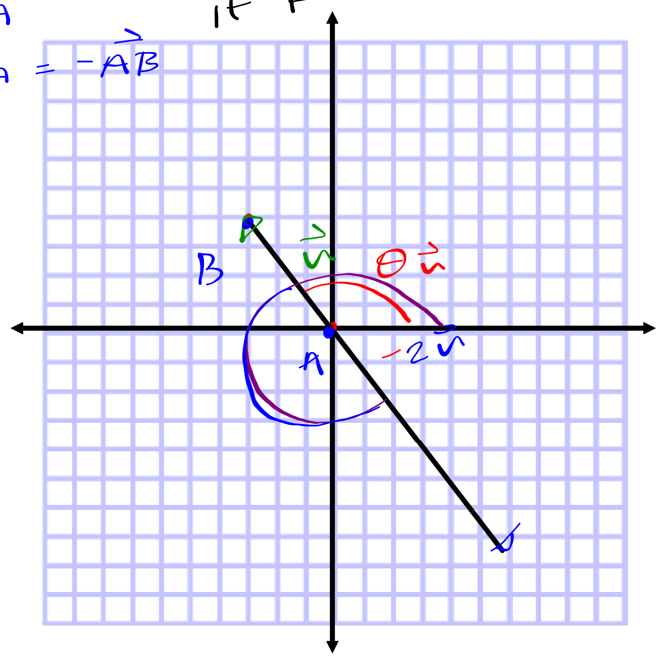
graph $k \cdot \vec{u}$ if $k = -2$

$\vec{BA} = -\vec{AB}$

$k\vec{u} = (k \cdot a, k \cdot b)$

$= (-2(-3), -2 \cdot 4)$

$k\vec{u} = (6, -8)$



Multiplication Continued
(Norm Angle Form)

$k \cdot \vec{u}$, where $k \in \mathbb{R}$ $\vec{u} = \|\vec{u}\|, \theta_{\vec{u}}$

$k \cdot \vec{u} = |k| \cdot \|\vec{u}\|$, $\theta_{k\vec{u}} = \theta_{\vec{u}}$ if k is positive

$\theta_{k\vec{u}} = \theta_{\vec{u}} + 180^\circ$ if k is negative

to find new angle, take original angle & add 180°

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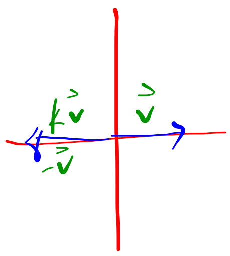
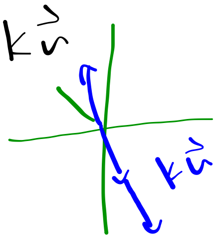
if $k = -1$ $\vec{v} = 3$ unit E

find $k\vec{v}$



if $k = -2$ $\vec{u} = 4$ units, 98°

find $k\vec{u}$



P 2.42

Properities of Vector Addition
& multiplication by a Scalar

$2 + 3 = 3 + 2$

$\vec{u} + \vec{v} = \vec{v} + \vec{u}$

$(2+3)+1 = 2 + (3+1)$

$(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$

$2 - 2 = 0$

$\vec{u} + (-\vec{u}) = \vec{O}$ $\vec{O} = (0,0)$

$\vec{AB} + \vec{BA} = \vec{AB} - \vec{AB} = \vec{O}$

$1 \cdot \vec{u} = \vec{u}$

$2 \cdot 3 \cdot 1 = 6$

$0 \cdot \vec{u} = \vec{O}$

$k_1(k_2\vec{u}) = (k_1 k_2)\vec{u}$

$2(x+2)$
$= 2x + 4$

$k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v}$

P 2.43 - 2.51 | unit 2

P 2.52 - 2.65 | unit 1 and 2

P 2.40 - 2.41 # 2 - 7

P 2.30 #2