Unit 5: Graphing Sinusoidal Function

Determining the Equation

(a/b/h/k)

Reeeall:

a - flips up/down
scale factor

Amplitude: $A = |a|$

$a = \frac{M - m}{2}$

b - flips left/right
scale factor

Period: $P = \frac{2\pi}{|b|}$

h - horizontal translation

phase shift

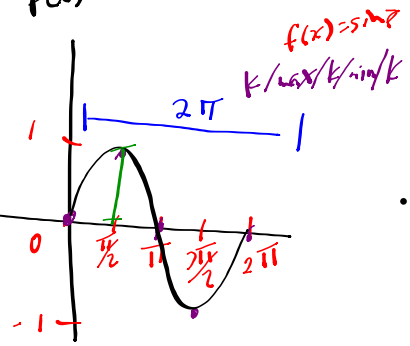
$D = h$

k - vertical translation

$y = k$ → central axis

$f(x) = a \sin b(x-h) + k$

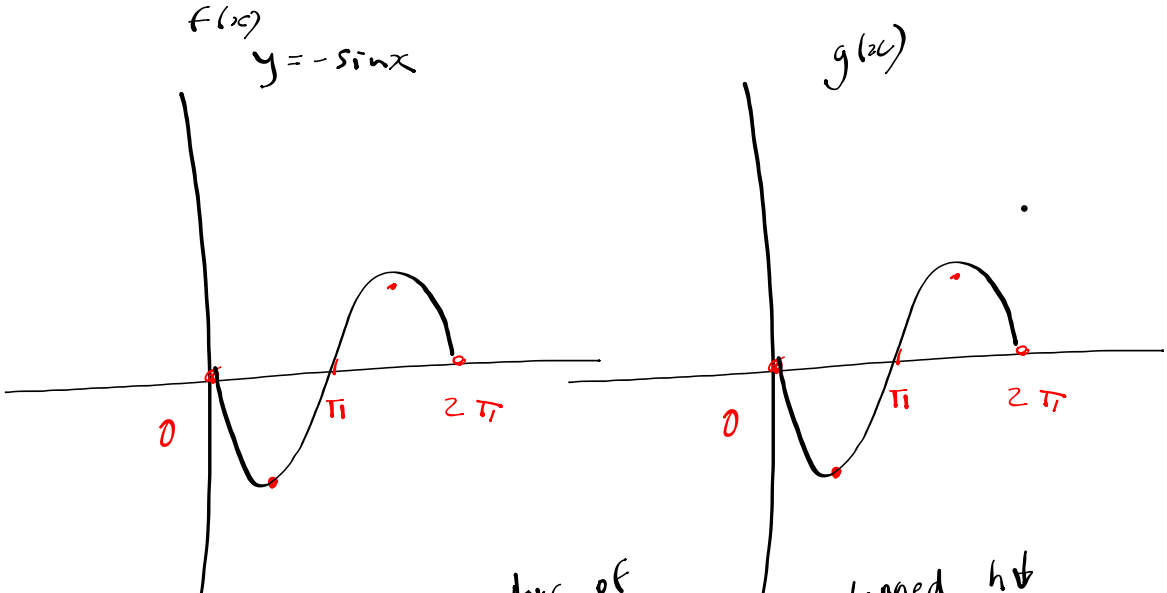
$f(x) = a \cos b(x-h) + k$



For sin x	for cos x
(h, k) starting point	(h, K + a) starting point
k/max/k/min/k for a·b > 0 or	max/k/min/k/max for a > 0
k/min/k/max/k for a·b < 0	min/k/max/k/min for a < 0

$max = k + |a|$

$min = k - |a|$



what happened to the parameters of $f(x)$ to $g(x)$.

a) h? sign of a changed
b) b changed d) sign of a changed
c) sign of a changed h
sign of b changed

p 5.31 graph $f(x) = 2\sin\frac{1}{2}(x+2\pi)+1$

$y = a\sin b(x-h)+k$

ex. 8

Step ① : Find all parameters and characteristics

$a = 2$ $A = 2$

$b = \frac{1}{2}$ $P = \frac{2\pi}{\frac{1}{2}}$

$h = -2\pi$ $P = 4\pi$

$k = 1$

max = k + |a|

max = 3

min = −1

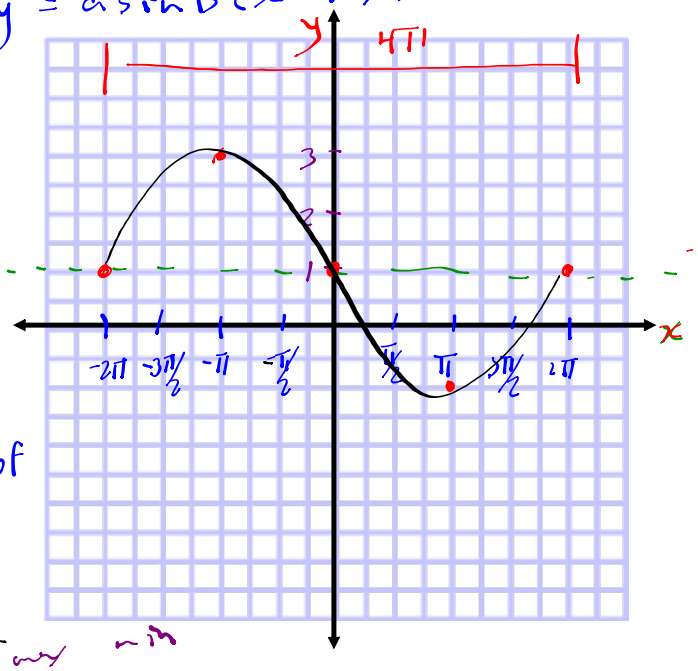
Starting point sin (h, k)

$(-2\pi, 1)$

Central axis $y = 1$

k/max/min/k

1/4 P 1/2



Step ② Construct Table of Value

sin

x	y
h	k (min)
$h+\frac{P}{4}$	k (max)
$h+\frac{P}{2}$	k (min)
$h+\frac{3P}{4}$	k
$h+P$	k

+P/4 +P/4 +P/4

cos

x	y
h	k (max)
	k
	k (min)
	k
	k (max)

$f(x) = 2\sin\frac{1}{2}(x+2\pi)+1$

$h = -2\pi$

$h+\frac{P}{4} = -2\pi + \frac{4\pi}{4}$

$h+\frac{P}{4} = -\pi$

$h+\frac{P}{2} = -2\pi + \frac{4\pi}{2}$

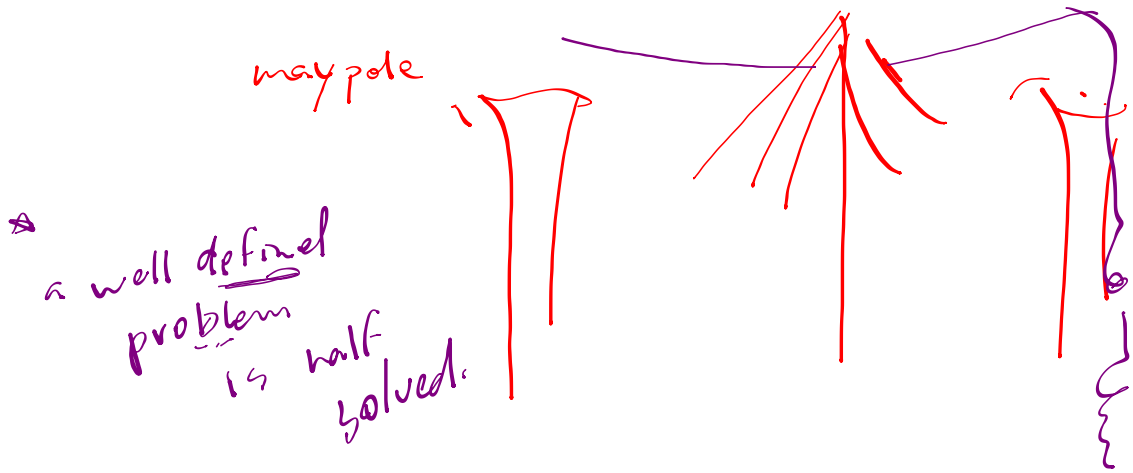
$h+\frac{P}{2} = 0$

x	y
-2π	1
$-\pi$	3
0	1
π	−1
2π	1

$y = 2\sin\frac{1}{2}(-2\pi+2\pi)+1$ ← Eliminate

along mirror line

1. Clearance


maypole
a well defined problem is half solved.

p 5.64 #5.8 graph $g(x) = 3\cos 3\left(x + \frac{2\pi}{3}\right) + 1$

Finding the x-ints $(x; 0)$

$g(x) = 3\cos 3\left(x + \frac{2\pi}{3}\right) + 1$

Step ①: Sub $y=0$ into equation

$0 = 3\cos 3\left(x + \frac{2\pi}{3}\right) + 1$

Step ②: Isolate the trig term

$-\frac{1}{3} = \frac{3\cos 3\left(x + \frac{2\pi}{3}\right)}{3}$

$-\frac{1}{3} = \cos 3\left(x + \frac{2\pi}{3}\right)$

Step ③: If the L.S. corresponds to a trig point on the unit circle, replace it w/ the sin/cos of that angle. $P(\theta) = (\cos\theta, \sin\theta)$

Step ④: Continue to isolate for x → $\sin^{-1}$ is opposite sin

$\cos^{-1}\left(-\frac{1}{3}\right) = \cos 3\left(x + \frac{2\pi}{3}\right)$

$\frac{1.91}{3} = \frac{3\left(x + \frac{2\pi}{3}\right)}{3}$

$x = \frac{1.91}{3} - \frac{2\pi}{3}$

$x = -1.457$

Step ⑤: Final answer should acknowledge the cyclical nature

$x = -1.457 + kP$

$x = -1.457 + k\frac{2\pi}{3}, k \in \mathbb{Z}$

find

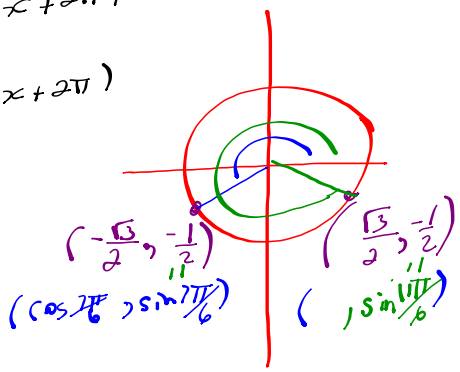
$f(x) = 2\sin\frac{1}{2}(x + 2\pi) + 1$

$P = 4\pi$

$0 = 2\sin\frac{1}{2}(x + 2\pi) + 1$

$-1 = 2\sin\frac{1}{2}(x + 2\pi)$

$-\frac{1}{2} = \sin\frac{1}{2}(x + 2\pi)$



$\sin^{-1}\sin\frac{7\pi}{6} = \sin^{-1}\sin\frac{1}{2}(x + 2\pi)$

$2 \cdot \frac{7\pi}{6} = \left(\frac{1}{2}(x + 2\pi)\right)^2$

$\frac{14\pi}{6} = x + 2\pi$ (×6, −2π)

$x = \frac{2\pi}{6}$

$x_1 = \frac{\pi}{3}$

$x_{II} = \frac{\pi}{3} + 4\pi$

$x_{III} = \frac{\pi}{3} + 8\pi$

$x = \frac{\pi}{3} + k4\pi$

$x\text{-int} \in \{0, 2\pi\}$

$\sin^{-1}\sin\frac{11\pi}{6} = \sin^{-1}\sin\frac{1}{2}(x + 2\pi)$

$x_1 = -1.457 + \frac{2\pi}{3}$

$x_1 = 0.636 \in [0, 2\pi]$

p 5.69 Determine the equation of the sinusoidal function (aka find all the parameters)

a / b / h / k

① First find $k = \frac{M+m}{2}$

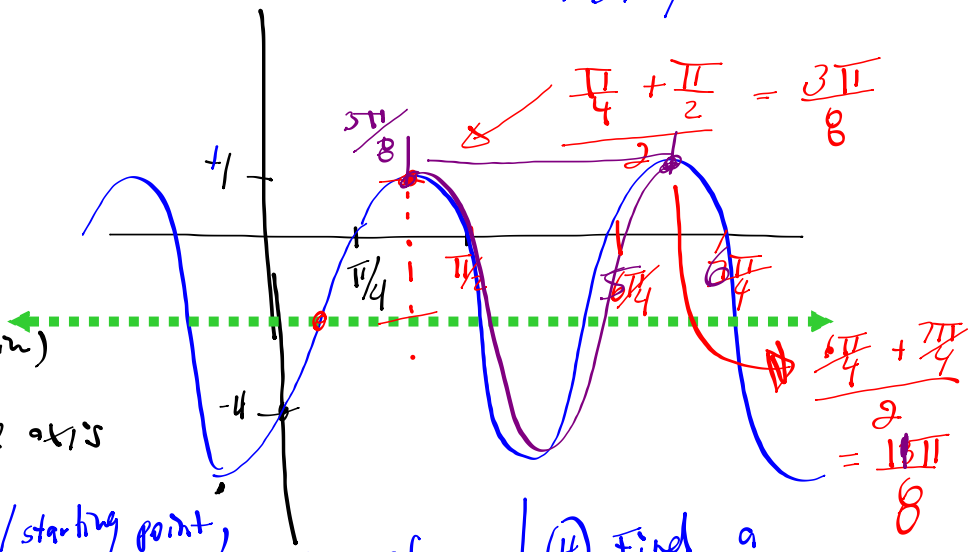
$k = \frac{1+(-5)}{2}$

$k = -2$

② Find h by choosing it!

cos → (h, k+a) (max or min)

sin → (h, k) on central axis



③ Based on h / starting point, pick cos or sin and pick sign of a.

max → cos

↳ a positive

④ Find a

$a = \frac{M-m}{2}$

$a = \frac{1-(-5)}{2}$

$a = 3$

⑤ Find b using $P = \frac{2\pi}{|b|}$.

Period is the distance between two consecutive max points.

$P = \frac{15\pi}{8} - \frac{3\pi}{8}$

$P = \frac{12\pi}{8} = \pi$

$P = \frac{2\pi}{b}$

$\pi = \frac{2\pi}{b}$

$b\pi = 2\pi$

$b = 2$

$y = 3\cos 2\left(x - \frac{3\pi}{8}\right) - 2$

Unit 6: Determining the value of an unknown defined by a trig function using trig identities

ex. Calculate the value of $\cos\theta$ given that $\sin\theta = \frac{3}{5}$ and $\theta \in [0, \frac{\pi}{2}]$

$\sin x$

$\sin^2 x + \cos^2 x = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2 x = 1$

$\cos^2 x = 1 - \left(\frac{3}{5}\right)^2$

$\cos x = \pm \frac{4}{5}$

$\sin x = \frac{1}{\csc x}$

$\tan x = \frac{\sin}{\cos x}$

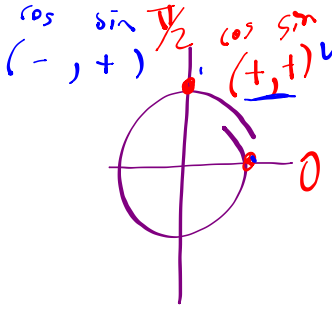
$\csc x = \frac{1}{\sin x}$

$\sin^2 x + \cos^2 x = 1$

Step ①: Pick a trig identity that has what you know (ex. sin) and what you want (or what will lead you to what you want) (ex. cos)

Step ②: sub in what you know and isolate for what you want.

Step ③: Pick final answer $\pm$ or $-$ or both by consider the unit and whether the function is +/- in the given interval.



$[0, \frac{\pi}{2}[$

Find ANS

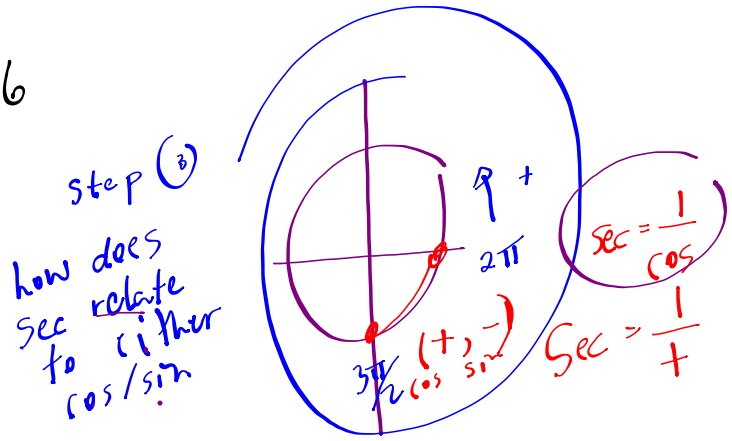
$\cos x = \frac{4}{5}$

Calculate the value of $\sec x$ given that $\tan x = -1.2$ and $x \in \left\{\frac{3\pi}{2}, 2\pi\right\}$

$\sec x = \pm 1.56$

step (3)

how does sec relate to either cos/sin



Calculate the value of $\tan x$ given that

$\csc x = -6.25$ $\quad x \in \left[-\pi, -\frac{\pi}{2}\right]$

$1 + \cot^2 x = \csc^2 x$

$1 + \cot^2 x = (-6.25)^2$

$\cot x = \sqrt{(-6.25)^2 - 1}$

$\cot x = \frac{\sqrt{609}}{4} = 6.16$

$\csc x = \frac{1}{\sin x}$

$1 + \cot^2 x = \csc^2 x$

$\cot = \frac{1}{\tan x}$

sub in $\cot x = \frac{\sqrt{609}}{4}$

$\cot x = \frac{1}{\tan x}$

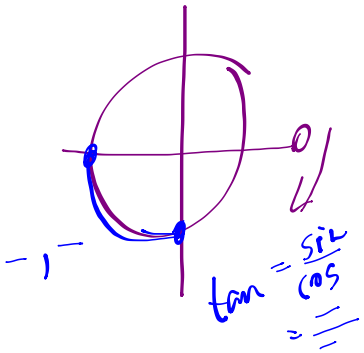
$x \in \left[-\pi, -\frac{\pi}{2}\right)$

$\frac{\sqrt{609}}{4} = \frac{1}{\tan x}$

$\frac{\tan x \sqrt{609}}{\sqrt{609}} = \frac{4}{\sqrt{609}}$

$\tan x = \pm 0.162$

Final ANS $\tan x = 0.162$



Calculate the value of $\csc x$ given that $\cos x = \frac{-13}{85}$ and that $x \in \left[-\pi, \frac{-\pi}{2}\right]$