

Determining

$$f(x) = a \sin b(x-h) + k$$

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$\boxed{x - \text{intg}}$

Unit 5: Graphing <sup>sin/cos</sup> Sinusoidal Functions  
and Determining their Equations  
 (a/b/h/k)

graph  $f(x) = 2 \sin \frac{1}{2}(x + 2\pi) + 1$

Recall:

a - up/down flip  
 - scale factor

Amplitude  $A = |a|$

$$A = \frac{\text{max} - \text{min}}{2}$$

$$\text{max} = k + |a|$$

$$\text{min} = k - |a|$$

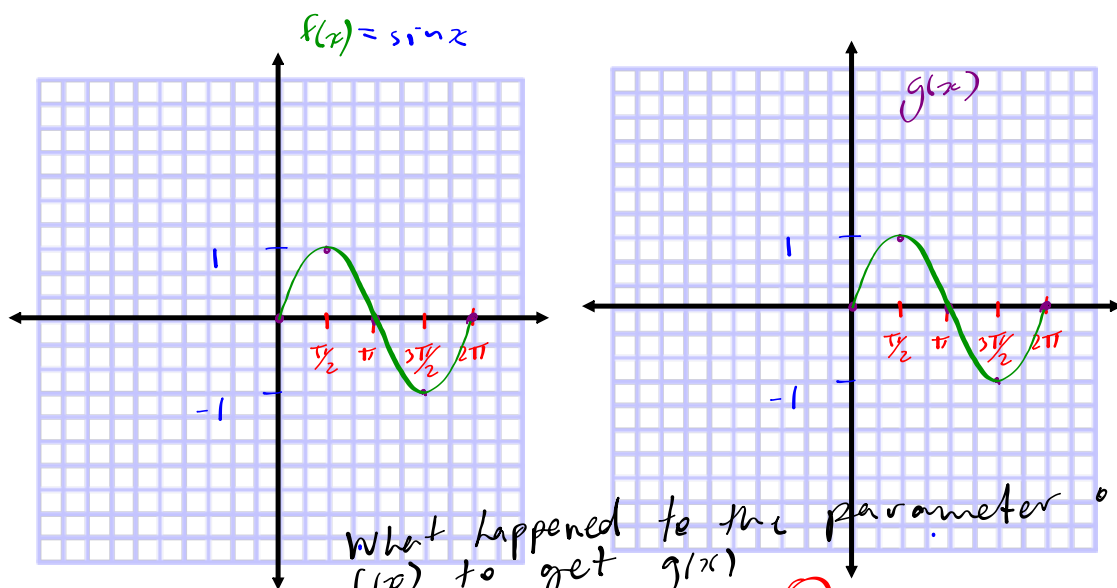
b - left/right flip  
 - scale factor

$$\text{Period} = \frac{2\pi}{|b|}$$

h - horizontal translation  
 phase shift  $D = h$

k - vertical translation  
 central axis  
 $y = k$

For $\sin x$ starting point (h, k)	For $\cos x$ starting point (h, k + a)
k/max/k/min/k a.b +	max/k/min/k/max if a +
k/min/k/max/k a.b -	min/k/max/k/min if a -



What happened to the parameter of  $f(x)$  to get  $g(x)$

a) increasing  $h$        c)  $a - b -$   
 b)  $a - k \downarrow$        d)  $\uparrow h \quad k \downarrow$

graph  $f(x) = 2 \sin \frac{1}{2}(x + 2\pi) + 1$   
 $f(x) = a \sin b(x - h) + k$

P 5.31

step i. find parameters

and info

$a = 2$   $h = -2\pi$

$b = \frac{1}{2}$   $k = 1$

Central axis

$y = k$

$y = 1$

Starting point

$(x, y)$

$(h, k)$

$(-2\pi, 1)$

$A = 2$

$P = \frac{2\pi}{|\frac{1}{2}|}$

$P = \frac{2\pi}{\frac{1}{2}} = 4\pi$

$P = 4\pi$

$\max = k + |a|$   
 $= 1 + 2$   
 $= 3$

$\min = k - |a|$   
 $= 1 - 2$   
 $= -1$

Step ii construct TOV.

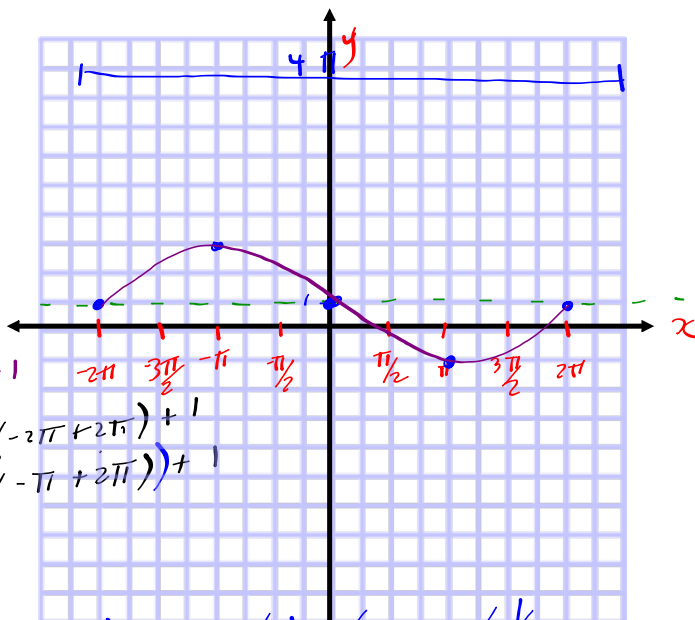
$y = 2 \sin \frac{1}{2}(x + 2\pi) + 1$

$y = 2 \sin \frac{1}{2}(-2\pi + 2\pi) + 1$

$y = 2 \sin(\frac{1}{2}(-\pi + 2\pi)) + 1$

$x$	$y$
$h$	$k$
	max
	min
	$k$

$x$	$y$
$-2\pi$	1
$-\pi$	3
0	1
$\pi$	-1
$2\pi$	1



$k / \max / k / \min / k$

graph  $y = 3 \sin -2(x + \frac{\pi}{2}) + 2$   
 pg 5.35

Finding the x-ints/zeros/solution

ex. find the x-ints of  
 $g(x) = 3\cos 3(x + \frac{2\pi}{3}) + 1$

step i. Do a graph!

$a=3$       $A=3$   
 $b=3$       $P = \frac{2\pi}{3}$   
 $h = -\frac{2\pi}{3}$      st.  $(h, k+a) = (-\frac{2\pi}{3}, 4)$   
 $k=1$       $y=1$   
 max = 4     max/k/min/k/max  
 min = 1-3  
 = -2

step ii put  $y=0$  and solve for  $x$   
 by doing opposite operations.

BE DMAS

$$0 = 3\cos 3(x + \frac{2\pi}{3}) + 1$$

$$-1 = 3\cos 3(x + \frac{2\pi}{3})$$

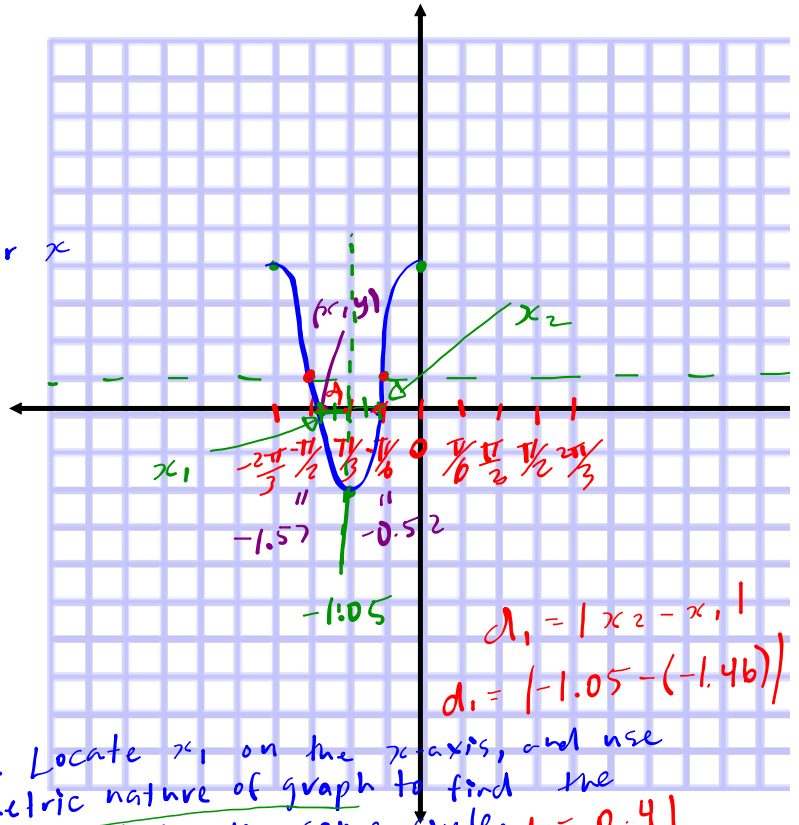
$$\frac{-1}{3} = \cos 3(x + \frac{2\pi}{3})$$

$$\cos^{-1}(\frac{-1}{3}) = \cos^{-1}(\cos 3(x + \frac{2\pi}{3}))$$

$$1.91 = 3(x + \frac{2\pi}{3})$$

$$\frac{1.91}{3} = x + \frac{2\pi}{3}$$

$$x_1 = -1.46$$



step iii Locate  $x_1$  on the  $x$ -axis, and use symmetric nature of graph to find the 2 x-int ( $x_2$ ) in the same cycle.  $d_1 = 0.41$

$$x_2 = \min_x + d_1$$

$$x_2 = -1.05 + 0.41$$

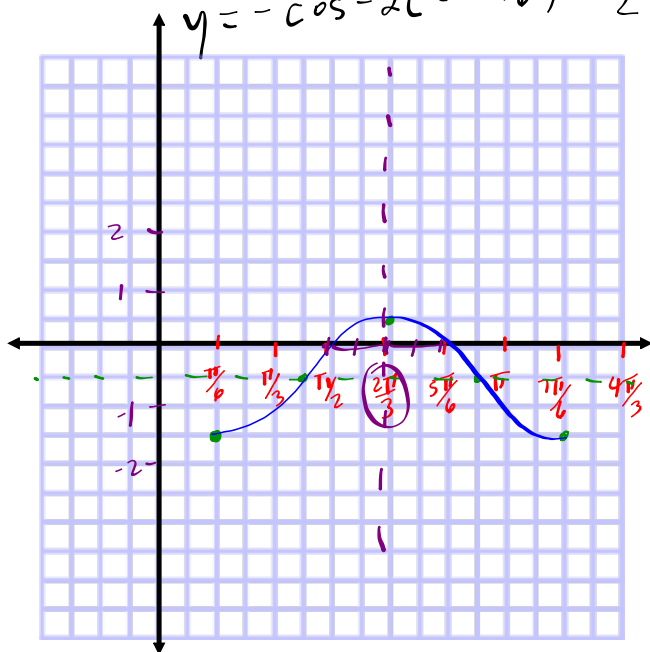
$$x_2 = -0.64$$

step iv. Express the cyclical nature of the sin function in final answer.

$$\left. \begin{aligned} x_1 &= -1.46 + kP \\ x_1 &= -1.46 + \frac{2k\pi}{3} \\ x_2 &= -0.64 + \frac{2k\pi}{3} \end{aligned} \right\} \text{ where } k \in \mathbb{Z}$$

Find the x-ints

$$y = -\cos - 2(x - \pi/6) - \frac{1}{2}$$



$$\left\{ \begin{array}{ll} a = -1 & A = 1 \\ b = -2 & P = \pi \\ h = \pi/6 & y = -1/2 \\ k = -1/2 & \text{s.p. } (h, k + a) \\ & (\pi/6, -1/2 - 1) \\ & (\pi/6, -1.5) \end{array} \right.$$

$$\begin{aligned} \text{max} &= -0.5 + 1 \\ &= 0.5 \\ \text{min} &= -1.5 \end{aligned}$$

min/k/max/k/w

$$x_1 = -0.5236$$

$$x_1' = -0.5236 + \pi$$

$$x_1'' = 2.62$$

P 5-69

Determining the equation of a sinusoidal function

step i. fill out x-axis

step ii. find  $|a|$

$A = |a|$

$|a| = \frac{\max - \min}{2}$

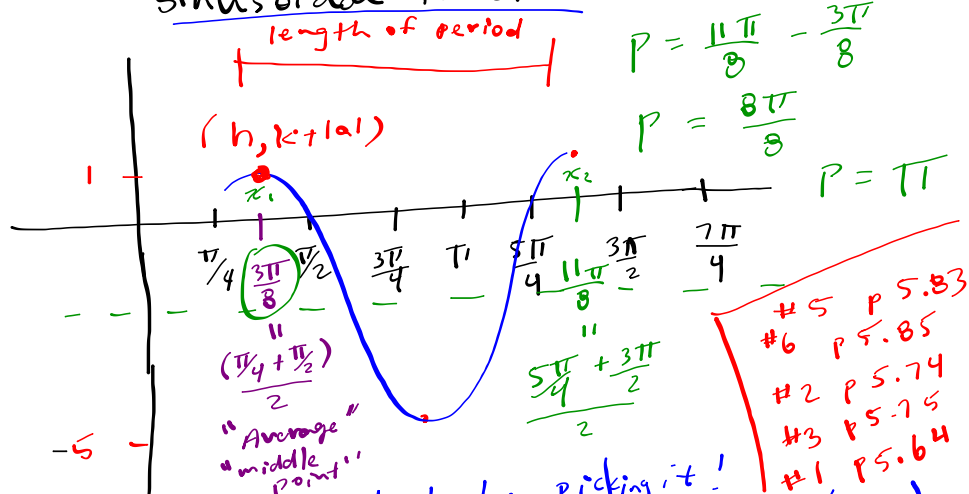
$|a| = \frac{1 - (-5)}{2}$

$|a| = 3$

step iii find  $k$

$k = \frac{\max + \min}{2}$

$k = \frac{1 + (-5)}{2} = -2$



step iv. Find  $h$  by picking it!  
(Either on central axis, or a max/min)

step v. Based on  $h$ /starting point, pick cos or sin and pick the sign of  $a$ .  
 $\therefore \cos$  and  $a +$   
 $a = 3$

$y = 3 \cos b \left( x - \frac{3\pi}{8} \right) - 2$

step vi. Find  $b$  by using

$P = \frac{2\pi}{|b|}$

$P$  is the length between 2 consecutive max

$d = |x_2 - x_1|$

$P = \pi$   
 $\pi = \frac{2\pi}{|b|}$

$\frac{|b|\pi}{\pi} = \frac{2\pi}{\pi}$   
 $|b| = 2$

$b = 2$   
 $y = 3 \cos 2 \left( x - \frac{3\pi}{8} \right) - 2$

