Unit 3 : Evaluating a Trig Function for a Number expressed in Radians

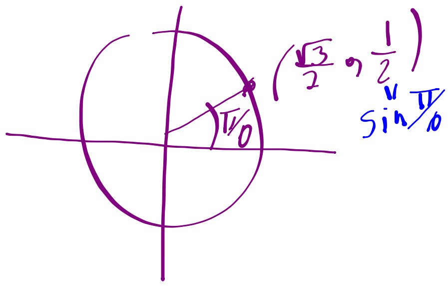
$x = \pi$

e.x. If $f(x) = \sin x$, find $f\left(\frac{\pi}{6}\right)$

$f\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6}$ ← evaluate

To evaluate wont giving a demcial refer to the points on the unit circle $(\cos\theta, \sin\theta)$

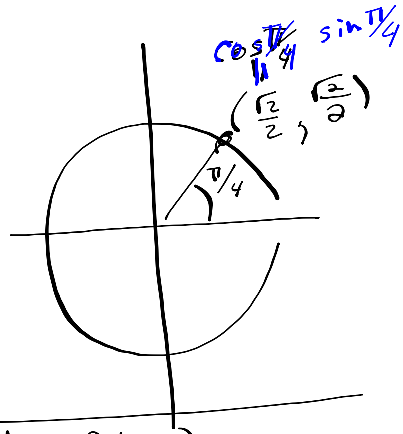
$f\left(\frac{\pi}{6}\right) = \frac{1}{2}$



ex. If $f(x) = \cos x$, find $f(\frac{\pi}{4})$

$f(\frac{\pi}{4}) = \cos(\frac{\pi}{4})$

$f(\pi/4) = \frac{\sqrt{2}}{2} = 0.7071$



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ex If $f(x) = \tan x$, find $f(\pi/6)$ ← for function that isn't apart of the wrapping function

$f(x) = \tan x$

$f(x) = \frac{\sin x}{\cos x}$

$f(\pi/6) = \frac{\sin \pi/6}{\cos \pi/6}$

Step ① Rewrite the function using the trig identities (something that ~~has what you want~~ have what you want) cos/sin and what you

$f(\pi/6) = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

Step ②: Evaluate wont decimal by refering to the unit circle (rationalize the denominator if need be).

To divide: flip the 2nd fraction and times

$f(\pi/6) = \frac{1}{2} \times \frac{2}{\sqrt{3}}$

$f(\pi/6) = \frac{\cancel{2}}{\cancel{2}\sqrt{3}}$

$f(\pi/6) = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$f(\pi/6) = \frac{\sqrt{3}}{(\sqrt{3})^2} = \frac{\sqrt{3}}{3}$ → 0.577

Check

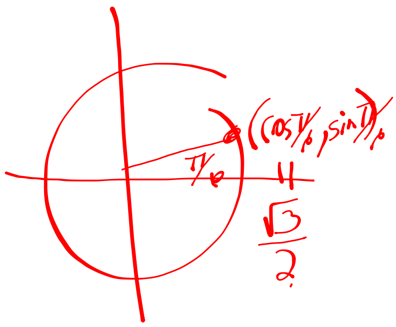
$\tan(\pi/6) = 0.577$

If $f(x) = \sec x$, find $f\left(\frac{\pi}{6}\right)$

$$f(x) = \sec x$$

$$f(x) = \frac{1}{\cos x}$$

$$f(\pi/6) = \frac{1}{\cos \pi/6}$$



$$f(\pi/6) = \frac{1}{\frac{\sqrt{3}}{2}}$$

$$f(\pi/6) = \frac{1}{1} \times \frac{2}{\sqrt{3}}$$

$$f(\pi/6) = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$f(\pi/6) = \frac{2\sqrt{3}}{3} = 1.15$$

$$\frac{1}{\cos \pi/6} = 1.15$$

If $g(x) = \csc x$, find $g\left(\frac{19\pi}{4}\right)$

4.75π

$\theta' = \theta - k2\pi$

4.75 $\theta' = \frac{19\pi}{4} - k2\pi$

$\theta' = \frac{19\pi}{4} - 4\pi$

$\theta' = \left(\frac{19}{4} - 4\right)\pi$

$\theta' = \frac{3}{4}\pi$

For angles $\notin [0, 2\pi]$

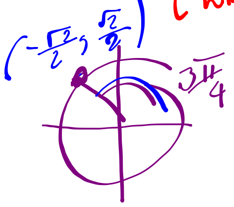
Step ① Find the reduced angle θ'

$\theta' = \theta - 2k\pi \quad k \in \mathbb{Z}$

$g(x) = g(x + 2\pi)$

Step ② Rewrite our function

Step ③: Evaluate with the reduced angle θ' (no decimals)



$g(x) = \csc x$

$g(x) = \frac{1}{\sin x}$

$g\left(\frac{3\pi}{4}\right) = \frac{1}{\sin \frac{3\pi}{4}}$

$g\left(\frac{3\pi}{4}\right) = \frac{1}{\frac{\sqrt{2}}{2}}$

$g\left(\frac{3\pi}{4}\right) = 1 \times \frac{2}{\sqrt{2}}$

$g\left(\frac{3\pi}{4}\right) = \frac{2}{\sqrt{2}} \frac{\times\sqrt{2}}{\times\sqrt{2}}$

$g\left(\frac{3\pi}{4}\right) = \frac{\cancel{2}\sqrt{2}}{\cancel{2}}$

$g\left(\frac{3\pi}{4}\right) = \sqrt{2}$

If $f(x) = \tan x$, find $f\left(\frac{29\pi}{4}\right)$ and $f\left(-\frac{5\pi}{4}\right)$

$f(x) = \tan x$

$f(x) = \frac{\sin x}{\cos x}$

7.25

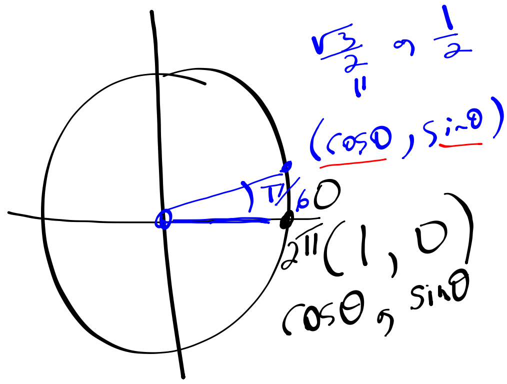
$f\left(\frac{29\pi}{4}\right)$

$\theta' = \theta - 2k\pi$

$\theta' = \frac{29\pi}{4} - 6\pi$

CB $\theta' = \left(\frac{29}{4} - 6\right)\pi$

$\theta' = \frac{5\pi}{4}$



$f\left(\frac{29\pi}{4}\right) = f\left(\frac{5\pi}{4}\right) = \frac{\sin\frac{5\pi}{4}}{\cos\frac{5\pi}{4}}$

$= \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$

$= 1$

$\tan\left(\frac{29\pi}{4}\right)$

$-\frac{5\pi}{4}$

$\theta' = -\frac{5\pi}{4} - 2k\pi$

$\theta' = -\frac{5\pi}{4} + 2\pi$

$\theta' = \frac{3\pi}{4} \in [0, 2\pi]$

Prove that $\tan\theta = \frac{\sin\theta}{\cos\theta}$

L.S. R.S.

Statement	Justification
R.S. $\frac{\sin\theta}{\cos\theta}$	
$\frac{\frac{opp}{hypo}}{\frac{adj}{hypo}}$	SOH CAH TOA
$\frac{opp}{hypo} \times \frac{hypo}{adj}$	fraction division
$\frac{opp \cdot hypo}{adj \cdot hypo}$	
$\frac{opp}{adj}$	SOH CAH TOA
$\tan\theta$ L.S.	p 3.12

Unit 4: Graphing Trig Function

(in their initial state

$f(x) = a \sin b(x-h) + k$

$f(x) = a \cos b(x-h) + k$

$a = 1$
$b = 1$
$h = 0$
$k = 0$

Periodic Function (Wave Function)

$f(x) = \sin x$
$g(x) = \cos x$
$h(x) = \tan x$

x-values come for the radians on the unit circle

$f(x) = f(x+P) = f(x+2P) = \ldots f(x+kP)$

$k \in \mathbb{Z}$

graph

To graph:
Identify parameters and all the info about the function

$f(x) = \sin x$

$f(x) = a \sin b(x-h) + k$

$a = 1$
$b = 1$
$h = 0$
$k = 0$

$y = \sin x$

x	y
0	$\sin 0 = 0 = k$
$\pi/2$	$\sin \pi/2 = 1 =$ max
π	$\sin \pi = 0 = k$
$3\pi/2$	$\sin 3\pi/2 = -1 =$ min
2π	$\sin 2\pi = 0 = k$

Amplitude

the length of the max height of a wave measured from equilibrium

$+\frac{P}{4}$ h

+P

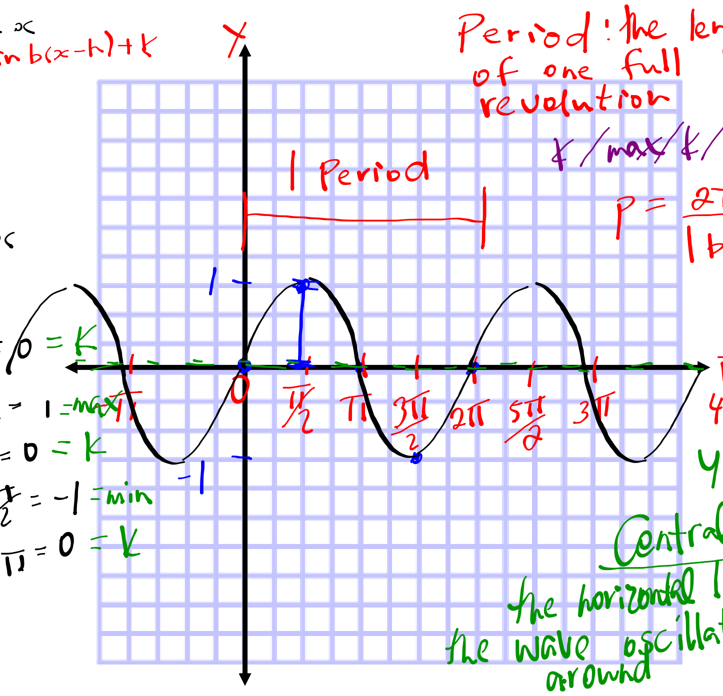
$A = |a|$

$A = 1$

Period: the length of one full revolution

k / max / k / min / k

$P = \frac{2\pi}{|b|}$



$y = 0$

$y = k$

Central Axis

the horizontal line the wave oscillates around

Info:

max = k + |a|

max = 0 + 1

max = 1

min = k − |a|

min = 0 − 1

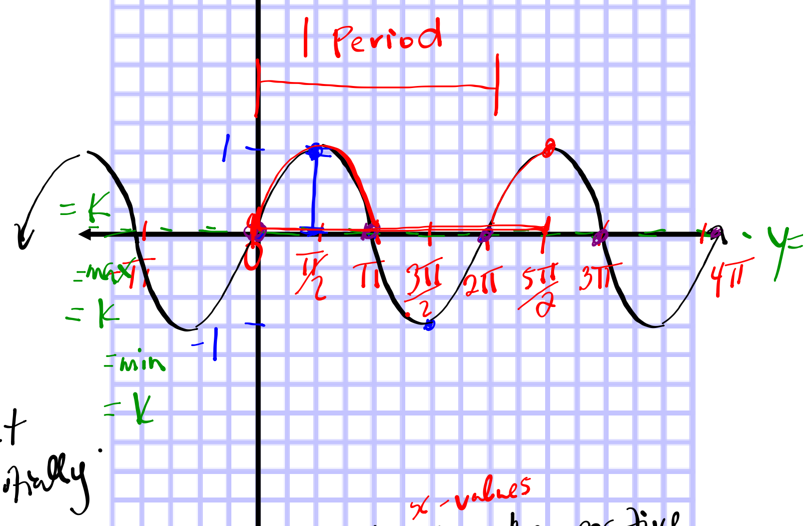
min = −1

(max/min are located at the halves of π initially.)

x-ints are located at the wholes of π initially.

Domain ℝ

Range [−1, 1]



Ex: Determine the positive interval of f(x) over the interval [0, 5π/2]

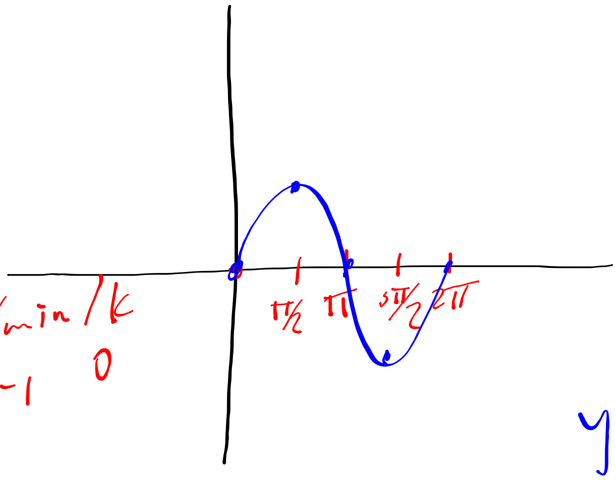
$[0, \pi] \cup [2\pi, \frac{5\pi}{2}]$

$y = \sin x$

$P = \frac{2\pi}{|b|}$

$P = 2\pi$

k	max	k	min	k
0	1	0	-1	0



$y = \sin x$

graph $g(x) = \cos x$

$g(x) = a\cos b(x-h)+k$

Period = max/k/min/k/max

$a = 1$

$b = 1$

$h = 0$

$k = 0$

$P = \frac{2\pi}{|b|}$

$P = 2\pi$

$A = |a|$

$A = 1$

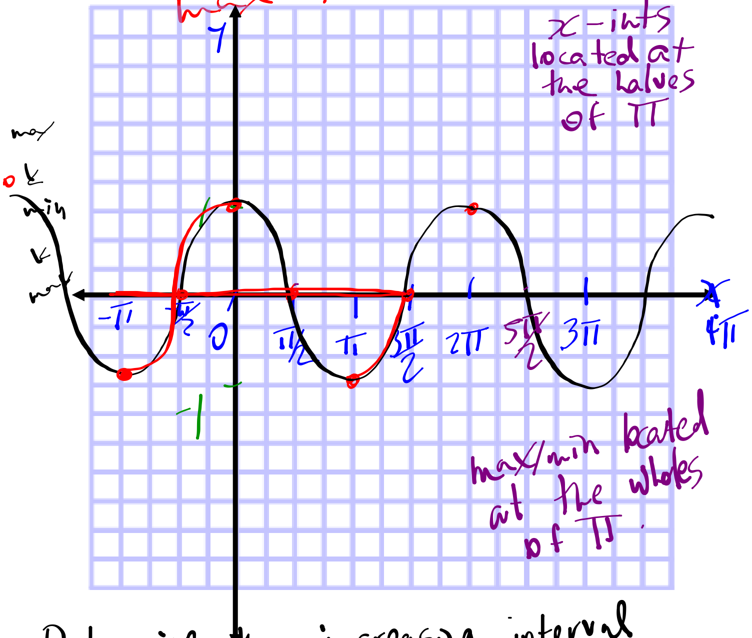
Central axis

$y = 0$

$y = \cos x$

x	y	
0	$\cos 0 = 1$	max
$\pi/2$	$\cos \pi/2 = 0$	k
π	-1	min
$3\pi/2$	0	k
2π	1	max

max $= k + |a|$



ex. Determine the increasing interval of $g(x)$ over the interval $\left[-\pi, \frac{3\pi}{2}\right]$. $[-\pi, 0) \cup \left[\pi, \frac{3\pi}{2}\right]$

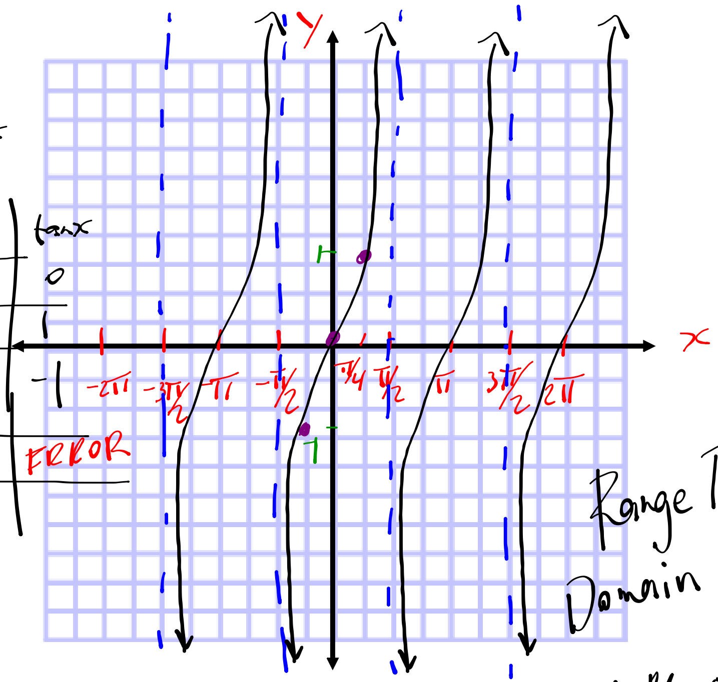
$h(x) = \tan x$

$h(x) = \tan x \rightarrow h(x) = \frac{\sin x}{\cos x}$

- There's asymptotes at the halves of π

x-ints located at $k\pi \quad k \in \mathbb{Z}$

x	$\sin x$	$\cos x$	$\tan x$
0	0	1	0
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$-\pi/4$	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1
$\pi/2$	1	0	ERROR



Range $\mathbb{R}$

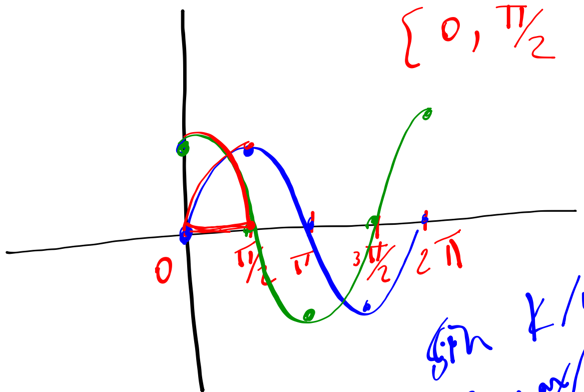
Domain $\mathbb{R} \setminus$ not the halves of π

ex Determine the negative interval of $f(x)$ over $[-\pi/2, \pi/2]$

$]-\pi/2, 0[$

ex. Determine the interval over which $f(x) = \sin x$ and $g(x) = \cos x$ are both positive for $[0, 2\pi]$.

$[0, \pi/2$



sin k / max / k / min / k

cos max / k / min / k / max