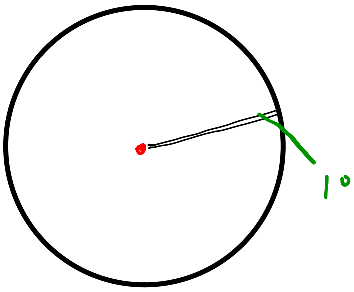
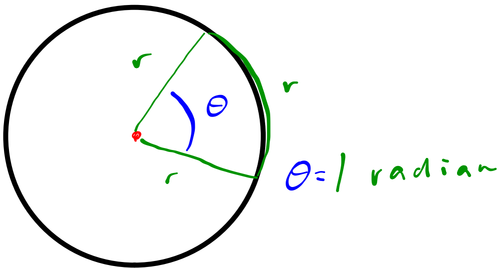
Unit 1: Converting Degrees into Radians and Vice Versa





360°	2π rad	6.28
180°	π rad	
90°	$\frac{\pi}{2}$ rad	
60°	$\frac{\pi}{3}$ rad	

Convert the following angle to radians.

$$\cancel{\pi}\left(\frac{\theta}{\cancel{\pi}}\right) = \left(\frac{57^\circ}{180^\circ}\right)\pi$$

step ①: Put ratio $\frac{180^\circ = \pi \text{ rad}}{}$ under equation. Degrees under degrees.

$$\theta = 0.99 \text{ rads.}$$

step ②: Solve for angle by isolating it. opposite operations

Convert to degrees

$$\cancel{180}\left(\frac{\alpha}{\cancel{180}}\right) = \left(\frac{2.17}{\pi}\right)^{180} \text{rads}$$

$$\alpha = 124.33^{\circ}$$

Nota Bene: One degree (1°) can be divided up into 60 minutes ($60'$) and one minutes ($1'$) can be divided up into 60 secs ($60''$)

ex

$\theta = 12^\circ 30'$

$\theta = 12.5^\circ$

ex

$\alpha = 12^\circ \underbrace{30' 30''}$

$\alpha = 12^\circ \underbrace{30.5'}$

$\alpha = 12.51^\circ$

ratio

$1^\circ = 60'$

$\frac{x}{1} = \frac{30.5'}{60'}$

Convert to radians

take angle and express it just in degrees before converting to radians

$\beta = 18^\circ 15' 30''$

$\beta = 18^\circ 15.5'$

$\beta = 18.26^\circ$

$\beta = 0.32$ rads

p 1.5
see calculator instruction
$\boxed{D^\circ m' s''} \rightarrow RAD$

$\frac{x}{1} = \frac{30''}{60''}$ $1' = 60''$

$x = 0.5'$

$\frac{y}{1} = \frac{15.5'}{60'}$ $1^\circ = 60'$

$y = 0.26^\circ$

$\cancel{\pi}\left(\frac{z}{\cancel{\pi}}\right) = \left(\frac{18.26^\circ}{180^\circ}\right)\pi$ $180^\circ = \pi$ rad

$z = 0.31$ rads

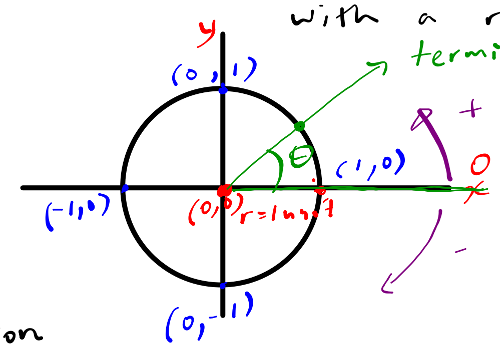
Convert to rads

$\theta = 20^\circ 30' 15''$

Unit 2: The Wrapping Function

$$P(\theta) = (\cos\theta, \sin\theta)$$

Definition: the unit circle : a circle centered at origin with a radius of one unit.



Definition:
The wrapping function tells us the coordinates of a point (x, y) on the unit circle that correspond to an given angle (θ)

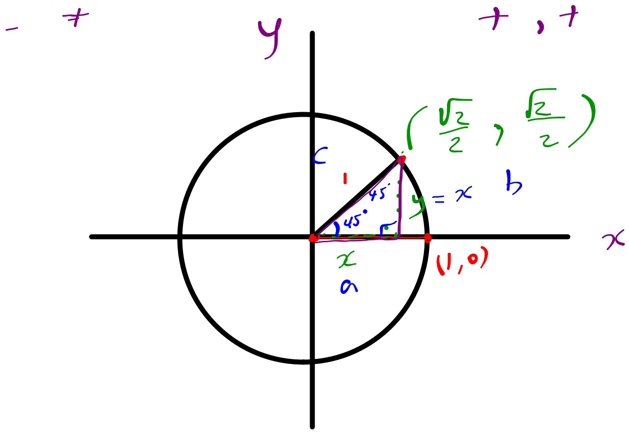
$P(90°) = (0, 1)$

$P(45°) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

0.7071

$P(225°)$ $P(0°)$

Evaluate $P(45°)$ under the wrapping function.



$$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

isosceles right triangle

$$c^2 = a^2 + b^2$$

$$1^2 = x^2 + x^2$$

$$\frac{1}{2} = \frac{\cancel{2}x^2}{\cancel{2}}$$

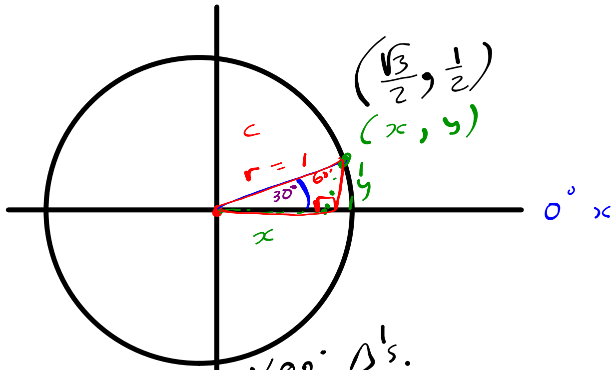
$$\sqrt{\frac{1}{2}} = \sqrt{x^{\cancel{2}}}$$

$$x = \frac{\sqrt{1}}{\sqrt{2}}$$

$$x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$x = \frac{\sqrt{2}}{2}$$

Evaluate $P(30°)$ under the wrapping function



for 30°/60°/90° Δ's.
the side opposite the 30° is one half the hypotenus.

$$P(30°) = \left(\underset{x}{\frac{\sqrt{3}}{2}}, \frac{1}{2}\right)$$

$$c^2 = a^2 + b^2$$

$$1^2 = x^2 + y^2 \; \cancel{- \left(\tfrac{1}{2}\right)^2}$$

$$1 \cancel{- \left(\tfrac{1}{2}\right)^2} = x^2 + \cancel{\left(\tfrac{1}{2}\right)^2}$$

$$4 \times \frac{1}{4} \; 1 - \frac{1}{4} = x^2 \Rightarrow \frac{4}{4} - \frac{1}{4} = x^2$$

$$\frac{4-1}{4} = x^2$$

$$\sqrt{\frac{3}{4}} = \sqrt{x^{\cancel{2}}}$$

$$x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\sqrt{3}}{\sqrt{4}}$$

$$x = \frac{\sqrt{3}}{2}$$

Evaluate $P(60^\circ)$ under the wrapping function (w/out using calculator nor formula sheet)

Evaluate for $P(\theta) = (\cos\theta, \sin\theta)$

$P(120^\circ) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = (\cos 120^\circ, \sin 120^\circ)$

$\frac{\sqrt{3}}{2} = 0.866$, $\cos 120^\circ = -0.5$, $\sin 120^\circ = 0.866$

no decimal

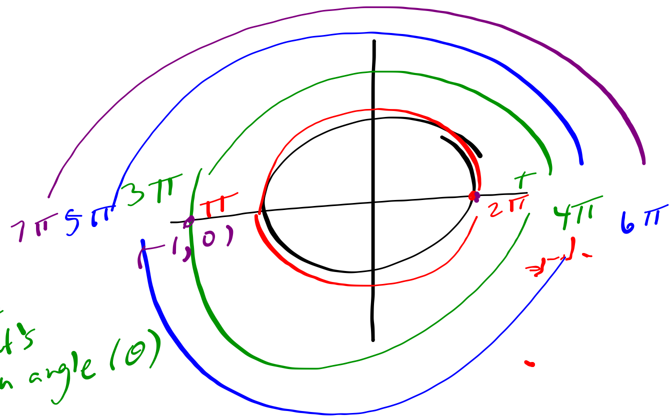
$P\left(\frac{3\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \left(\cos\left(\frac{3\pi}{4}\right), \sin\frac{3\pi}{4}\right)$ calculator in radian

$-\frac{\sqrt{2}}{2} = -0.7071$, $\cos\left(\frac{3\pi}{4}\right) = -0.7071$

$P(-30^\circ) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Evaluate $P(25\pi)$ under the wrapping function $P(\theta)$

step i. Since θ is "off" the unit circle, first find its corresponding reduced angle $\theta' \in [0, 2\pi[$



will have an even coefficient

$$\theta' = \theta - 2k\pi \quad , k \in \mathbb{Z}$$

subtract the highest multiple of 2π that's still lower than given angle (θ)

if $\theta = 25\pi$ find θ'

$$\theta' = 25\pi - 2k\pi$$
$$\theta' = 25\pi - 24\pi$$
$$\theta' = (25-24)\pi$$
$$\theta' = \pi$$

step ii: find $P(\theta')$

cuz $P(\theta) = P(\theta')$

$$P(\pi) = (-1, 0)$$

so $\therefore$ $P(25\pi) = (-1, 0)$

Nota Bene: Correspondings have the same trig point (x, y) under the wrapping function.

if θ is the angle

then $\theta' = \theta + \underbrace{2k\pi}$, where $k \in \mathbb{Z}$

↳ a mulitple of the period / one cycle / one revolution / 2π

corresponding angle

$\theta \neq \theta + 2\pi$

$\pi \neq \underset{3\pi}{\pi + 2\pi} \neq \underset{5\pi}{\pi + 2(2\pi)} \neq \underset{7\pi}{\pi + 3(2\pi)}$

$P(\theta) = P(\theta + 2\pi) = P(\theta + 4\pi) = P(\theta + 6\pi) = \dots$

$P(\theta + 2k\pi)$

$k \in \mathbb{Z}$

$P(\pi) = P(3\pi) = P(5\pi) = P(7\pi) = (-1, 0)$

Evaluate $\frac{37\pi}{3}$ under the wrapping function
θ

i. find θ'
ii. find $P(\theta')$

$\theta' = \theta - 2k\pi \quad , k \in Z$

$\theta' = \frac{37\pi}{3} - 2k\pi$

$\theta' = 12.33\pi - 2k\pi$

$\theta' = \frac{37}{3}\pi - 12\pi$

$\theta' = \left(\frac{37}{3} - \frac{12 \cdot 3}{1 \cdot 3}\right)\pi$

$\theta' = \left(\frac{37}{3} - \frac{36}{3}\right)\pi$

$\theta' = \frac{1}{3}\pi$

$P\left(\frac{1}{3}\pi\right) =$

$P\left(\frac{\pi}{3}\right) = \boxed{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$

so $\therefore$ $P\left(\frac{37\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Evaluate $\frac{-29\pi}{4}$ under the WF

θ

Same steps!!

$\theta' = \theta - 2k\pi$

$\theta' = -\frac{29\pi}{4} - 2k\pi$

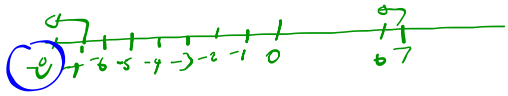
$\theta' = -7.25\pi - 2k\pi$

$\theta' = -\frac{29\pi}{4} - 2(-4)\pi$

$\theta' = -\frac{29\pi}{4} + 8\pi$

$\theta' = \left(-\frac{29}{4} + \frac{8\cdot 4}{4}\right)\pi$

$\theta' = \frac{3\pi}{4}$

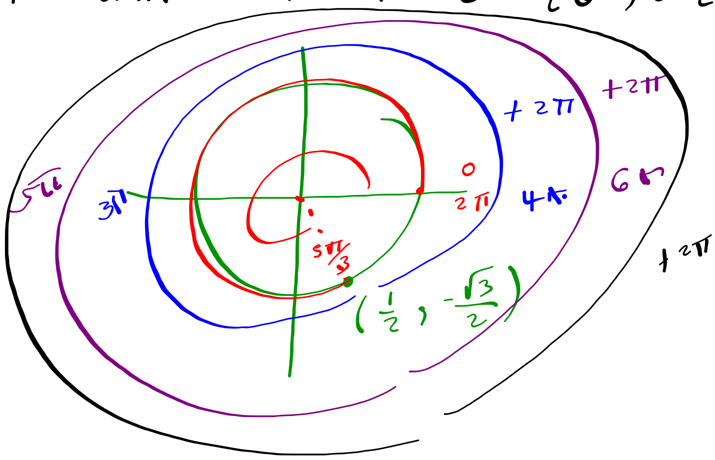


$k = -4$

$P(\theta') =$

$P\left(\frac{3\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

Find θ corresponding to trig point $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ on the unit circle if $\theta \in [6\pi, 8\pi[$



step i. Find θ' corresponding to point $\theta' \in [0, 2\pi[$

$$\theta' = \frac{5\pi}{3}$$

step ii. Find θ by adding $k2\pi$ so that $\theta \in [6\pi, 8\pi[$ (given interval)

$$\theta = \theta' + 2k\pi$$

$$\theta = \frac{5\pi}{3} + 6\pi$$

$$\theta = \left(\frac{5}{3} + \frac{3 \cdot 6}{3}\right)\pi$$

$$\theta = \frac{23}{3}\pi \overset{?}{\in} [6\pi, \overset{10\pi}{8\pi}[$$

verify

yes since $\frac{23}{3} = 7.667$

$$\boxed{\text{ANS } \frac{23\pi}{3}}$$

step iii. verify θ belongs to the interval and that there's no more multiple of θ that belong.

$$\theta'' = \theta + 2\pi \overset{?}{\in} [6\pi, 8\pi[$$

$$\theta'' = \left(\frac{23}{3} + \frac{6}{3}\right)\pi$$

$$\theta'' = \left(\frac{29}{3}\right)\pi \notin [6\pi, 8\pi[$$

9.667

HMWK
P 2.39
2 a) - j)

HMWK
P 2.41
3 a) - j)