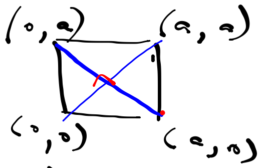
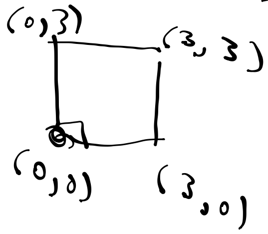
Intro Proofs

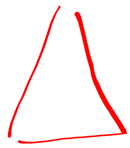
→ shows





Unit 10: Congruent
Similar

Proof

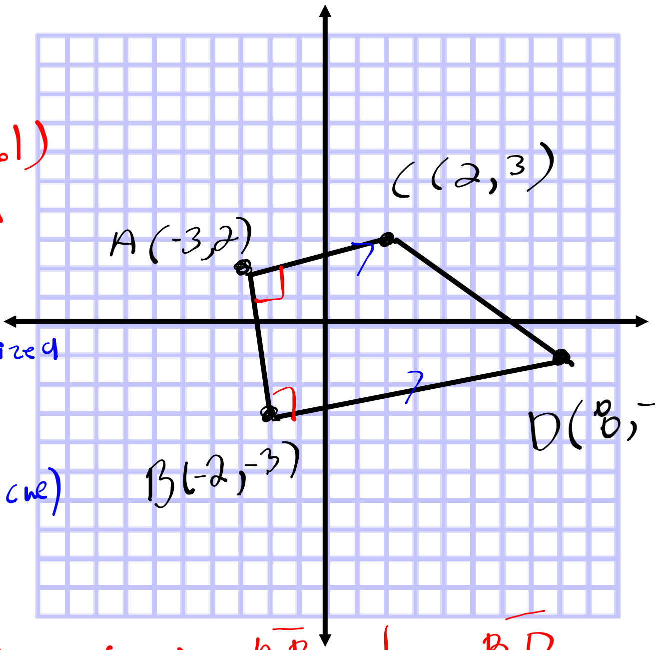


Unit 9: Intro to Proofs (Shows)

A quadrilateral is formed by joining the following points:

Using the formulas and 2) (choose the right tool) principles related to the slope formula, distance formula slopes and measures of line segments, show that this quadrilateral is a right trapezoid. → Note: you must have memorized 1) the properties of each shape. (use the diagram as a visual cue)

90°



Show $\overline{AC} \parallel \overline{BD}$

$A(\overset{x_1}{-3},\overset{y_1}{2})$
$C(\overset{x_2}{2},\overset{y_2}{3})$

$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{AC} = \frac{3-2}{2-(-3)}$

$m_{AC} = \frac{1}{5} = 0.2\overline{2}$

$B(\overset{x_1}{-2},\overset{y_1}{-3})$
$D(\overset{x_2}{8},\overset{y_2}{-1})$

$m_{BD} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{BD} = \frac{-1-(-3)}{8-(-2)}$

$m_{BD} = \frac{2}{10} = 0.2\overline{2}$

$m_{BD} = \frac{1}{5}$

∴ $\overline{AC}$ is indeed $\parallel$ to $\overline{BD}$

Show $\overline{AB} \perp \overline{BD}$

$m_{BD} = \frac{1}{5}$

$A(\overset{x_1}{-3},\overset{y_1}{2})$
$B(\overset{x_2}{-2},\overset{y_2}{-3})$

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{AB} = \frac{-3-2}{-2-(-3)}$

$m_{AB} = \frac{-5}{1}$

∴ Since the slopes are negative reciprocals $\overline{BD}$ is indeed $\perp$ to $\overline{AB}$

∴ ABCD is indeed a right trapezoid

How to show:

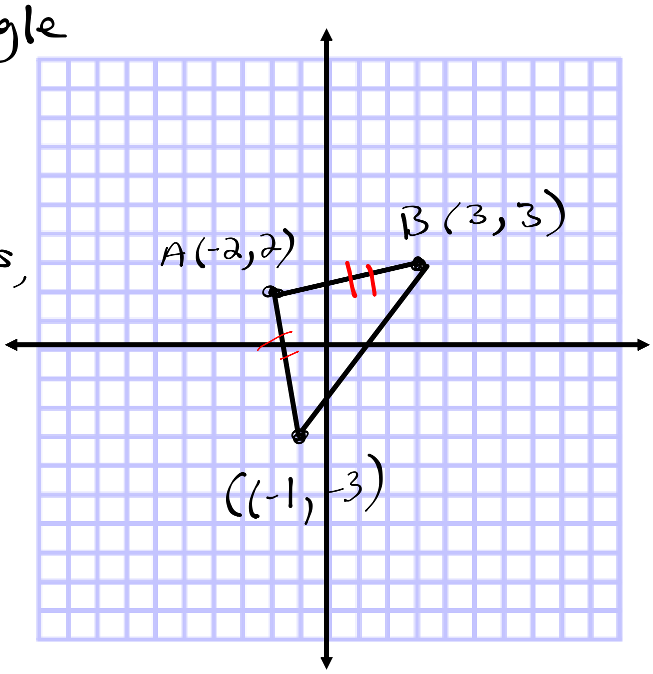
two lines are parallel: Calculate the slopes of each line independantly and verify that they are equal.

two lines are perpendicular: Calculate the slopes of each line independantly and verify that they are negative recipercals (ex. $\frac{2}{3}$ and $\frac{-3}{2}$)

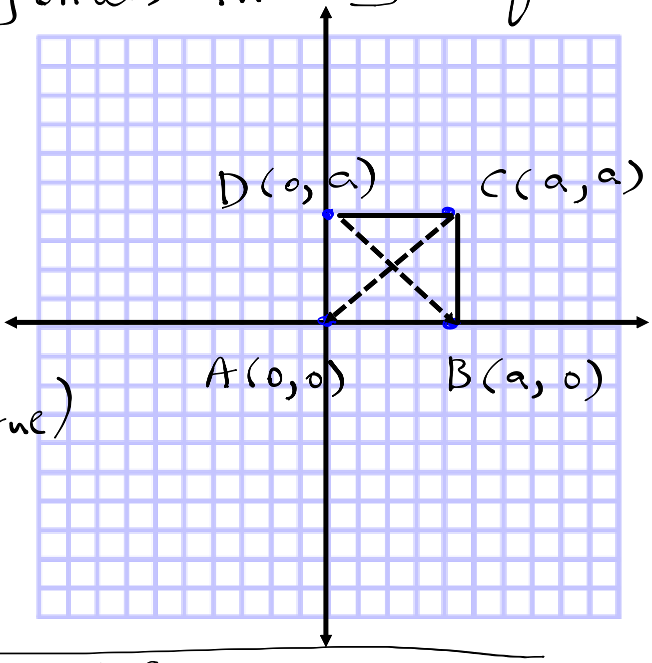
two lines are congruent (same measure/length/distance)
Calculate the distance (w/ distance formula) of each independantly and verify that they're equal.

Right isosceles triangle

Using the formulas related to slopes and measure of line segments, show ABC is a right isosceles triangle.



Prove that the diagonals in any square are perpendicular.



Hypothesis (What you know is true)

ABCD is square

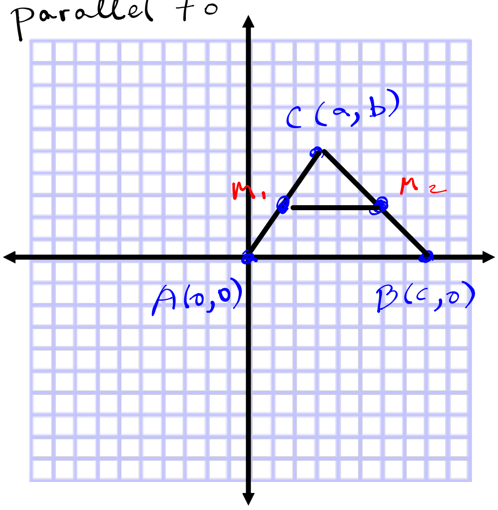
A(0, 0)
B(a, 0)
C(a, a)
D(0, a)

Conclusion: (What you must prove to be true)

Prove $\overline{AC}$ is $\perp$ $\overline{BD}$

Statement (where you show you're work)	Justification
$A(\overset{x_1}{0}, \overset{y_1}{0})$ $C(\overset{x_2}{a}, \overset{y_2}{a})$	
$m_{AC} = \dfrac{y_2 - y_1}{x_2 - x_1}$	slope formula
$m_{AC} = \dfrac{a - 0}{a - 0}$	
$m_{AC} = \dfrac{a}{a} = 1$	
$B(\overset{x_1}{a}, \overset{y_1}{0})$ $D(\overset{x_2}{0}, \overset{y_2}{a})$	
$m_{BD} = \dfrac{y_2 - y_1}{x_2 - x_1}$	
$m_{BD} = \dfrac{a - 0}{0 - a}$	Slope formula
$m_{BD} = \dfrac{a}{-a}$	
$m_{BD} = -1$	
$\therefore \overline{AC} \perp \overline{BD}$	$\perp$ lines have negative reciprocal slope (which is the case here)

Prove that the segment joining the midpoints of two sides of a triangle is parallel to the third side.



Hypothesis:

- M_1 and M_2 are midpoints
- ABC is a triangle

Conclusion to be proved:

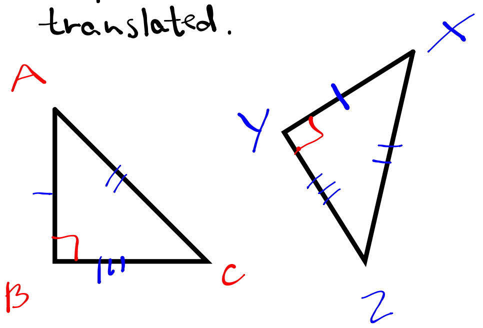
Segment $\overline{M_1M_2}$ is parallel to $\overline{AB}$

Statements	Justification
The coordinates of M_1 $\left(\frac{a}{2}, \frac{b}{2}\right)$	Midpoint formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
The coordinates of M_2 $\left(\frac{a+c}{2}, \frac{b}{2}\right)$	" "

Unit 10 : Characteristics of Congruent and Similar Shapes 2D / Figures 3D

(Triangles)

Congruent Shapes: They are the same shape just rotated or translated.



→ their corresponding sides (the longest sides of each triangle) are congruent (same distance)

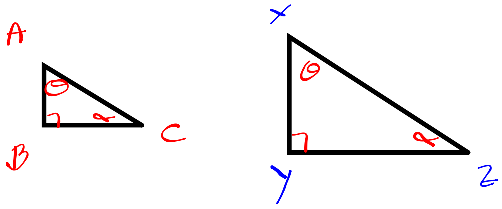
→ their corresponding angles are congruent (angle has same measure)

→ same perimeter

→ same area

→ same volume (3D)

Similar Shapes: Porportionally the same shape, but one's smaller



Th 50 b) → their corresponding angles are congruent (angles have same measu.)

$\frac{90}{90} = 1$

Th 50 a) $\frac{AC}{XZ} = \frac{BC}{YZ} = \frac{AB}{XY} = K$

→ the ratio of the corresponding side are all equal. They equal k - the ratio similitude
→ or - the scale factor

→ the ratio of the perimeter is equal to K

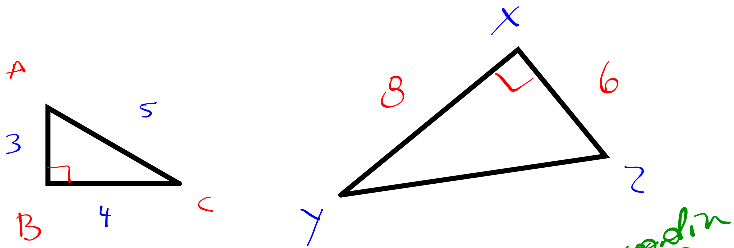
$\frac{S_{small} + S_{small} + S_{small}}{S_{big} + S_{big} + S_{big}} = K = \frac{P_1}{P_2}$

Th 50 c) → the ratio of their areas is equal to K^2

$\frac{A_1}{A_2} = K^2$

Th 50 d) → the ratio of their volumes is equal K^3

$\frac{V_1}{V_2} = K^3$



$k < 1$

$\frac{4}{8} = \frac{1}{2}$ $\frac{3}{6} = \frac{1}{2} = k$ corresponding sides $\frac{S_{small}}{S_{Big}} = k$

$k > 1$

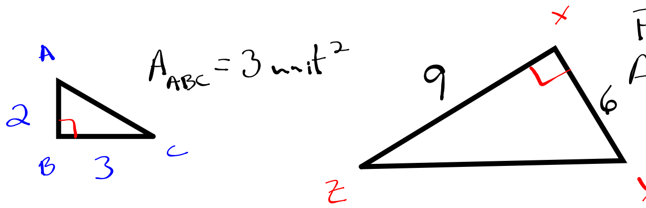
$\frac{S_{Big}}{S_{small}} = k$

$\frac{AC}{YZ} = k$

$\frac{5}{YZ} = \frac{1}{2}$

$YZ = 10$

$\triangle ABC \sim \triangle XYZ$



$A_{ABC} = 3\ unit^2$

Find $A_{XYZ} =$

* k is determined by constructing a (one) ratio of corresponding side

$\frac{2}{6} = \frac{1}{3} = k$ | $\frac{6}{2} = 3 = k$

$$\frac{A_1}{A_2} = k^2$$

$$\frac{A_{XYZ}}{A_{ABC}} = k^2$$

$$\frac{A_{XYZ}}{3} = 3^2$$

$$\cancel{3}\cdot\left(\frac{A_{XYZ}}{\cancel{3}}\right) = 9 \cdot 3$$

$$A_{XYZ} = 27\ unit^2$$

The two triangles are similar.
The ratio of the areas is equal to 4.
What's the perimeter of the 2nd triangle? (△XYZ)

$$\frac{S}{S} = k$$

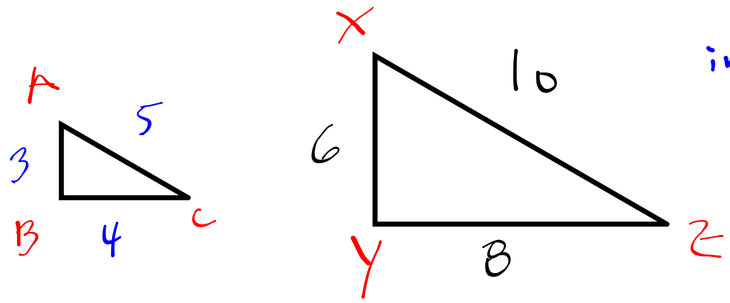
$$\frac{A}{A} = k^2$$

Translate sentences into equations.

$$\frac{A}{A} = 4$$

$$\sqrt{4} = \sqrt{k^{\cancel{2}}}$$

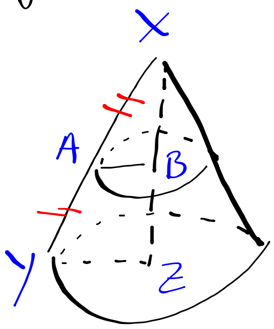
$$k = 2$$



$P_{ABC} = 12$ units

$P_{XYZ} = 24$ units

Shannon bought two birthday hats for herself (XZY) and her baby (XAB). The smaller hat can fit into the larger as the diagram shows.



similar

$\frac{S}{S} = K$

$\frac{A}{A} = K^2$

$\frac{V}{V} = K^3$

The volume of the small cone is 70 unit³.
If you know A is the midpoint of $\overline{XY}$, what's the volume of the big hat?

$XA = AY$

$\frac{S_{Big}}{Small} = K$

$\frac{XA + AY}{XA} = K$

$\frac{XA + XA}{XA} = K$

$\frac{2XA}{XA} = K$

$2 = K$