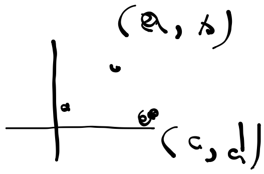
Proofs :

→ slope formula

→ distance

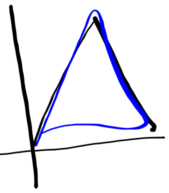
→ midpoint



1)

→ proprieties of congruent triangles

→ proprieties similar triangles



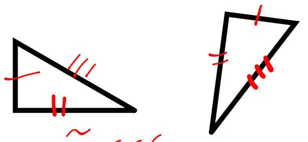
2)

Unit 13: Proofs involving congruent or similar triangles

Nota Bene: For proof questions with triangles without coordinates, you must first prove the triangles are congruent (or similar) using only the information you're sure.

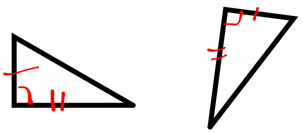
You know 2 triangles are congruent if:

Th 15:



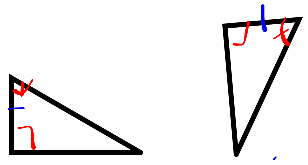
if SSS ≅ SSS
∴ Δ ≅ Δ

Th16



if SAS ≅ SAS
∴ Δ ≅ Δ

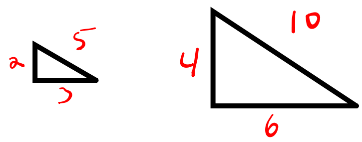
Th17



if ASA ≅ ASA
then Δ ≅ Δ

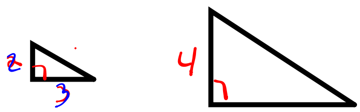
You know 2 triangles are similar if:

Th 19



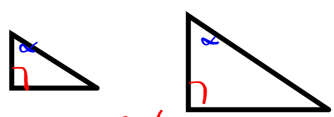
$$\frac{S}{S} = \frac{S}{S} = \frac{S}{S} = 2$$

Th 20



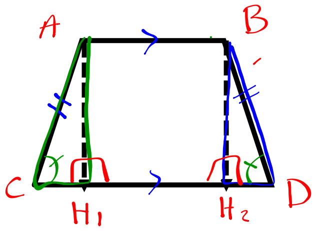
$\frac{S}{S} = \frac{S}{S}$ A ≅ A $\frac{S}{S}$ A $\frac{S}{S}$

Th18



90 α   90 α
AA ≅ AA then the Δ's are similar.

Prove that in any isosceles trapezoid the altitudes/heights are congruent.



Hypothesis:
(the information you know is true)

$\overline{AB} \parallel \overline{CD}$

$m\angle C \cong m\angle D$

$m\,\overline{AC} \cong m\,\overline{BD}$

Conclusion to be proved: $m\overline{AH_1} \cong m\overline{AH_2}$

~~SSS~~
SAS
ASA

Pick one tool based on the info you know.

Statements	Justification
A $m\angle C \cong m\angle D = \alpha$ (measure) (angle) congruent	hypo (property of an isosceles trapezoid)
S $m\,\overline{AC} \cong m\,\overline{BD}$	hypo. "
$m\angle CH_1A = 90°$	property of a height
$m\angle DH_2B = 90°$	" " "
$m\angle CAH_1 = 180 - 90 - \alpha$	180° in a triangle
$m\angle DBH_2 = 180 - 90 - \alpha$	" " " "
A $m\angle CAH_1 = m\angle DBH_2$	(they equal $180-90-\alpha$
$\triangle ACH_1 \cong \triangle BDH_2$ green tri blue tri	ASA $\cong$ ASA
$\therefore m\overline{AH_1} \cong m\overline{BH_2}$	corresponding sides are congruent in congruent triangles

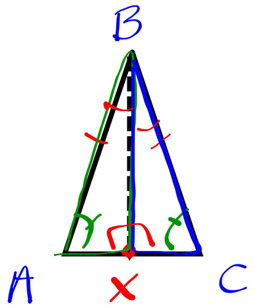
$\blacksquare$ Q.E.D

Prove that in any isosceles Δ the altitude from the vertex of the congruents sides is also the median.

Hypothesis:

$m\angle A = m\angle C$

$m\overline{AB} = m\overline{BC}$

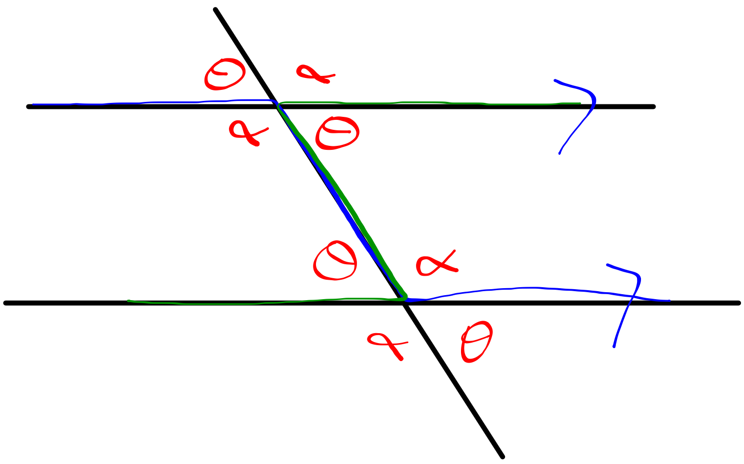


SSS
SAS
ASA

Conclusion: $m\overline{AX} \cong m\overline{XC}$

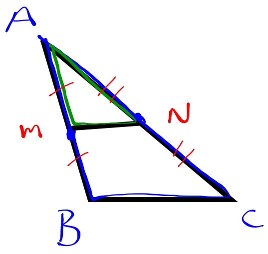
Statements	Justification
S $m\overline{AB} \cong m\overline{BC}$	hypo
$m\angle A = m\angle C = \alpha$	hypo
$m\angle AXB = m\angle BXC = 90$	height
A $m\angle ABX = m\angle XBC = 180 - 90 - \alpha$	180° in a Δ
S $m\overline{BX} = m\overline{BX}$	- shared side
$\Delta AXB \cong \Delta BXC$	SAS
$mAX \cong mXC$ ☒	congruent sides in congruent Δs

Recall :



alternate interior

Prove that in any triangle, the line segment joining the midpoints of two sides creates a similar triangle.



Hypothesis:

M and N are midpoints

$m\overline{AM} = m\overline{MB}$

$m\overline{AN} = m\overline{NC}$

Conclusion: $\triangle ABC \sim \triangle AMN$

do you have to i-bout

$\frac{S}{S} = \frac{S}{S} = \frac{S}{S}$

$\frac{S}{S} A \frac{S}{S}$

~~AA~~

Statements	Justification
A $m\angle BAC = m\angle MAN$	Shared angle
$\frac{AB}{AM} = \frac{AM+MB}{AM}$ $= \frac{AM+AM}{AM}$ $= \frac{2\cancel{AM}}{\cancel{AM}}$ $= 2$	hypo (subbed in what $\overline{mB}$ is equal to)
$\frac{AC}{AN} = \frac{AN+NC}{AN}$ $= \frac{AN+AN}{AN}$ $= \frac{2\cancel{AN}}{\cancel{AN}} = 2$	hypo
$\frac{S}{S}$ $\frac{S}{S}$. $\frac{AC}{AN} = \frac{AB}{AM} = 2$	$\frac{S}{S} A \frac{S}{S}$

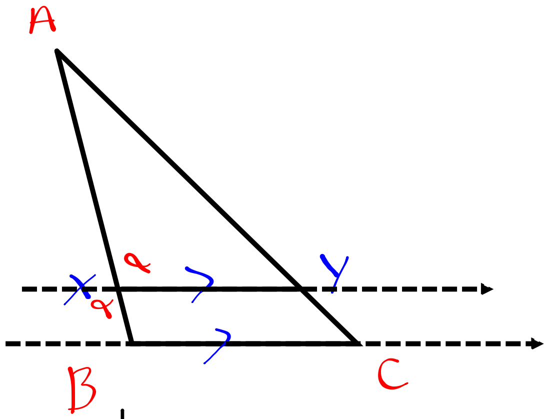
$\therefore \triangle ABC \sim \triangle ANM$

Tip for proofs:

- first construct the ratios of coresponding and then try to work w̄ the ratios

(rewrite)
- talk about a whole as a sum of its parts.

Prove that a line segment drawn parallel to a side in a triangle creates a similar triangle.



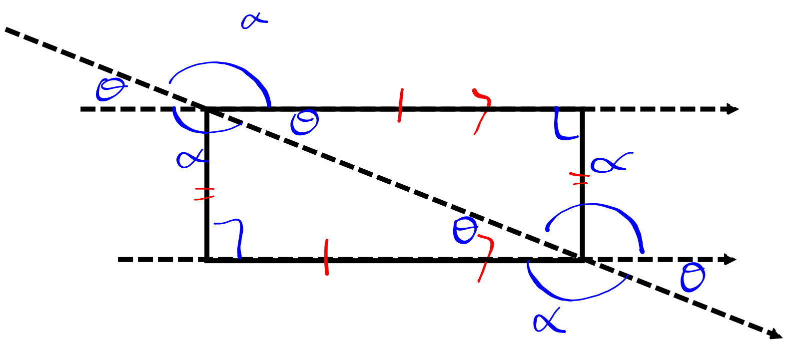
Hypothesis

$\overline{XY} \parallel \overline{BC}$

Conclusion: $\triangle ABC \sim AXY$

Statements	Justification.

tip : extend your parallel
so you can
visualize 'Z' pattern
better.



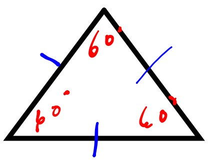
Definition : Regular Polygon

↳ a polygon whose sides and angles are all equal.

e.x.

$\frac{180^\circ}{3 \text{ sides}}$

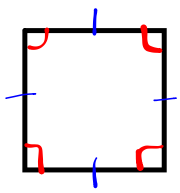
$= 60^\circ$



e.x.

$\frac{360}{4 \text{ sides}}$

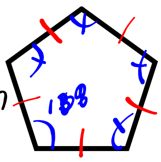
$= 90^\circ$



e.x.

$\frac{540^\circ}{5 \text{ sides}}$

$= 108^\circ$



→ To calculate the area of weriods shapes, break them up in triangles.

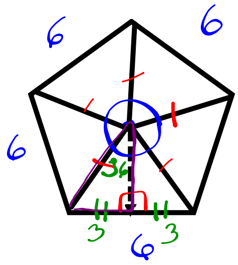
SOH CAH TOA

A regular hexagon has side length of 6. Calculate it's area.

(the circle)

$\frac{360^\circ}{5 \text{ angles}} = \frac{72^\circ}{2} = 36$

SOH CAH TOA

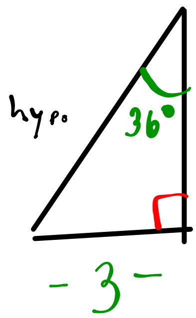


Note:

- Any line segment from a vertex to the centre of a polygon is a radii.

The radii are all equal to each other.

$\tan\theta = \frac{opp}{adj}$



$\tan 36^\circ = \frac{3}{h}$

$h \tan 36^\circ = 3$

$h = \frac{3}{\tan 36^\circ}$

$h = 4.13$ units

$A_{Isosceles} = \frac{b \cdot h}{2} = \frac{6 \cdot 4.13}{2}$

$A_{Isosceles} = 12.39 \text{ units}^2$

$A_{hexagon} = 5 \cdot A_{Isosceles}$

$A_{hex} = 5 \cdot 12.39$

$A_{hex} = 61.94 \text{ units}^2$