Unit 8: Functions

- a function is a relation with one more condition!

Recall $R = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = 2x + 1\}$

↳ is a function

$R = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid x^2 + y^2 = 9\}$

↳ not a function!

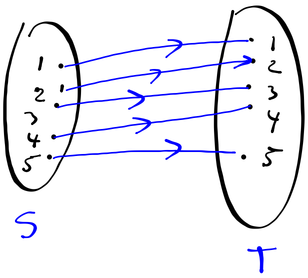
N.B. A function is a relation in which each element of the source set corresponds to (is paired with) only <u>one</u> element of the target set.

NOT COOL  FINE It's cool 

e.x (3, 2) (1, 2)

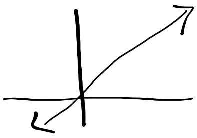
NOT A FUNCTION → (3, 5) (4, 2) Sure, a FUNCTION

Relation

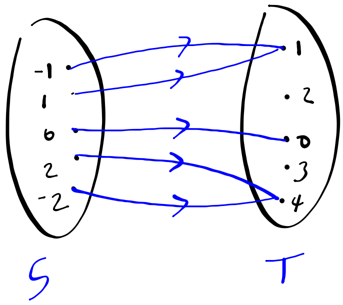


Function? YES

Rule: $y = x$

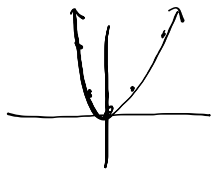


Relation

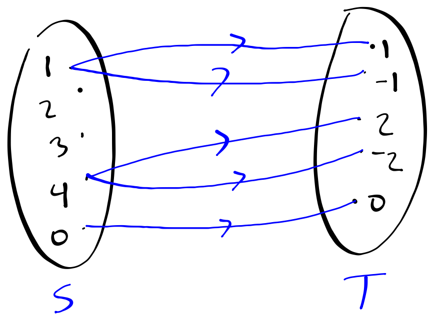


Function? YES

Rule: $y = x^2$



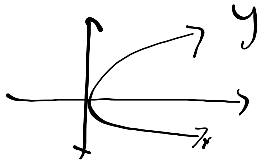
Relation



Function? NO

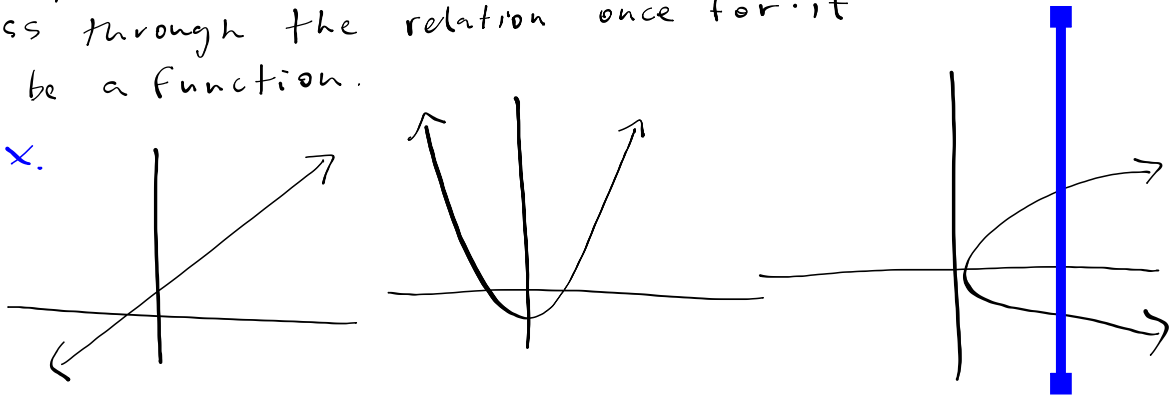
Rule: $y = \pm\sqrt{x}$

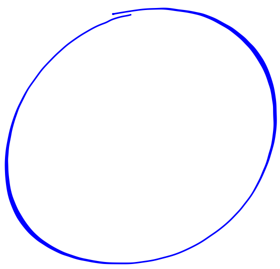
$y^2 = x$



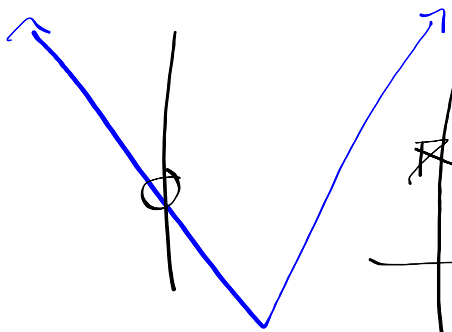
Use the Vertical Line Test to quickly determine whether Relation is a Function. VLT: No matter where you put a vertical line, it can only pass through the relation once for it to be a function.

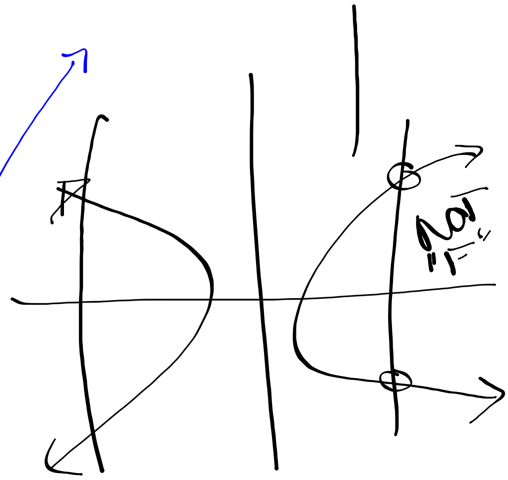
e.x.





$x^2 + y^2 = r^2$





Ways to Represent Functions

Set Builder's Notation

$$f = \{ (x,y) \in \mathbb{R} \times \mathbb{R} \mid y = 2x - 4 \}$$

→ only used when y is completed isolated!

Functional Notation

source set

target set

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$

$$x \longrightarrow 2x - 4$$

$= y$

Write the following function in Functional Notation

$$f = \{ (x, y) \in \mathbb{R} \times \mathbb{R} \mid y = x^2 + 2 \}$$

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$

$$x \longrightarrow x^2 + 2$$

For functions, you can rewrite the equation:

$$y = 2x - 4$$

equivalent!

$$f(x) = 2x - 4$$

$x = 0$
$y = -4$

$f(0) = -4$

"f at x"

the y value when x is defined.

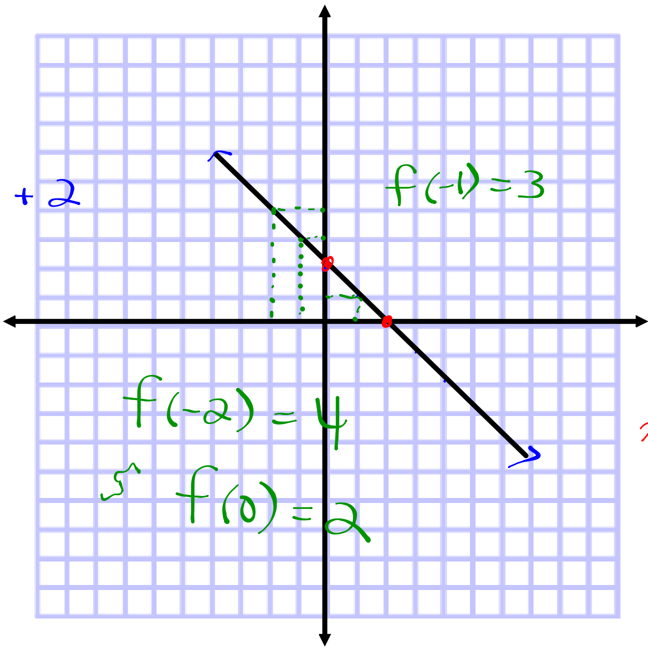
ex. $f(2)$

f at 2

The value of y when $x = 2$.

e.x.

$f(x) = -x + 2$



Find x, y

$f(1) = 1$ $(1,1)$

$f(-1) = 3$ $(-1,3)$

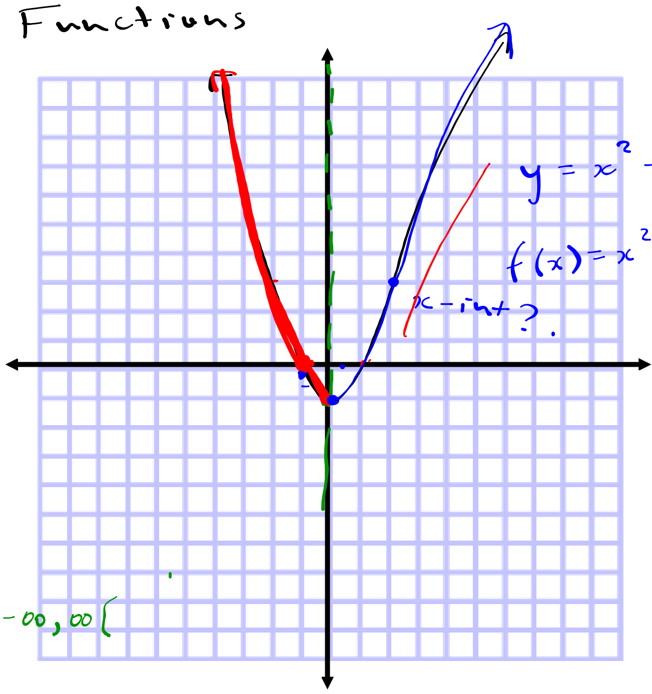
y-int $f(0) = 2$ $(0,2)$

x-int $f(2) = 0$ $(2,0)$

"

$f(0) = $ y-int

Unit 9 : Characteristics of Functions

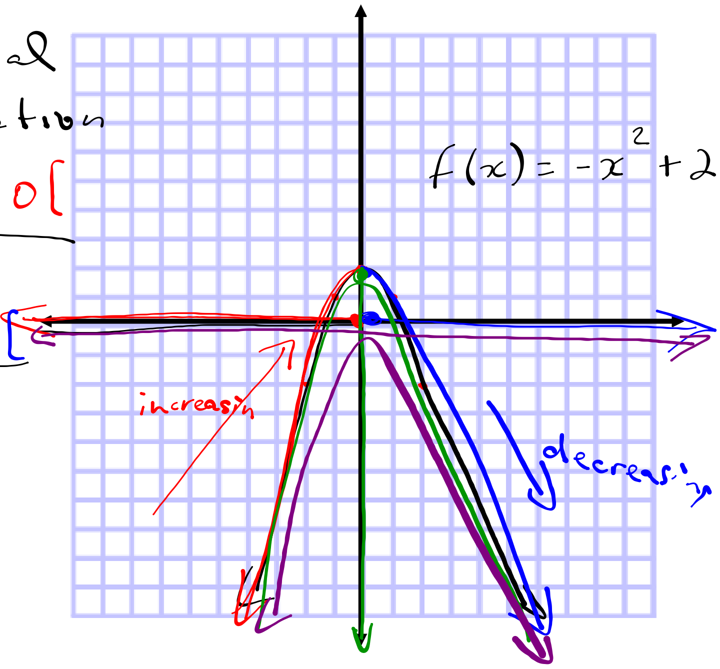


Recall :

- y-int : the point where the function intercepts the y-axis. $f(0)$ $(0, y)$
- Zeros / x-int(s) / $f(x)=0$ / solutions roots
  the point where the function intercepts the x-axis.
  $(1, 0)$ $f(1)=0$
  $(-1, 0)$ $f(-1)=0$
- max/min point / vertex
- axis of symmetry $x=4$
- the domain of a function: ex $]-\infty, \infty[$
  ↳ an interval $[\ ,\ ]$ that is a subset of source set.
  The interval of x-values the function uses.
- the range of a function: $[-1, \infty[$
  ↳ subset of the target set.
  4 interval of y-values the function uses.
- the increasing interval of a function: $]0, \infty[$
  ↳ in terms of x-values.
  ↳ when reading the graph left to right, the graph is going up. As x increases, y also increases.
- the decreasing interval of a function $]-\infty, 0[$
  ↳ in terms of x-values
  ↳ When reading the graph left to right, the graph is going down. As x increases, y decrease.

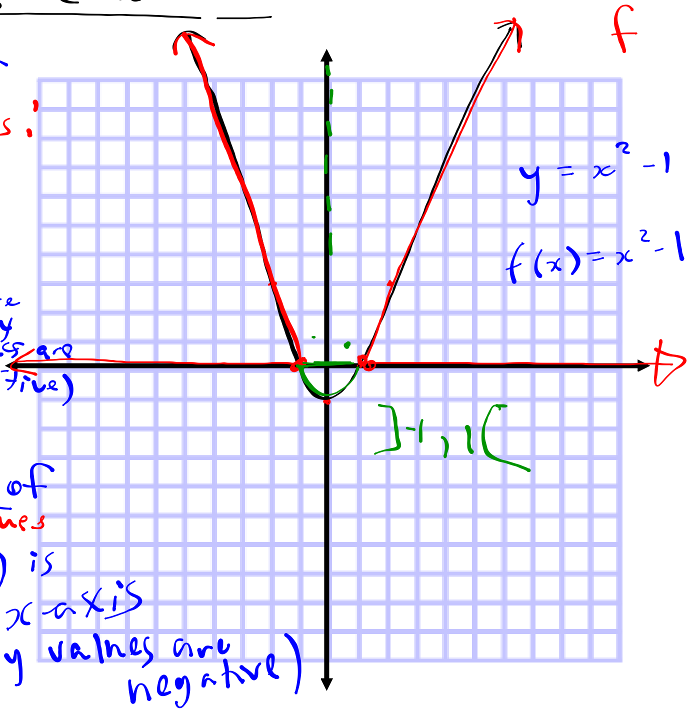
Determine the interval over which this function

- is increasing:]-∞, 0[
- is decreasing]0, ∞[
- Range:]-∞, 2]
- Domain:]-∞, ∞[
- f(0) : 2
- f(-1) : 1



Characteristics Continued

- positive interval of a function ↳ x-values;
  ↳ the function (f/y) is positive when it's above the x-axis (where the y values are positive)
  ex. $]-\infty, -1[\cup]1, \infty[$
- negative interval of a function ↳ x-values
  ↳ the function (f/y) is negative below the x-axis (where the y values are negative)



Power Function

$f(0) = 3$

x-ints: $(-5, 0)$
$(1, 0)$
$(5, 0)$

Range: $]-\infty, \infty[$

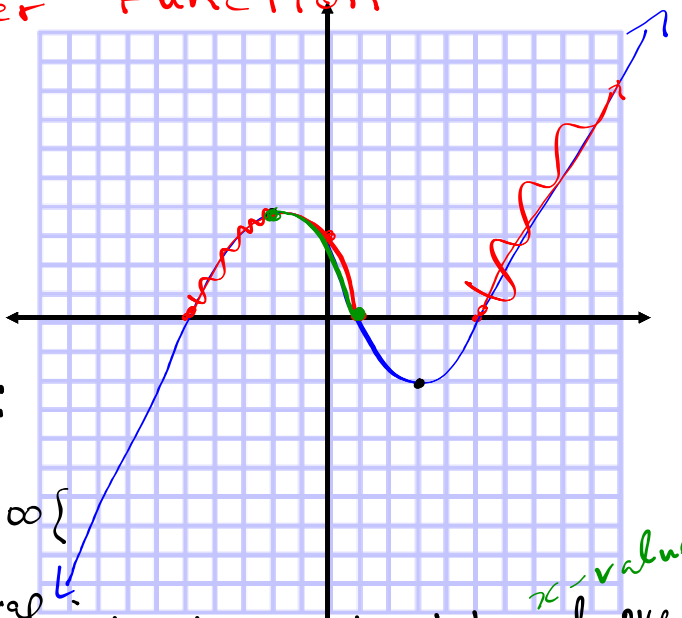
Domain: $]-\infty, \infty[$

Positive interval:

$]-5, 1[\cup]5, \infty[$

Decreasing interval:

$]-2, 3[$



Determine the interval over which the function is positive and decreasing.

$]-2, 1[$

Absolute Value Function

min : (0, −1)

negative interval:
]−1 , 1[

range:
[−1 , ∞[

increasing interval:
]0, ∞[

Determine the interval over which the function is negative and increasing