

$$A \setminus B = \{1, 2, 3\}$$

$$A \cup B = \{1, 2, 3, 4\}$$

Set

$$A : \{1, 2, 3\}$$

$$B : \{4\}$$

$$A \cap B = \{\}$$

A'

$$A = \{x \in \mathbb{N} \mid 0 < x \leq 4\}$$

$$A \cap B = \{\}$$

• listing

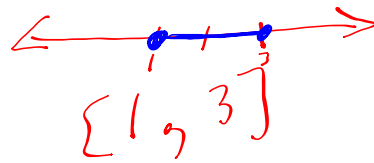
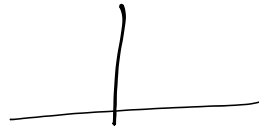
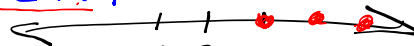
• Set builder not.

• venn diagrams

• graphing

• interval notation

$$A' = \{x \in \mathbb{N} \mid x \geq 5 \text{ and } x = 0\}$$

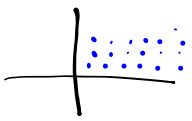


\mathbb{R}

Unit 5: Cartesian Product

↳ the product of two sets
 ↳ a set of ordered pairs (x, y)

(Cartesian Plane)



e.x.

$$A \times B = \{ (x, y) \in A \times B \} \quad | \quad \begin{array}{l} x \in A \\ \text{and } y \in B \end{array}$$

ordered pair
Source set
target set

$$A = \{ 0, 1, 2 \}$$

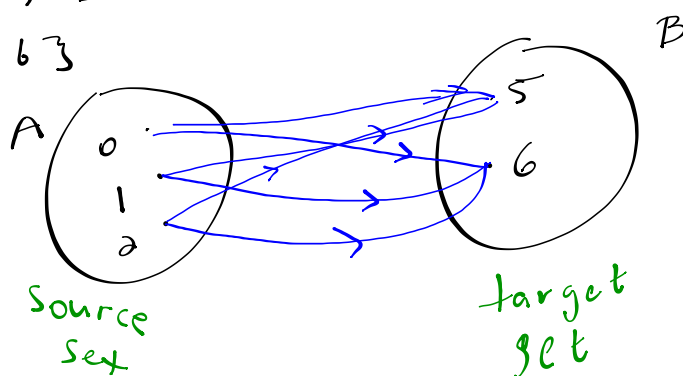
$$B = \{ 5, 6 \}$$

$$A \times B = \{ (0, 5), (0, 6), (1, 5), (1, 6), (2, 6), (2, 5) \}$$

Arrow Diagrams: a graphical representation of the Cartesian Product

$$A = \{0, 1, 2\}$$

$$B = \{5, 6\}$$



$\{0, 5\}$

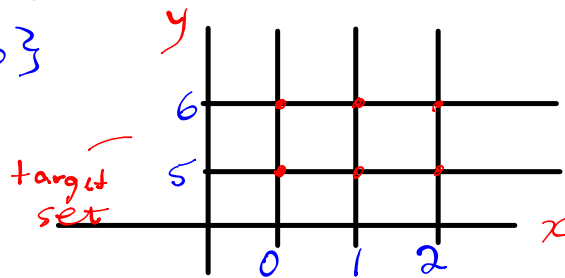
Cartesian Graph

↳ a more practical graphical way to represent the cartesian product

$$A = \{0, 1, 2\}$$

$$B = \{5, 6\}$$

$A \times B$

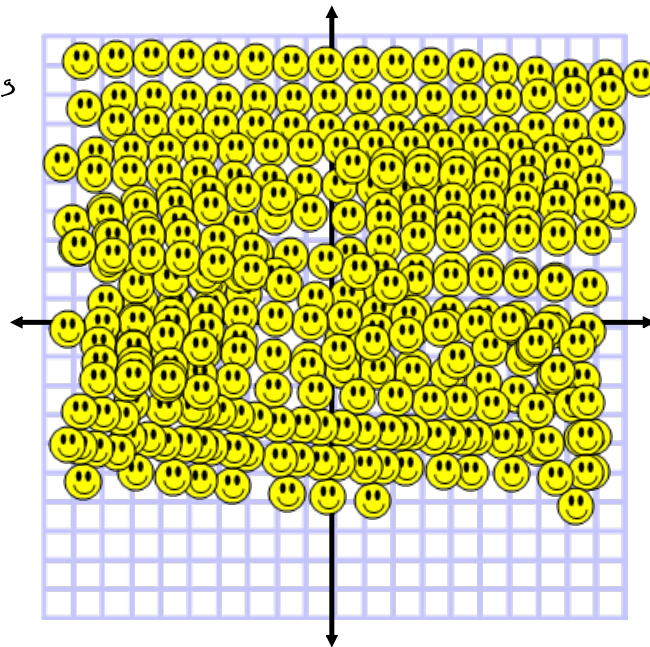


$0, 5$

↳ source set

$\mathbb{R} \times \mathbb{R}$ \rightarrow the cartesian product
most often used

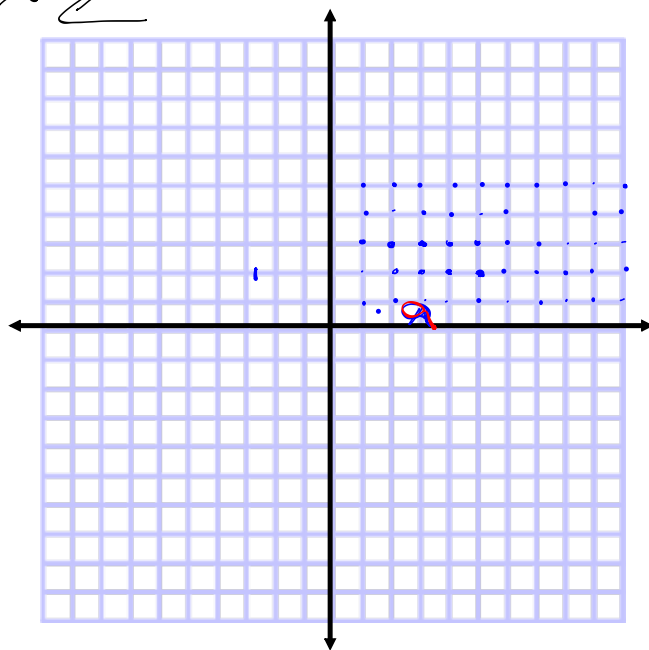
$$\mathbb{R} \times \mathbb{R} = \left\{ \begin{array}{l} (1, 2), \\ (0, 1), \\ (-1, 5), \\ \left(\frac{1}{2}, \sqrt{2}\right), \\ \left(-\frac{3}{2}, 25\right), \\ (-2, 4), \\ \dots \end{array} \right\}$$



The elements
of $\mathbb{R} \times \mathbb{R}$
are
everywhere.
They cover
all of the
graph.

$\mathbb{Z} \times \mathbb{Z}$

$a \in \mathbb{Z} \times \mathbb{Z}$



$\mathbb{N} \times \mathbb{N}$

Unit 6: Relations

Definition: A relation is a subset of the cartesian product. A set of ordered pairs that satisfy a given rule of correspondance (formula/equation)

e.x. $R = \{ (x, y) \in A \times B \mid y = 2x \}$

$$R \subseteq A \times B$$

$$R = \{ (1, 2), (2, 4), (3, 6), (4, 8), (0, 0) \}$$

$$A = \{ x \in \mathbb{N} \mid x < 10 \}$$

$$A = \{ 0, 1, 2, 3, 4, 5, \dots, 9 \}$$

$$B = \{ y \in \mathbb{N} \mid y < 10 \}$$

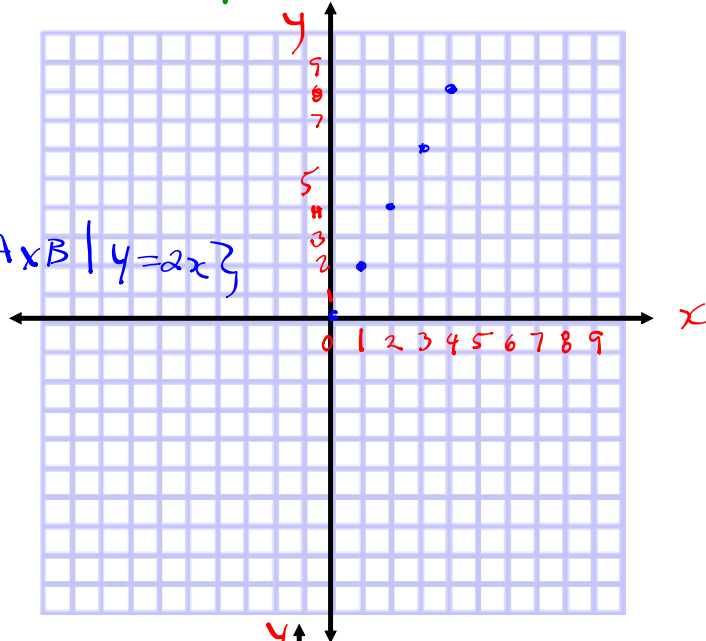
$$A \times B = \{ (1, 2), (3, 1), (6, 0), (4, 6), \dots \}$$

Graphing

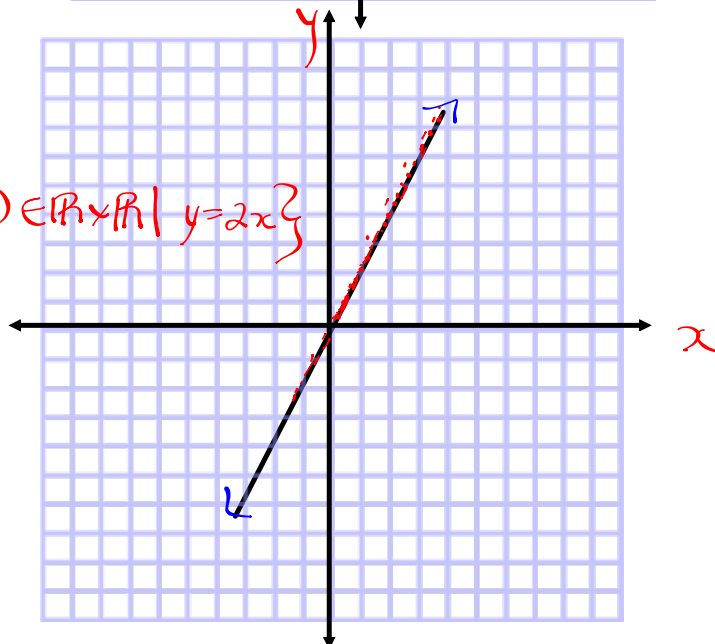
↳ a graphical way to represent a relation.

$$y = 2x$$

$$R = \{(x, y) \in A \times B \mid y = 2x\}$$



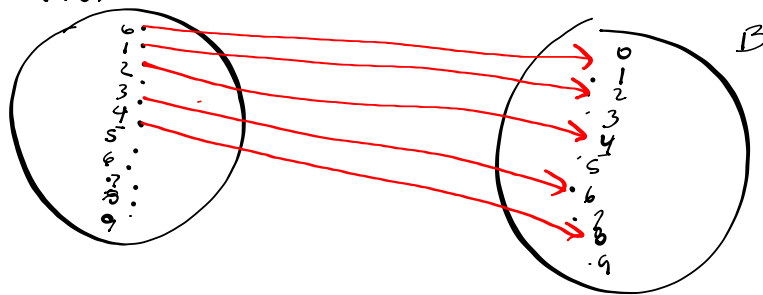
$$R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = 2x\}$$



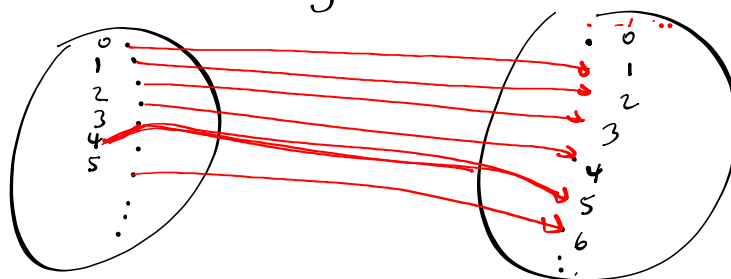
Arrow Diagrams

↳ a graphical way to represent a relation

$$R = \{(x, y) \in A \times B \mid y = 2x\}$$
$$A = \{x \in \mathbb{N} \mid x < 10\}$$



Put the following Relation in set Builder Notation



(x, y)

≥ 0
 (\mathbb{N})

$$R = \{(x, y) \in A \times B \mid y = x + 1\}$$

$$A = \{x \in \mathbb{N} \mid x \geq 0\}$$

$$B = \{y \in \mathbb{N} \mid y \geq 0\}$$

$$R = \{(x, y) \in \mathbb{N} \times \mathbb{N} \mid y = x + 1\}$$

Graph the following relation

$$R = \{ (x, y) \in \mathbb{R} \times \mathbb{R} \mid 2x + 3y = 6 \}$$

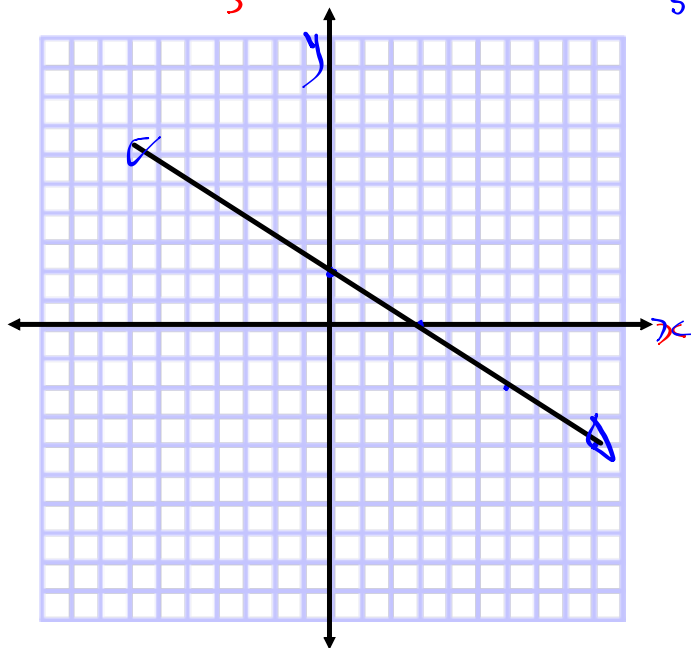
isolate
y:
perform
the
opposite
operations
to both
sides.

$$2x + 3y = 6 \quad -2x$$

$$\frac{3y}{3} = \frac{-2x + 6}{3}$$

$$y = -\frac{2}{3}x + 2 \quad \rightarrow \quad m = -\frac{2}{3} \quad \frac{\text{rise}}{\text{run}}$$

To graph
① put equation
in $y = mx + b$
↑ ↑
Slope y-int



Relations as Inequalities

"<" "≤" "dotted"
"≤" "≥" "solid"

↳ graphed it's the shading on one side of a line.

e.x. $\mathbb{R} = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x - y \geq 2\}$
 $x - y = 2$

Steps to graph inequalities

Step ①: Graph the corresponding equality (i.e. change the sign to an equal sign)

e.x. $x - y = 2$

$x = 2 + y$

$x - 2 = y$

$y = x - 2$

$b = -2$

$m = \frac{1}{1}$

rise
run



Step ②: To figure out where to shade, use a test point and see if it satisfies the inequality. If the test point satisfies the inequality shade where the point is. If not, shade where the point is not.

use the original inequality

$x - y \geq 2$ sub in (0, 0)

$0 - 0 \geq 2$

$0 \geq 2$

FALSE
(0, 0) does not satisfy.

Graph the following inequality

$$R = \{ (x, y) \in \mathbb{R} \times \mathbb{R} \mid 2y - 4x > 4 \}$$

* since there's no line under the inequality sign, you draw a dotted line for step ①.

$$\begin{array}{r} 5.5 - 1.572 \\ 472 \end{array}$$

$$R = \{ (x, y) \in \mathbb{N} \times \mathbb{N} \mid x - y > 2 \}$$

Step ①:

$$x - y = 2 \quad \begin{array}{l} -2 + y \\ -2 + y \end{array}$$

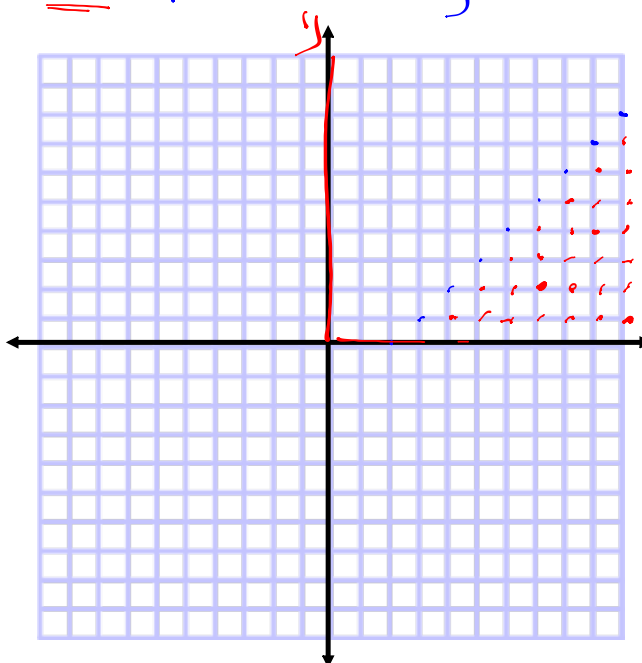
$$x - 2 = y$$

$$y = x - 2$$

Step ②: Test (0, 0)

$$0 - 0 > 2$$

$$0 > 2 \quad \text{FALSE}$$



Graph the following Relation

$$R = \left\{ (x, y) \in \mathbb{R} \times \mathbb{R} \mid \frac{x}{2} - \frac{y}{4} < 1 \right\}$$

$$\frac{x}{2} - \frac{y}{4} = 1$$

$$-\frac{y}{4} = 1 - \frac{x}{2}$$

$$y = 2x - 4$$

$$-\frac{x}{2} - \frac{-4}{4} = 1 - \frac{x}{2}$$