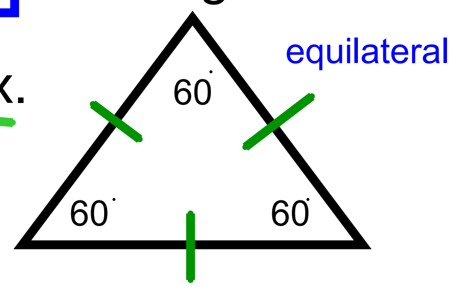
Review/Recap

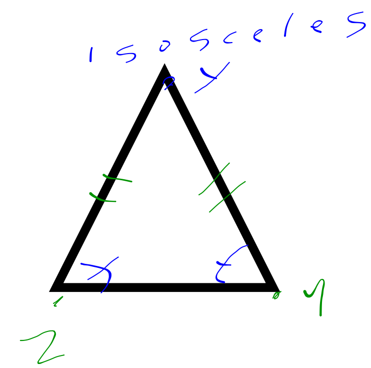
MTH 4153 - Geometric Representation

Definitions: **Equilateral Triangle** - A triangle with three congruent sides and three angles equalling 60 degrees

ex.



ex.

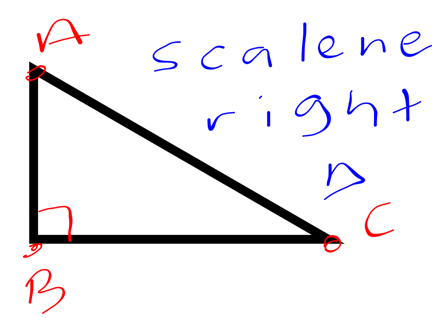


Isosceles Triangle - A triangle with two congruent sides and angles

Scalene Triangle - A triangle where all sides and angles are different measures

Right Triangle - A triangle where one angle measures 90 degrees

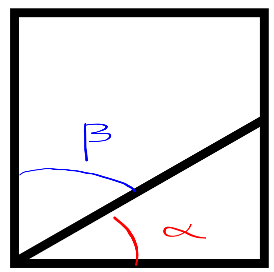
ex.



Describing/Determining the Value of Angles

Definitions: **Complementary Angles** - Two angles that add up to 90 degrees

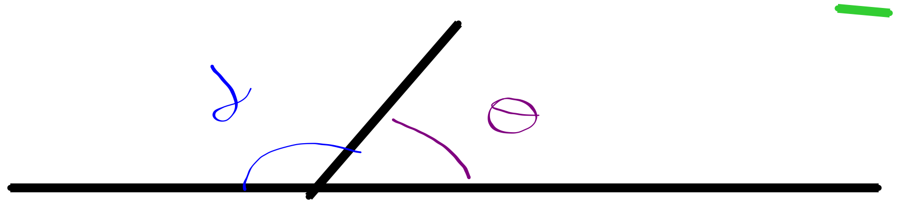
ex.



$\alpha + \beta = 90^\circ$

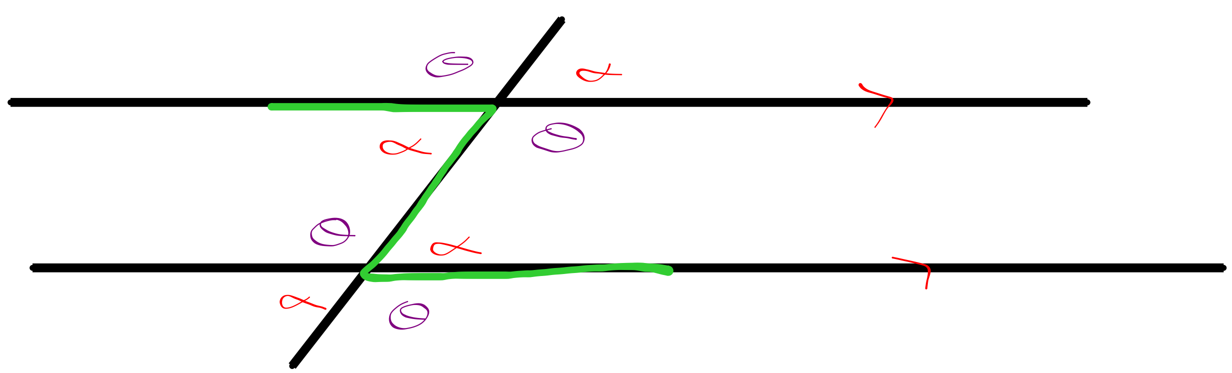
Supplementary Angles - Two angles that add up to 180 degrees

ex.



$\gamma + \theta = 180^\circ$

Angles Formed by Two Parallel Lines and a Transversal - Angles that are either congruent or supplementary depending on their position



ex.

$\alpha + \theta = 180^\circ$

Congruent Triangles

Definition: **Congruent Triangles** - Two triangles that are the same, that is, triangles that have the same sides lengths and angles

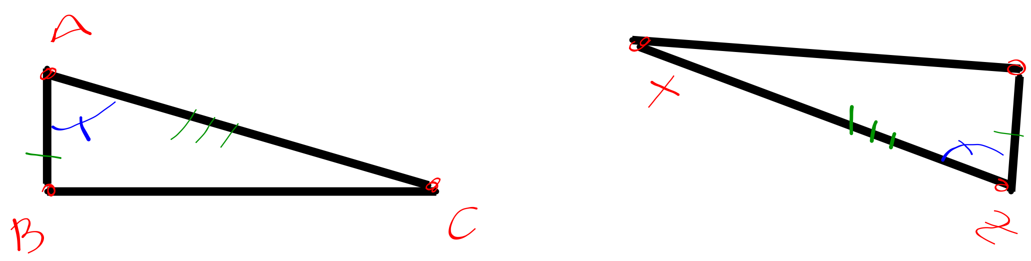
To determine if triangles are congruent ... draw them and see if one of following principles is applicable

ex. Congruency Criterion

Principle 1:

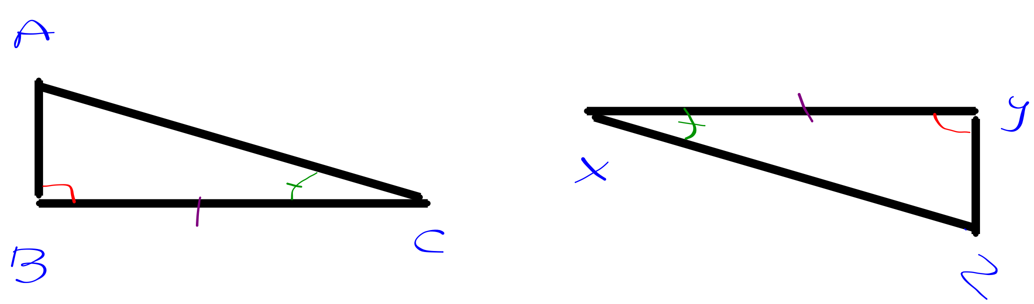
if
SSS $\cong$ SSS
then $\triangle \cong \triangle$

Principle 2:



if
SAS $\cong$ SAS
then $\triangle \cong \triangle$

Principle 3:



if
ASA $\cong$ ASA
then $\triangle \cong \triangle$

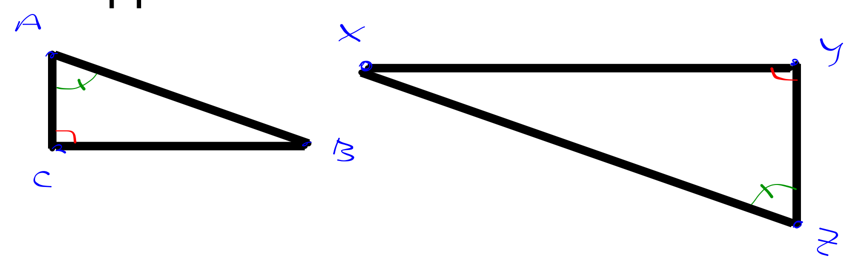
Similar Triangles

Definition: **Similar Triangles** - Two triangles that are proportionally the same, just smaller or larger

To determine if triangles are similar … draw them and see if one of following principles is applicable

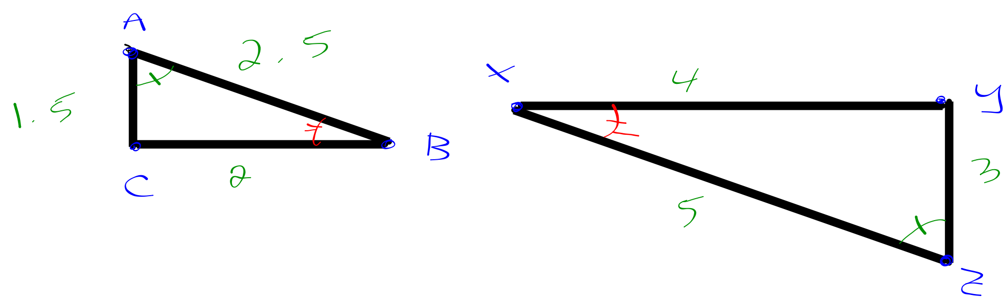
ex. Similarity Criterion

Principle 4:



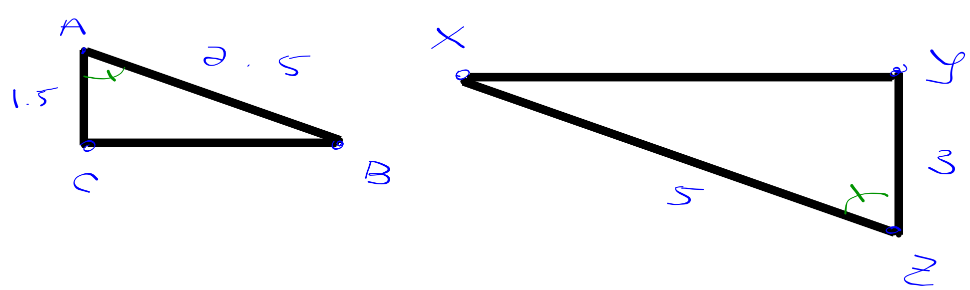
if AA ≅ AA
then △ ~ △

Principle 5:



if $\frac{S}{S} = \frac{S}{S} = \frac{S}{S}$
then △ ~ △

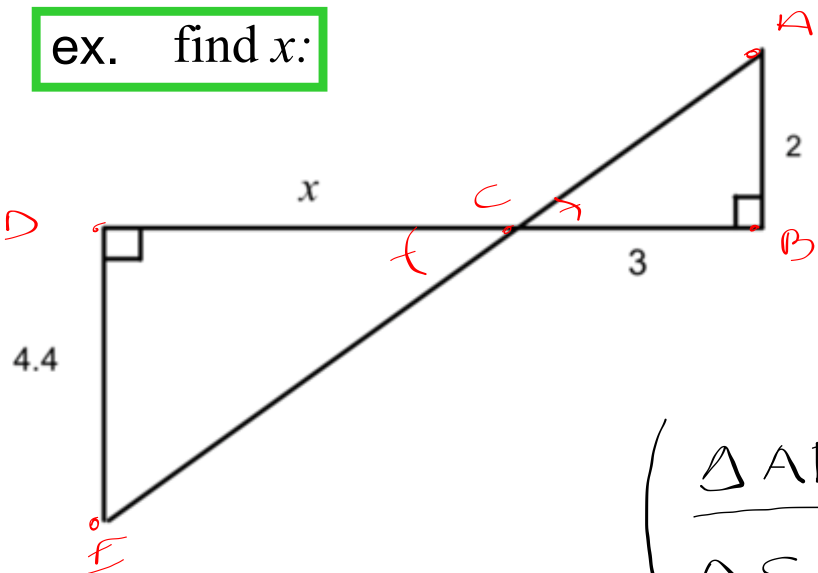
Principle 6:



if $\frac{S}{S} = \frac{S}{S}$
and contained A ≅ A then △ ~ △

Determining an Unknown Side Length in Similar Triangles

ex. find x:



$$\frac{\text{small side}}{\text{small}} = \frac{\text{big side}}{\text{big}} = \frac{\text{medium side}}{\text{medium}}$$

$$\left(\frac{\triangle ABC \rightarrow}{\triangle CDE \rightarrow}\right) \quad \frac{\overline{AB}}{\overline{ED}} = \cancel{\frac{\overline{AC}}{\overline{CE}}} = \frac{\overline{BC}}{\overline{DC}}$$

$$\frac{\overline{AB}}{\overline{ED}} = \frac{\overline{BC}}{\overline{DC}}$$

$$\frac{2}{4.4} = \frac{3}{x}$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{13.2}{2}$$

$$x = 6.6$$

STEPS

Construct ratios of corresponding sides

Pick 2 ratios with given and wanted info

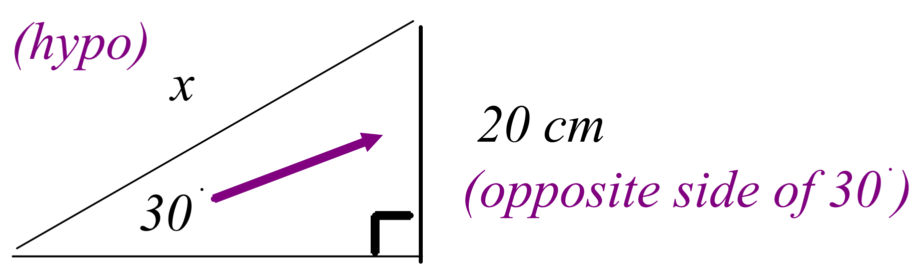
Cross multiply and solve with opposite operations

Determining an Unknown Side Length in a Right Triangle

nota bene: before using SOH CAH TOA, you must confirm that you have a right triangle, that is, a triangle with 90 degrees,

SOH use when know/looking for opposite side of an angle as well as hypotenuse

ex. find x:



STEPS

$$\sin\theta = \frac{opp}{hypo}$$

sub in values

$$\frac{\sin 30^\circ}{1} = \frac{20}{x}$$

cross multiply

$$\frac{x \cancel{\sin 30^\circ}}{\cancel{\sin 30}} = \frac{20}{\sin 30}$$

solve for *x* with opposite operations

$$x = 40\text{ cm}$$

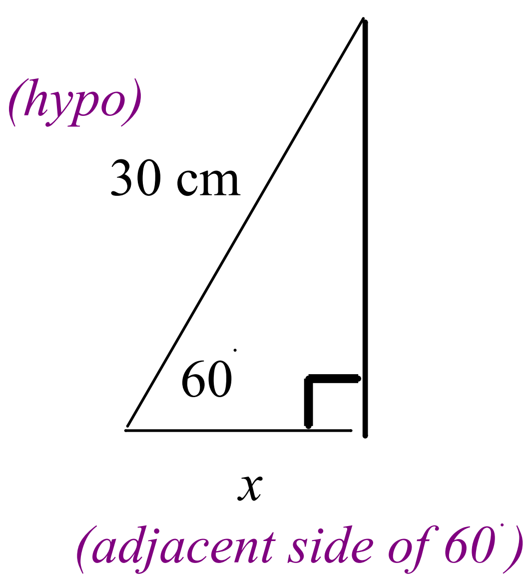
(make sure calculator is in degrees)

Determining an Unknown Side Length in a Right Triangle

nota bene: before using SOH CAH TOA, you must confirm that you have a right triangle, that is, a triangle with 90 degrees,

CAH use when know/looking for adjacent side of an angle as well as hypotenuse

ex. find x:



$$\cos\theta = \frac{adj}{hypo}$$

$$\cos 60^\circ = \frac{x}{30}$$

$$\frac{\cos 60^\circ}{1} = \frac{x}{30}$$

$$30\cos 60^\circ = x$$

$$15\text{ cm} = x$$

$$x = 15\text{ cm}$$

STEPS

sub in values

cross multiply

x is isolated, so just evaluate

(make sure calculator is in degrees)

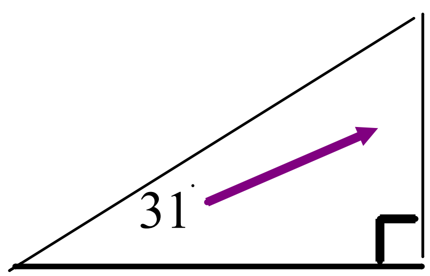
Determining an Unknown Side Length in a Right Triangle

nota bene: before using SOH CAH TOA, you must confirm that you have a right triangle, that is, a triangle with 90 degrees,

TOA

* use when know/looking for opposite side of an angle as well as adjacent *

ex. find x:



STEPS

$$\tan\theta = \frac{opp}{adj}$$

sub in values

$$\frac{\tan 31^\circ}{1} = \frac{15}{x}$$

cross multiply

$$\frac{x \cancel{\tan 31^\circ}}{\cancel{\tan 31^\circ}} = \frac{15}{\tan 31^\circ}$$

solve for x with opposite operations

$$x \approx 25 \text{ cm}$$

(make sure calculator is in degrees)

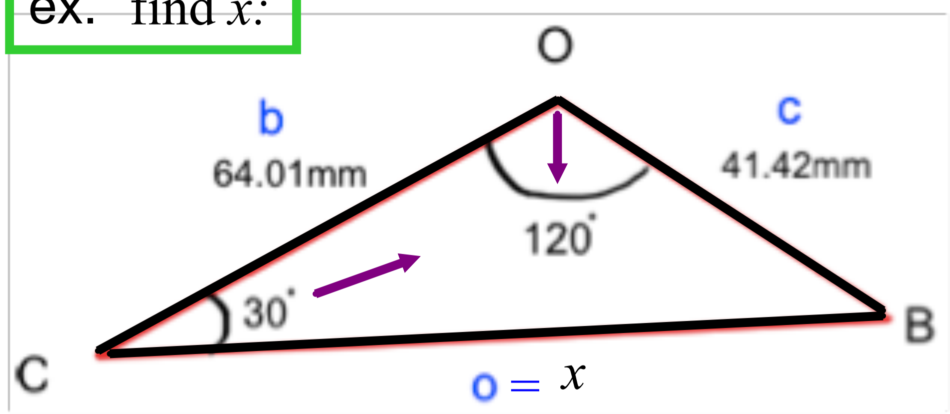
Determining an Unknown Side Length in ANY type of Triangle

nota bene: when your triangle is NOT a right triangle, your only tool is $\boxed{\text{Sine Law}}$

Principle 8:

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

ex. find x:



$$\cancel{\frac{b}{\sin B}} = \frac{c}{\sin C} = \frac{o}{\sin O}$$

$$\frac{41.42}{\sin 30^\circ} = \frac{x}{\sin 120^\circ}$$

$$\frac{41.42 \sin 120}{\sin 30^\circ} = \frac{x \cancel{\sin 30^\circ}}{\cancel{\sin 30^\circ}}$$

$$71.7\text{mm} = x$$

$$x = 71.7\text{mm}$$

STEPS

pick 2 ratios with given and wanted info

sub in values

cross multiply

solve for x with opposite operations

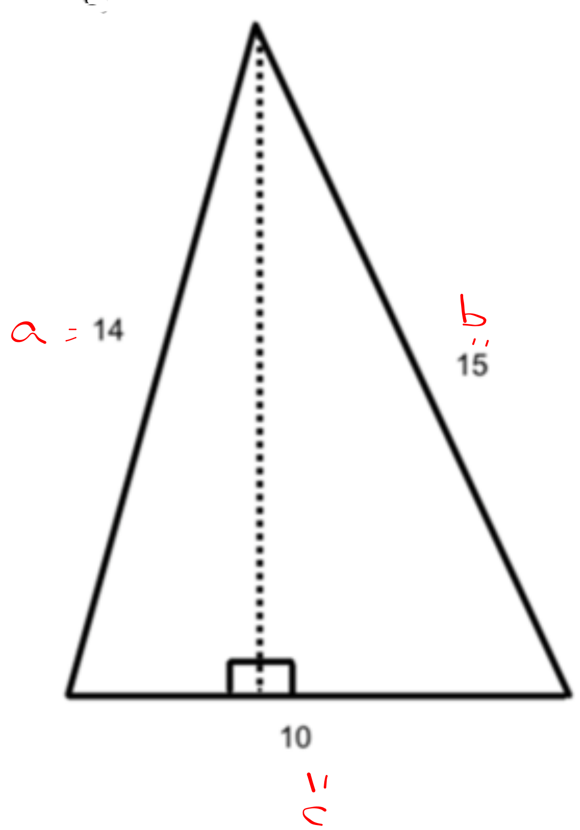
(make sure calculator is in degrees)

Determining the Area of a Triangle with Hero's Formula

nota bene: answering a task question involving area requires you to simply understand and apply Hero's Formula for area.

Principle 9: $A = \sqrt{p(p - a)(p - b)(p - c)}$ *, where p is half the perimeter, and a,b, and c are the side lengths of the triangle*

ex. find the area:



$p = \frac{a + b + c}{2}$

$p = \frac{14 + 15 + 10}{2}$

$p = 19.5$

$A = \sqrt{p(p-a)(p-b)(p-c)}$

$A = \sqrt{(19.5(19.5-14)(19.5-15)(19.5-10))}$

$A \approx 67.7 \text{ unit}^2$

STEPS

find half perimeter

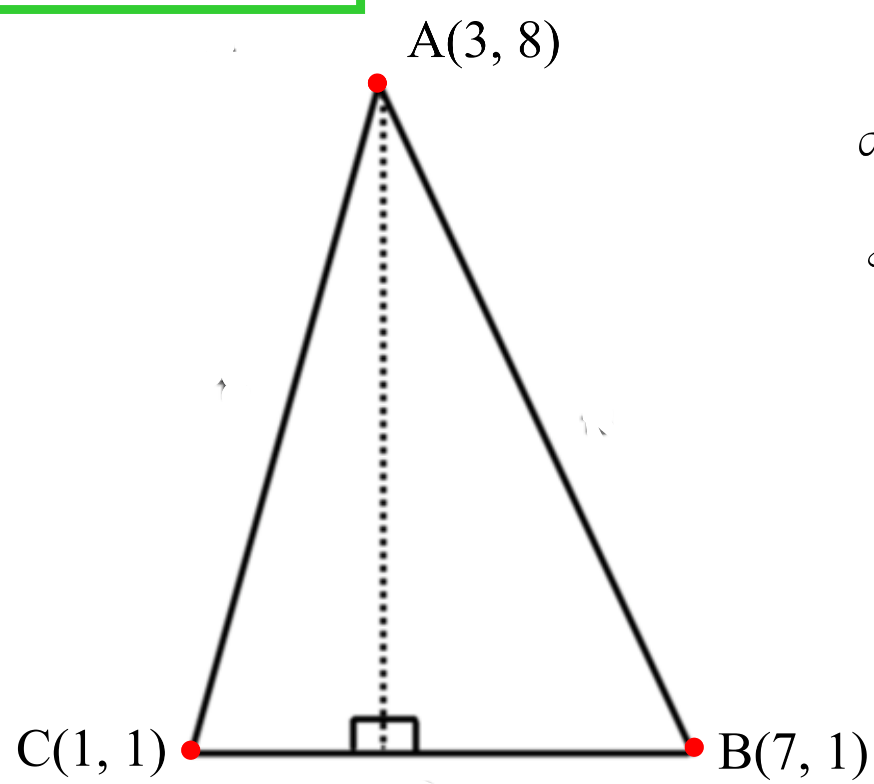
sub info in area formula and evaluate

Determining the Side Lengths of a Triangle with Distance Formula

nota bene: some area task questions could be made "harder" by only providing you with the coordinates of a triangle's vertices instead of the side lengths. You need only ask yourself then, which tool calculates the side lengths given two points: the Distance Formula

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

ex. find the area:



$A(3, 8)$, $B(7, 1)$

$A(x_1, y_1)$, $B(x_2, y_2)$

$d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d_{AB} = \sqrt{(7 - 3)^2 + (1 - 8)^2}$

$d_{AB} \approx 8.06$ unit

$d_{CB} = \ldots$

$d_{CA} = \ldots$

$A = \ldots$

STEPS

find each distance/side length being careful to label

sub in info and evaluate

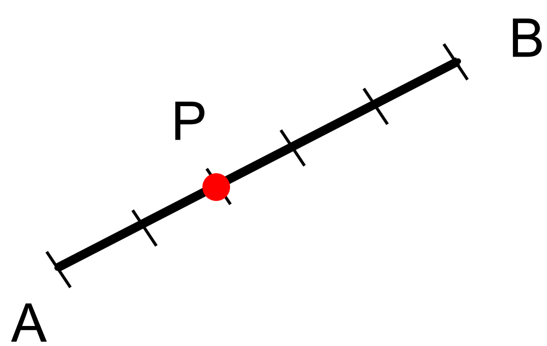
find other distances

sub info in area formula and evaluate

Determining the Location Ratio of a Point of Division

nota bene: answering an explicit knowledge question involving Point of Division is also as easy as understanding and applying a formula. However, *attention*, you must work with the Location Ratio

ex. describe point P along line segment $\overline{AB}$:



P divides $\overline{AB}$ in a ratio of two to three (2/3)

P is located two fifths (2/5) of the way along $\overline{AB}$

(needed for formula)

STEPS

algebraically convert ratios by keeping same numerator (2=2)

new denominator is the addition of the old numerator and denominator (2+3=5)

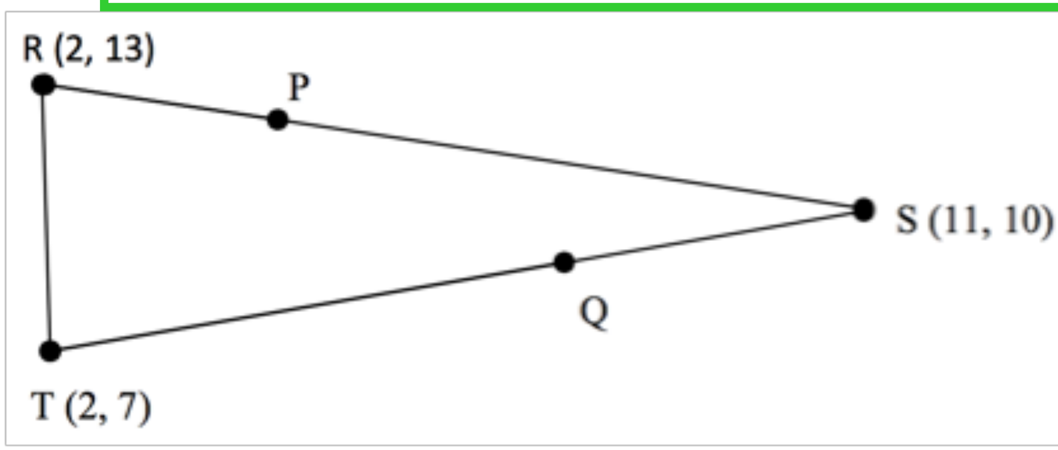
Determining the Coordinates of a Point of Division

nota bene: order matters in the Point of Division Formula. The first letter written in line segment is point 1, that is, (x_1, y_1)

$$P = \left(x_1 + \frac{m}{n}(x_2 - x_1)\ ,\ y_1 + \frac{m}{n}(y_2 - y_1) \right)$$

, *where* $\frac{m}{n}$ *is the location ratio*

ex. find the coordinates of Q that divides line segment $\overline{TS}$ into a ratio of 2:1



$\frac{2}{2+1} = \frac{2}{3} = \frac{m}{n}$

T(2, 7)   S(11, 10)

$P_1(x_1, y_1)$ $P_2(x_2, y_2)$

$x = x_1 + \frac{m}{n}(x_2 - x_1)$

$x = (2 + \frac{2}{3}(11 - 2))$

$x = 8$

$y = y_1 + \frac{m}{n}(y_2 - y_1)$

$y = (7 + \frac{2}{3}(10 - 7))$

$y = 9$

$\therefore Q = (8, 9)$

STEPS

find location ratio

determine which is point 1 and label

sub in info into x coordinate formula and evaluate

do the same for y coordinate

La fin !

(You got this!)