Unit 9 : Solving word problem w 2 unknowns that can be written as a 2nd degree equation

ex. Find 2 non-negative integers whose difference is 18 and whose product is 544.

Step ① : Identify the 2 unknowns.

- x : the 1st number
- y : the 2nd number

Step ② : Construct 2 equations involving the unknowns

[Strategy : Translate sentences into equations. ex. 'is' = '=', 'product' × , 'difference' −]

[Strategy: Think of the formula that applies to the context.]

$x - y = 18$

$x \cdot y = 544$

Step ③ : make one (2nd degree) equation w one unknown by using comparison method.

① Isolate one variable in both of the equations.

② make the equation equal to each other

③ Cross multiple (2 fractions w equal sign between) and solve ⇒ quad formula!

$x - y = 18$ $+y$ $+y$

$x = 18 + y$

$\frac{x \cdot y}{y} = \frac{544}{y}$

$x = \frac{544}{y}$

$\frac{18 + y}{1} = \frac{544}{y}$

$y(18 + y) = 544$

$18y + y^2 = 544$ -544 -544

$y^2 + 18y - 544 = 0$

$a = 1$

$b = 18$

$c = -544$

$\Delta = b^2 - 4ac$

$\Delta = (18)^2 - 4(1)(-544)$

$\Delta = 2500$

$y = \frac{-b \pm \sqrt{\Delta}}{2a}$

$y = \frac{(-18 + \sqrt{2500})}{(2(1))}$ or $y = \frac{(-18 - \sqrt{2500})}{(2(1))}$

$y = 16$

$y = -34$ (must be non-negative)

Step ④ : Find the other unknown by subbing in the found unknown into one of the original equations constructed.

$y = 16$ sub ⇒

$x - y = 18$

$x - 16 = 18$ $+16$ $+16$

solve for x!

$x = 34$

∴ the 2 number are 16 and 34!

2 #. x y 1st 2nd

The 1st # plus 4 times the 2nd # is 55.

The 1st # is 20 units longer than the 2nd #.

x + 4y = 55

x = 20 + y

P 9.8

Step ①: unknowns

x : cost per ball

y : # of balls

Step ②: equations (what formula applies here?)

- here we one equation but 2 cases

cost per thing × total # of things = total cost.

'total cost' equation

original case

$x \cdot y = 60$

altered case

$(x-1)(y+5) = 60$

Step ③ make one equation w/ comp. method

$x \cdot y = 60$

$\frac{x \cdot y}{y} = \frac{60}{y}$

$x = \frac{60}{y}$

$(x-1)(y+5) = 60$

$\frac{(x-1)(y+5)}{(y+5)} = \frac{60}{(y+5)}$

tip: start by isolating the x bracket

$x - 1 + 1 = \frac{60}{(y+5)} + 1$

$x = \frac{60}{(y+5)} + 1$

$\frac{60}{y} = \frac{60}{(y+5)} + \frac{1}{1} \cdot \frac{(y+5)}{(y+5)}$

TIP: make one fraction on the right side.

$\frac{60}{y} = \frac{60 + (y+5)}{(y+5)}$

$\frac{60}{y} = \frac{y+65}{y+5}$

$(y+5)60 = y(y+65)$

$60y + 300 = y^2 + 65y$

$-y^2 - 60y$

- Put everything to one side
- quad formula

$-y^2 - 5y + 300 = 0$

$a = -1$

$b = -5$

$c = 300$

$y = \frac{-b \pm \sqrt{\Delta}}{2a}$

$\Delta = b^2 - 4ac$

$\Delta = (-5)^2 - 4(-1)(300)$

$\Delta = 1225$

$y = \frac{-(-5) + \sqrt{1225}}{2(-1)}$ or $y = \frac{-(-5) - \sqrt{1225}}{2(-1)}$

$y = 20$ $y = 15$

Step ④

$x \cdot y = 60$

$y = 15$

$\frac{x \cdot 15}{15} = \frac{60}{15}$

$x = 4$

∴ he has 15 balls for 4 ¢ change!

P 9.24
12