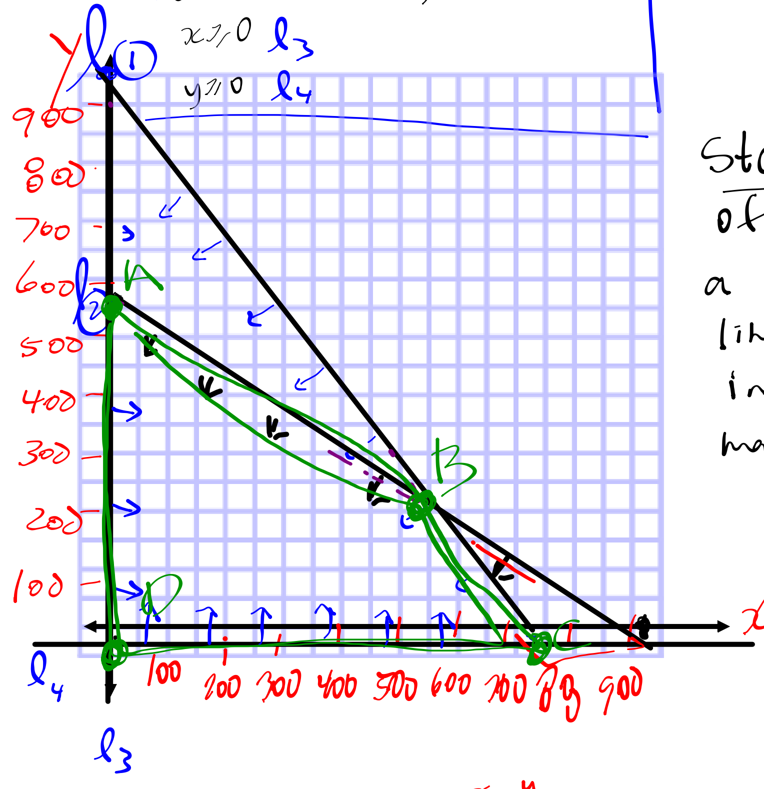
Unit 3: Graphing the Polygon of Constraints AND Finding its vertices

3.1/pl.1 Grand Slam Rackets.

1. x: # of tennis rackets
   y: # of badminton rackets
2. $P = 5 \cdot x + 6y$
3. l_1: $20 \cdot x + 15y \leq 14\,400$
   l_2: $20 \cdot x + 30 \cdot y \leq 18\,000$
   $x \geq 0$ l_3
   $y \geq 0$ l_4



$y = 0$

$y \geq 0$

$1 \geq 0$ True

Sub $(0, 1)$

To graph inequalities

Step 1: graph the corresponding equality

$20x + 15y = 14\,400$

$y = mx + b$
slope, y-int

table of value

x	y
0	960
720	0

Step 2: To see what side of the line to shade, use a test point not on the line. ex sub $(0,0)$ into inequality and see if it makes a true statement.

$20x + 15y \leq 14\,400$

$20(0) + 15(0) \leq 14\,400$

$0 \leq 14\,400$ True

so put arrows where test is.

graph

$20x + 30y = 18\,000$

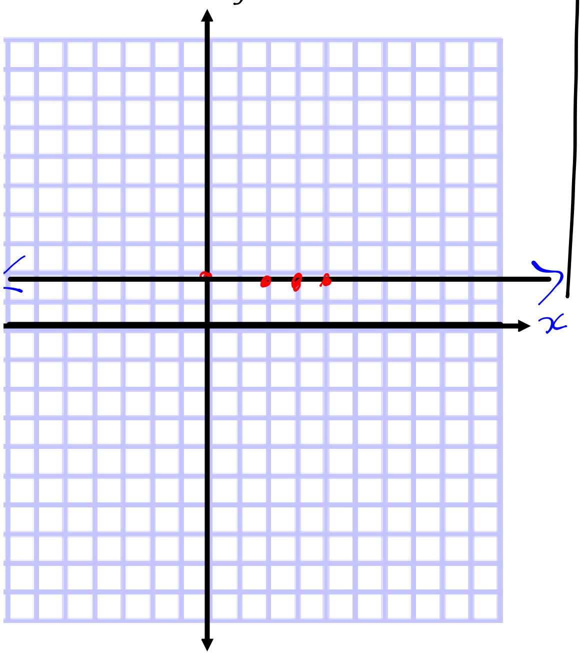
x	y
0	
	0

Note: There's only one true polygon of constraints. The one that has all the arrows pointing to it.

Horizontal Line

$(0, 2)$

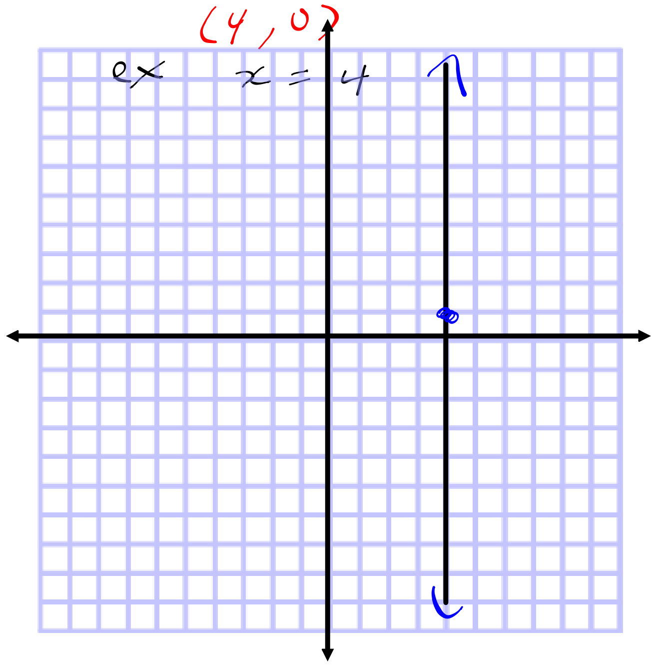
ex $y = 2$



Vertical Line

$(4, 0)$

ex $x = 4$



How many 27" televisions and how many 32" televisions should they sell if they want to maximize their profit?

The owner of an electronics store states that she cannot purchase more than 60 televisions of two different models for her store as she does not want to overstock items. At most, each 27" television occupies 0.7 m3 and each 32" model uses 1.2m3 . She has a maximum storage space of 50 m3 in the warehouse. Since the 32" model sells better than the 27" model, she keeps at least 30 in inventory. She makes a profit of 150$ per 27" and 250$ per 32".

① Define the unknowns

x = # 27" television
y = # 32" television

② Construct the optimization Equation (Profit/Cost Equation) $P = 150x + 250y$

③ Construct the inequalities

④ Graphing the inequalities & identify the POC

① $0.7x + 1.2y \le 50$
② $x + y \le 60$
③ $y \ge 30$
④ $x \ge 0$, $y \ge 0$ } trivial constraint

② $x + y = 60$

x	y
0	60
60	0

test $0 + 0 \le 60$
$0 \le 60$ true

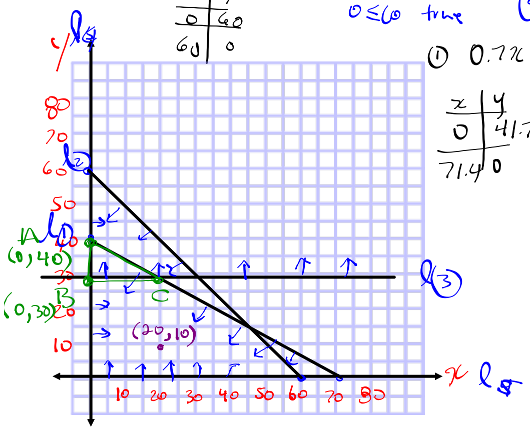
① $0.7x + 1.2y = 50$

x	y
0	41.7
71.4	0

$0.7(0) + 1.2y = 50$
$\frac{1.2y}{1.2} = \frac{50}{1.2}$
$y = 41.7$

$0.7x + 1.2(0) = 50$
$\frac{0.7x}{0.7} = \frac{50}{0.7}$
$x = 71.4$

test (0,0)
$0.7(0) + 1.2(0) \le 50$
$0 + 0 \le 50$
$0 \le 50$ true



$y \ge 30$ (10,0)
$0 \ge 30$ FALSE (arrows away from)

To determine the coordinates of the vertices of the POC, use either substitution, comparison, elimination.
(POI)
(x, y)

C $\ell_1 + \ell_3$

① $y = 30$
② $0.7x + 1.2y = 50$

Step ① isolate one unknown in one equation

Step ② sub what y equals into other equation

Sub $y = 30$ into ②

$0.7x + 1.2(30) = 50$
$0.7x + 36 - 36 = 50 - 36$
$\frac{0.7x}{0.7} = \frac{14}{0.7}$
$x = 20$

Step ③: find x by isolating by performing opposite operations to both sides.

Step ④: find y by subbing $x = 20$ into ① or ②

Sub $x = 20$ into ②

$0.7(20) + 1.2y = 50$
$14 - 14 + 1.2y = 50 - 14$
$\frac{1.2y}{1.2} = \frac{36}{1.2}$
$y = 30$

A(0, 40)
B(0, 30)
C(20, 30)

$$x + y = 60$$

x	y
0	60
60	0

sub $x=0$ into equation to find y.

$$(0) + y = 60$$
$$y = 60$$

$$x + 0 = 60$$
$$x = 60$$

Unit 4: Verifying that a point belongs to the Polygon of constraints

Note: A points belongs if it satisfies all the constraints (aka makes a true statement)

Points	$x \geq 90$	$y \geq 30$	$x+y \leq 150$	$x \geq 2y$	Belongs?
(100, 35)	$100 \geq 90$ True	$35 \geq 30$ true	$100+35 \leq 150$ True	$100 \geq 2(35)$ True	Yes
(100, 20)					
(100, 50)				$100 \geq 100$ True	

p 2.12

Also graph and find the vertices of the polygon of constraints

The Ideal Kitchen Company hires 12 cabinetmakers and 4 cabinetmaker-trainees to manufacture different quality cupboards: a luxurious model and a standard model. The manufacturing of a luxurious cupboard needs 4 hours of work per cabinetmaker and 2 of work per trainee. The manufacturing of a standard cupboard needs 8 hours of work per cabinetmaker and 2 of work per trainee. The collective agreement specifies that an employee can't work more than 8 hours a day. If the employer realizes a $20 profit per luxurious model and $12 on a standard model, how many cupboards of each model will he need to manufacture per day to maximize his profit?