Lesson 9 : Areas and Volume of Prisms (medium tasks)

April 9th 2024

mini

v.s.

prisms

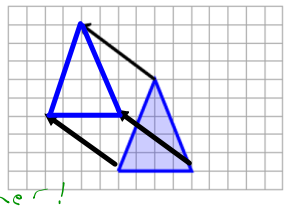
→ results from translating a polygon through 3D space

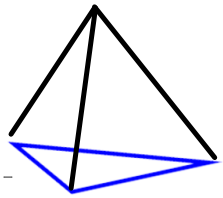
pyramid

→ results from connecting all of the vertices of a polygon to a single point (the apex)

(one vanishing point)

- triangular prism
- the bases are parallel to one another!

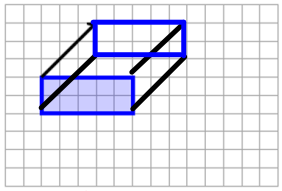




triangular pyramid

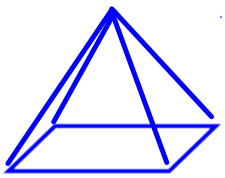
You draw and answer: How many bases does the prism have? How many does the pyramid have?

- rectangular prism



prisms

- 2 bases



- rectangular pyramid

pyramid

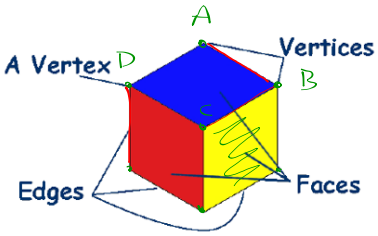
- 1 base

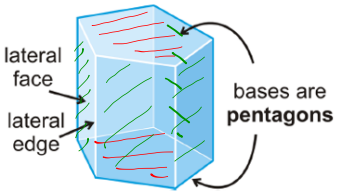
nota bene: we name prisms and pyramids according to their bases.

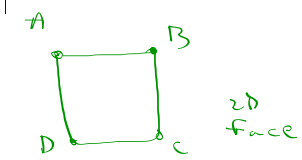
A a

1.1 The Total Surface Area of a Prism

Review Terminology: Before beginning, let's review some terminology





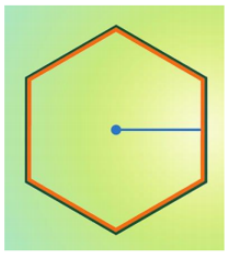


Total Area ≠ Lateral Area

Total area = 2 bases + lateral faces

Lateral Area = lateral faces

Review of The Area of any regular polygon: We will need to be able to find the area of any regular polygon.



Proof and Example:

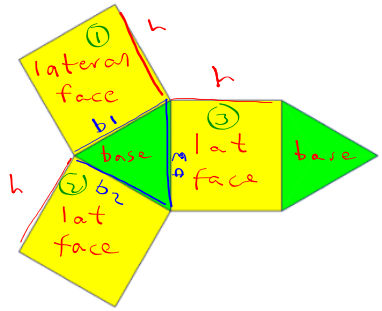
For any regular polygon, the area can be computed as follows:

$$A = \frac{p \times a}{2}$$

- p is the perimeter of the polygon
- a is the apothem

1.2 Discovering the Formula for the Lateral Area (A_l)

Let's consider the case of a triangular prism. We will see that calculating the lateral area for any prism is done in exactly the same way.



the net

legend:

A_L = lateral area

P_B = perimeter of base

h = height

A_B = Area of the base

Lateral Area = Area ① + Area ② + Area ③

$A_L = b_1 \times h + b_2 \times h + b_3 \times h$ (gcf = h)

$A_L = h(b_1 + b_2 + b_3)$

$A_L = h P_B$ ← perimeter of base

h - height of 3D solid

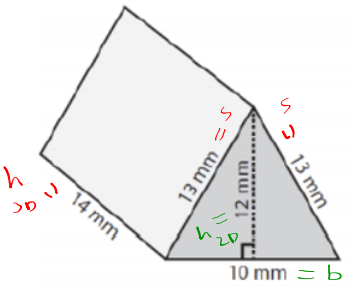
$A_L = P_B \times h$

$A_T = A_L + 2A_B$

$A_T = P_B \times h + 2A_B$

medium task

1.4 Example: Robin wants to make a toy tent in the shape of a triangular prism. He bought 3.5 bags of canvas. Each bag covers 170 mm^2 and cost him \$2.50. Will the 3.5 bags be enough to make his toy tent?



in notebook, write:
refer to lesson 9, ho 1, pg 2
q. 1.4

WANT:
i. total area of tent (Review)

ii. amount of canvas we have (new) mm^2

TOOL:

$A_T = A_L + 2A_B$

sub in:

$A_L = P_B \times h$

$A_B = \frac{b \times h}{2}$

$3.5 \text{ bags} \times \frac{170 \text{ mm}^2}{1 \text{ bags}}$

mm^2 per bag
mm^2 each bag

$\frac{\text{bags}}{1} \times \frac{\text{mm}^2}{\text{bags}}$

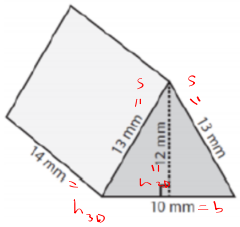
- label diagram
- sub in values

595 mm^2 of canvas we have

$A_T = P_B \times h_{3D} + 2\left(\frac{b \times h_{2D}}{2}\right)$

$A_T = (13 + 13 + 10) \times 14 + 2\left(\frac{10 \times 12}{2}\right)$

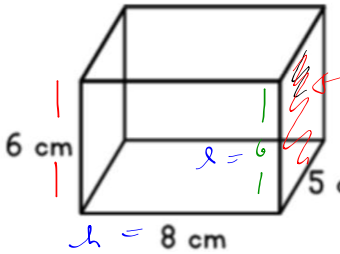
$A_T = 624 \text{ mm}^2$ we need



∴ No, the 3.5 bags won't be enough since $624 \text{mm}^2 > 595 \text{mm}^2$

You do: Practice 1.7.

1.7 Practice: Youssef makes boxes of matches in the shape of a rectangular prism shown below. He purchases 1.5 packs of cardboard to make one box. Each pack will cover 160 cm^2. Each box of matches will be made with the cardboard and will contain 25 matches. Will Youssef have enough cardboard to make a box?



ANS: Yes Youssef does have enough since $240\ cm^2 > 236\ cm^2$

$1.5\ \text{packs} \times 160\ cm^2/\text{pack}$

$240\ cm^2$ of cardboard we have

we need:

sub in

$A_T = A_L + 2A_B$

$A_L = P_B \times h$

$A_B = l \cdot w$

$P_B = 2l + 2w$

$P_B = l + l + w + w$

$A_T = P_B \times h + 2 \cdot l \cdot w$

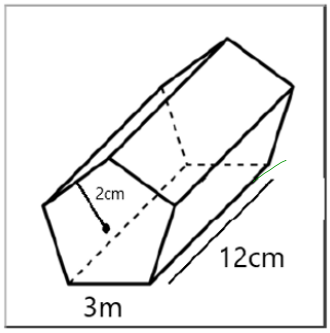
$A_T = (2l + 2w) \times h + 2l \cdot w$

$A_T = (2 \cdot 6 + 2 \cdot 5) \times 8 + 2 \cdot 6 \cdot 5$

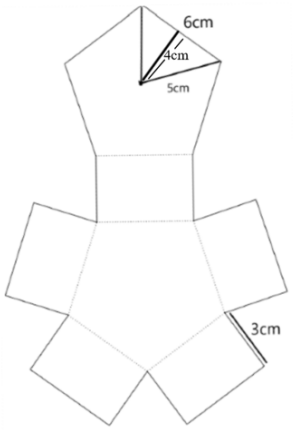
$A_T = 236\ cm^2$

$\therefore$ Youssef does have enough since $240\ cm^2 > 236\ cm^2$ we have we need.

1.5 Example: Paulina has a company that makes erasers in the form of pentagonal prims. She thinks 2.2 sheets of plastic should be enough to wrap them. She pays \$0.10 per sheet and each one covers $100cm^2$. Does she have enough plastic?



1.6 Practice: Paulina has a new design for the eraser and buys 5 sheets of paper wrapper that cost \$0.50 more. Below is the net of the new pentagonal prisms. If each sheet of paper cover $50cm^2$, will the 5 sheets be enough to wrap the eraser?



stuff to put on memory aid:

$A_L = P_B \times h$

$A_T = A_L + 2A_B$

$A_T = P_B \times h + 2A_B$

$V = A_B \times h$

legend

A_L = lateral area (3D)

P_B = perimeter of base (2D)

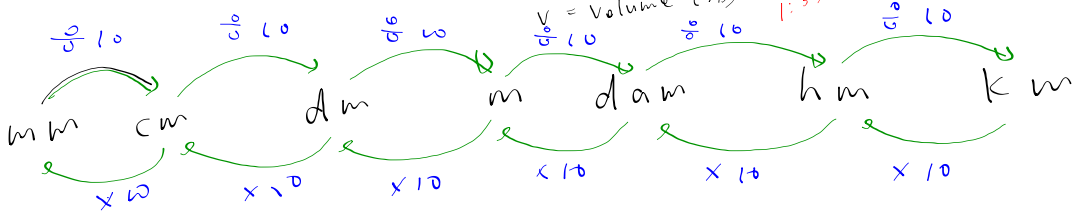
h = height of prism (3D)

A_T = total area (3D)

A_B = area of base (2D)

V = Volume (3D)

back @ 1:37



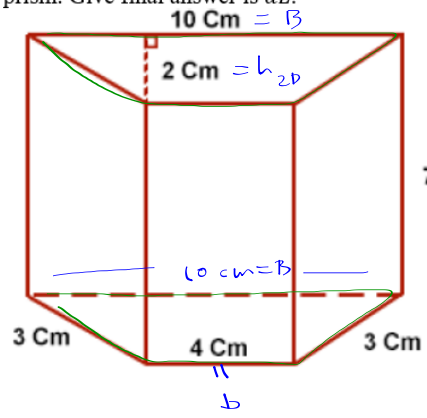
mL cL dL L daL hL kL

$1\,cm^3$

$1\,dm^3$

$1\,m^3$

1.10 Example: Find the volume of the following prism. Give final answer is dL.



trapezoidal prism

$V_{3D} = A_B \times h_{3D}$: $A_B = \frac{(b+B)\times h_{2D}}{2}$

$V = \frac{(b+B)\times h_{2D}}{2} \times \frac{h_{3D}}{1}$: label : diagram

$V = \frac{(4+10)\times 2\times 7}{2}$

final answer in dL

$V = 98\ cm^3$ } since $1cm^3 = 1mL$

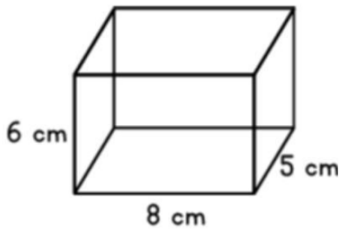
$V = 98\ mL$

$98 \div 10 \div 10$

$V = 0.98\ dL$

You do practise 1.11 and 1.12

1.11 Practice: Find the volume of the following prism. Give the final answer in *cL*.

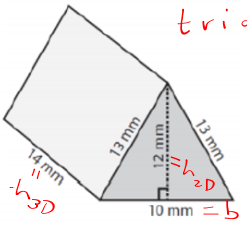


$V = 240\ cm^3$

$V = 240\ mL$

$V = 24\ cL$

1.12 Practice
Find the volume of the following prism. Give the final answer in *mL*.



triangular prism

h_{3D} = 14 mm

h_{2D} = 12 mm

b = 10 mm

$V = A_B \times h_{3D}$

$A_B = \frac{b \times h_{2D}}{2}$ (D)

generally

$V = \frac{b \times h_{2D} \times h_{3D}}{2}$

$V = \frac{10 \times 12 \times 14}{2}$

$V = 840\ mm^3$

$840 \frac{0}{0} 10^3$

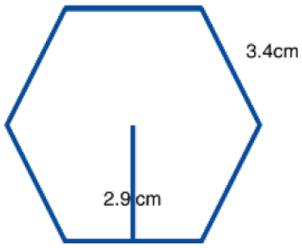
$V = 0.84\ cm^3$

$V = 0.84\ mL$

Answer
mL

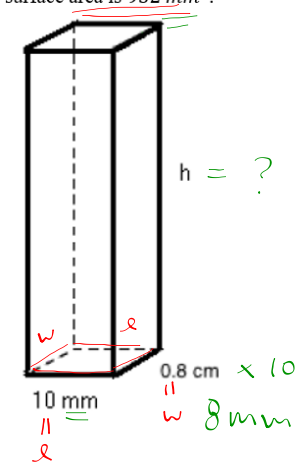
1.13 Practice

Calculate the volume of a prism whose height is 17 cm and whose base is the regular hexagon as shown below. How much water (in L) could this prism contain?



medium-level task

2.1.1 Example: Determine the volume (in mL) of the following rectangular prism if its total surface area is 952 mm^2.



Which tool to find h:?

Total Area

memory aid:

Pick the equation (tool) based on given info.

$A_T = A_L + 2A_B$

$A_T = P_B \times h + 2\ell \cdot w$

$A_L = P_B \times h$

$A_B = \ell \cdot w$

perimeter of base

$P_B = \ell + \ell + w + w$

$P_B = 2\ell + 2w$

BEDMAS

simplify

isolating

$A_T = (2\ell + 2w) \times h + 2(\ell \cdot w)$

$952 = (2(10) + 2 \cdot 8) \times h + 2(10) \cdot 8$

sub

$\ell = 10mm$

$w = 8m$

$A_T = 952\ mm^2$

sub, simplify, solve

$952 = 36h + 160$

$-160 \quad -160$

$\frac{792}{36} = \frac{36h}{36}$

$22 = h$

$h = 22mm.$

$V = A_B \times h$

$V = \ell \cdot w \cdot h$

$V = 10 \times 8 \times 22$

$V = 1760\ mm^3$

$1760 \div 10^3$

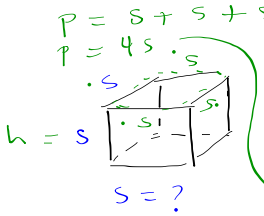
$V = 1.76\ cm^3$

$V = 1.76\ mL$

final answer in mL

You do 2.1.2

2.1.2 Practice: Determine the volume (in L) of a cube if its total surface area is 216 m^2.



Find s! Which tool?

$A_T = A_L + 2A_B$

$A_t = 6s^2$ ✓

$A_T = P_B \times h + 2s^2$

$A_T = 4s \times s + 2s^2$

$A_T = 4s^2 + 2s^2$

$A_T = 6s^2$

$\frac{216}{6} = \frac{6s^2}{6}$

$\sqrt{36} = \sqrt{s^2}$

$s = 6$

squaring x^2

square rooting $\sqrt{x}$

$V = A_B \times h$

$V = \ell \cdot w \cdot h$

$V = s \cdot s \cdot s$

$V = s^3$

$V = 6^3$

$V = 216\ m^3$

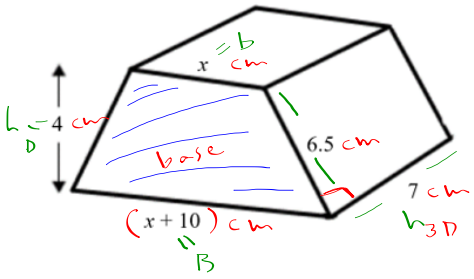
$V = 216\ kL$ → in L

$216 \times 10 \times 10 \times 10$

$V = 216000\ L$

$a^m \times a^n = a^{m+n}$

2.2.1 Example: Determine the lateral area of the following isosceles trapezoidal prism if the below units are in cm and its volume is 0.224 *litres*.



- Find x 1st
- which tool?

$$V = A_B \times h_{3D} \quad \vdots \quad A_B = \frac{(b+B)\,h_{2D}}{2}$$

$$V = \frac{(b+B)\,h_{2D}}{2} \times h_{3D}$$

$$V = \frac{\left(x + (x+10)\right) \times 4 \times 7}{2}$$

$$V = \frac{(1x + 1x + 10) \times 4 \times 7}{2}$$

$$V = (2x + 10) \cdot 14$$

solve

$$\frac{224}{14} = \frac{(2x+10)\cdot 14}{14}$$

$$16 = 2x + 10$$

$$-10 \quad -10$$

$$\frac{6}{2} = \frac{2x}{2}$$

$$x = 3$$

- must convert L to cm^3

$V = 0.224$ L

$V = 0.224\ dm^3$

$\times 1000$

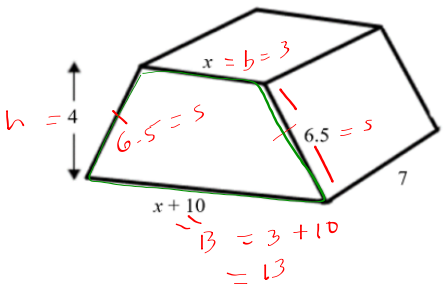
$V = 224\ cm^3$

$A_L = P_B \times h$

$A_L = (b + B + 2s) \times h$

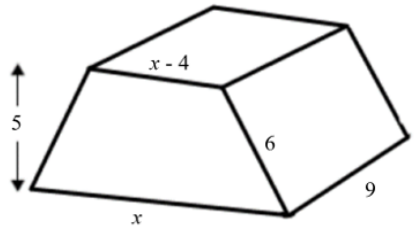
$A_L = (3 + 13 + 2(6.5)) \times 7$

$A_L = 203\ cm^2$



You do:

2.2.2 Practice: Determine the lateral area of the following isosceles trapezoidal prism if the below units are in cm and its volume is 0.0315 daL.



- HMWK 2.2.2 + 2.2.3 (from handout)
- from textbook pg 91 #2.21 / 2.23 pg 109 #3.8

Bonus pg 92 #2.25 pg 96 #2.31