(lesson 6 in syllabus)

Lesson 5 : Solving Equations and Pythagorean Theorem

April 2nd 2024

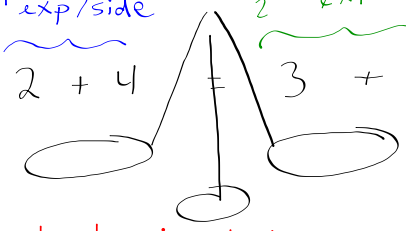
Recall

an expression

ex. $2 + 4$

an equation

1st exp/side $2 + 4$ $=$ 2nd exp/side $3 + 3$



nota bene: what you do new to one side, you must do the exact same thing to the other side.

$2 + 4 = 3 + 3$ (+1 to each side) — True or False?

$6 = 6$

simplify

ex $-2x + 3x + 1$

$x + 1$

- after simplifying exp.'s we solve equations

ex.

$-2x + 3x + 1 = 10$

$x + 1 = 10$

"Solve" (riddle) (puzzle)

find the value of x that makes a balanced equation

One-step Solving in a Geometric Context

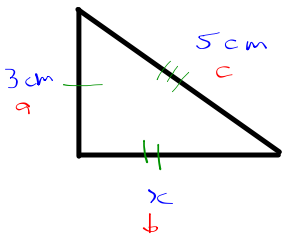
Recall: 2D shapes
- Area formulas
- Perimeter equations

ex.

The perimeter of the below (no sides are equal/congruent) scalene triangle is 12 cm (centimetres)

Find x. → WANT

TOOL: 1 equation (total edges)



$P = a + b + c$

Recall: Perimeter is the addition of the outer side lengths

step i. sub in values into eq.

$P = 12$ cm

$a = 3$ cm

$c = 5$ cm

$b = x$

$12 = 3 + x + 5$

1 side, 2nd side

$12 = 8 + x$

step ii. simplify the expressions. (make alone)

step iii. Solve by isolating x on one side of the eq. by doing opposite/inverse operations to both sides

$12 = 8 + x$

$-8 \quad -8$

$4 = x$

$x = 4$

$+ \longleftrightarrow -$

$\times \longleftrightarrow \frac{\circ}{\circ}$

$x^2 \longleftrightarrow \sqrt{x}$

Check:

$12 = 8 + x$

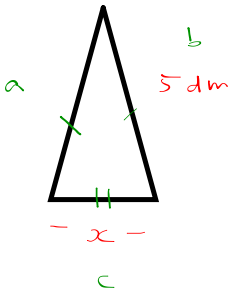
$12 = 8 + 4$

$12 = 12$

True ✓

ex The perimeter of the below isosceles $\triangle$ is 13 dm (decimetre)
2 congrment side

Find x



$P = a + b + c$

$13 = 5 + 5 + x$

$13 = 10 + x$

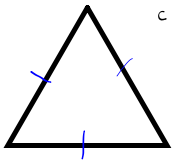
$-10 \quad -10$

$3 = x$

$x = 3$

equilateral $\triangle$

3 congrment side



INFO

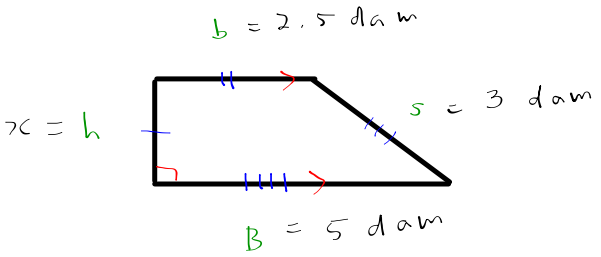
$P = 13$ dm

$b = 5$ dm

$a = 5$ dm

You do:

The perimeter of the below trapezoid is 12 dam (decametres). Find x.



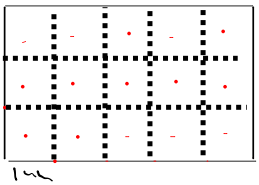
$P = h + b + B + s$

$x =$ dam

> (arrow that say the bases are parallel)

ex. The area of the below rectangle is 15 mm^2 (millimetres squared)

Find x.



$A = l \cdot w$

$\frac{15}{5} = \frac{5 \cdot x}{5}$

$3 = x$

$x = 3$

Recall Area is the amount of space (squares) inside a shape.

$A = 15\ mm^2$

$l = 5\ mm$

$w = x$

check

$15 = 5 \cdot x$

$15 = 5 \cdot 3$

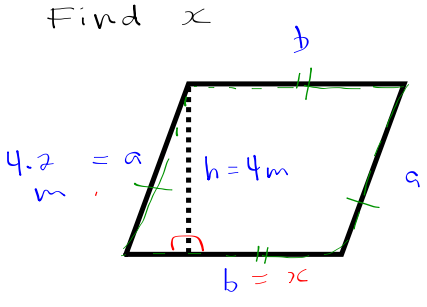
$15 = 15$ ✓ true

You do: handout 1:

AND:

The area of the below parallelogram is 20 m^2 (metres squared)

Find x



$A = b \cdot h$

$\frac{20}{4} = \frac{x \cdot 4}{4}$

$5 = x$

$x = 5\ m$

Bonus: Find the perimeter

$P = a + a + b + b$

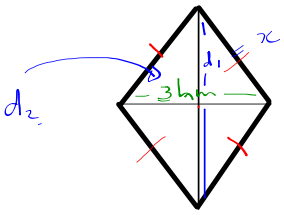
$P = 2a + 2b$

$P = 2(4.2) + 2(5)$

$P = 18.4\ m$

Multi-step Solving in Geo. context

. The area of the below rhombus is 12 hm² (hectometres squared)



Find x.

$$A = \frac{d_1 \times d_2}{2}$$

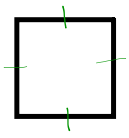
$$12 = \frac{(x \times 3)}{2}$$

$$2 \times 12 = \frac{(x \cdot 3)}{2} \times 2$$

$$\frac{24}{3} = \frac{3 \cdot x}{3}$$

$$8 = x$$

$$x = 8 \text{ hm}$$



step i. same as before

$A = 12 \text{ hm}^2$

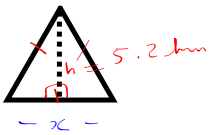
$d_1 = x$

$d_2 = 3 \text{ hm}$

step ii solve/isolate x. which operation 1st?

You do:

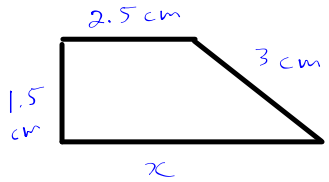
ex The area of the equilateral Δ is $15.6\ hm^2$. Find x.



$A_{\Delta} = \frac{b \times h}{2}$

Bonus: find perimeter.

ex The area of a trapezoid is $5.625\ cm^2$. Find x.



$A = 5.625$ (4) B
(3) E
(2) D M
(1) A S

$b = 2.5$

$h = 1.5$

$B = x$

$A = \frac{(b+B)\cdot h}{2}$

$2 \times 5.625 = \left(\frac{(2.5+B)\cdot 1.5}{2}\right) \times 2$

$\frac{11.25}{1.5} = \frac{(2.5+B)\cdot 1.5}{1.5}$

$7.5 = 2.5 + B$

$-2.5 \quad -2.5$

$5 = B$

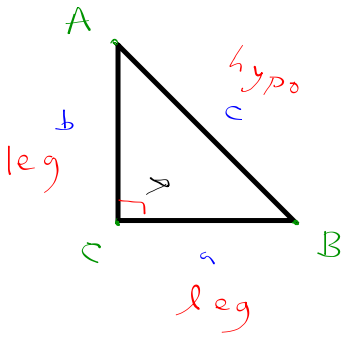
$B = 5$

Bonus: find perimeter

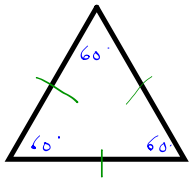
When Done, do handout 2

a new tool: Pythagorean Theorem: $c^2 = a^2 + b^2$ (only for right triangles)

Terminology: a right triangle is a Δ w/ a $90°$.

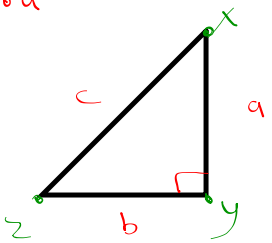


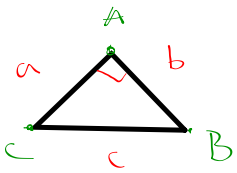
$\angle C = 90°$

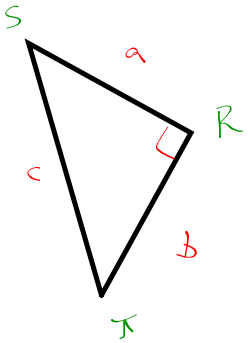


N.B.: $c^2 = a^2 + b^2$, where 'c' is the hypotenuse, that is, the longest side in the right Δ (< is opposite the 90°) legs hypo

You label the Δ's w/ a, b, c.

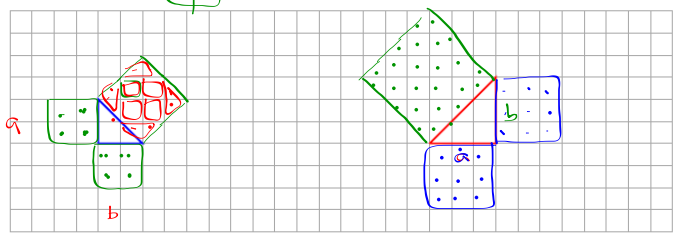






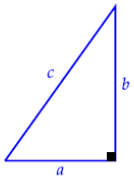
Discover Pythagoras Theorem

Consider the following right-angled triangles in the grid below. Is there are any relationship between the sides and the squares formed by those sides?



$a^2 + b^2 = c^2$

The Pythagorean Theorem



For any **right triangle**:

$$a^2 + b^2 = c^2$$

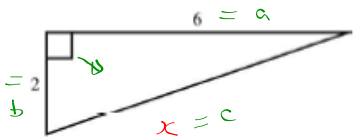
Where c is the *hypotenuse* of the right triangle.

1.2 Applications of the Pythagorean Theorem

1.2.1 Example

label a, b, c

Use the Pythagorean theorem to determine the hypotenuse of the following triangle:



Find x : ← WANT TOOL : 1 eq

$$c^2 = a^2 + b^2$$

$$x^2 = 6^2 + 2^2$$

$$x^2 = 36 + 4$$

$$x^2 = 40$$

$$\sqrt{x^2} = \sqrt{40}$$

$$x = 6.32 \text{ units.}$$

check:

$$c^2 = a^2 + b^2$$

$$6.32^2 = 6^2 + 2^2$$

$$39.9 = 40$$

INFO.

$c = x$. sub

$a = 6$. simplify

$b = 2$

~~Area~~

~~Perimeter~~

$c^2 = a^2 + b^2$

solve

isolat x.

do o.o.

$+ \leftrightarrow -$

$\times \leftrightarrow \div$

$x^2 \leftrightarrow \sqrt{x}$

You do

LABEL

Practice

1.2.2

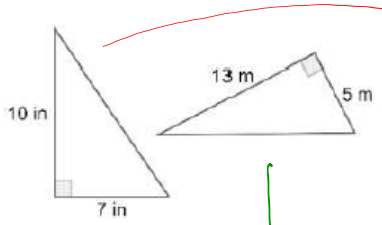
1.2.3

Bonus

1.2.4

1.2.2 Practice

Use the Pythagorean theorem to determine the hypotenuse of the following triangles:



$a^2 + b^2 = c^2$

$7^2 + 10^2 = c^2$

$\sqrt{149} = \sqrt{c^2}$

$c = 12.20$ ✗ −1%

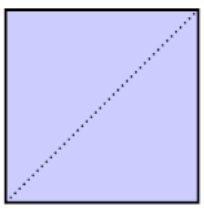
$c = 12.21$ ✓

ans

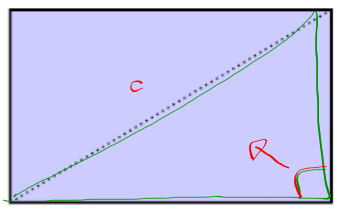
$c = 13.93\,m$

1.2.3 Practice

Use the Pythagorean theorem to determine the measure of the *diagonals* of the following square and rectangle. Round your answer to the nearest hundredth, if necessary:



$c = 7.07\,cm$

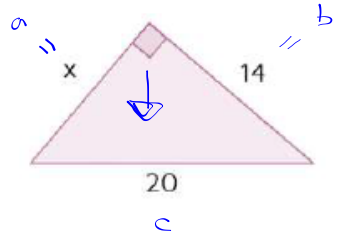


$c = 8.06\,cm$

$c^2 = a^2 + b^2$ to find the legs (a/b)

1.2.4 Example

Use the Pythagorean theorem to find the missing side of the following triangle:



$c^2 = a^2 + b^2$

$20^2 = x^2 + 14^2$ (side 1, side 2)

$400 = 1.x^2 + 196$

$400 = x^2 + 196 - 196$

$\sqrt{204} = \sqrt{x^2}$

$14.28 = x$ units

- sub
- simplify:

$r \cdot a^n + s a^n = (r+s) a^n$

- solve: isolate x, do o.o.

BEDMAS (simplifying) / solving

You do:

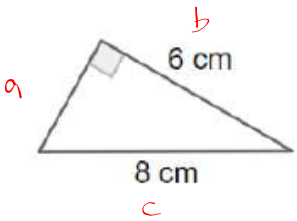
1.2.5 p93

Bonus

1.2.9

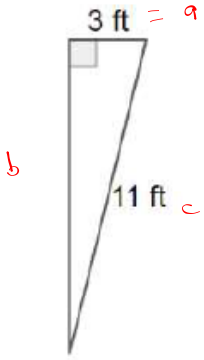
1.2.5 Practice

Use the Pythagorean theorem to find the missing side of the following triangles:



$a = 5.29\text{ cm}$

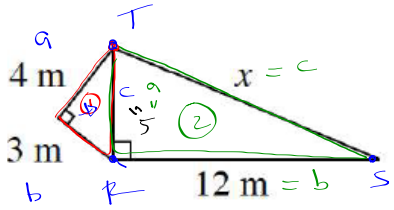
You do bonus 1.2.9



$b = 10.58\text{ ft}$

1.2.9 Example

Determine the measure of x in the following:



#1:

for Δ (1)

$c^2 = a^2 + b^2$

$c^2 = 4^2 + 3^2$

$\sqrt{c^2} = \sqrt{25}$

$c = 5$

step #2 for Δ (2)

$c^2 = a^2 + b^2$

$\sqrt{x^2} = \sqrt{5^2 + 12^2}$

$x = 13$

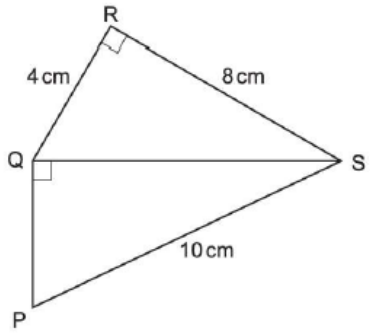
Solving Tips:

- use formula/tool you have info for!
- start w/ Δ's you have most info for.

You do later: 1.2.10

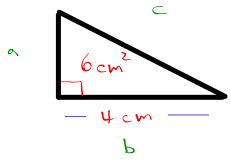
1.2.10 Practice

Determine the measure of $\overline{QP}$:



Constructing Steps of a Task

ex the area of the below right Δ is 6 cm². Find its Perimeter.



step #1.
find height = a

$A = \frac{b \times h}{2}$ $A = 6\,cm^2$ $b = 4$ $h = a$

$2 \times 6 = \frac{(4 \times h)}{2} \times 2$ sub solve

$\frac{12}{4} = \frac{4 \times h}{4}$

$3 = h$

$h = 3\,cm$

BEDMAS

step #3

WANT: P

TOOL: $P = a + b + c$

INFO: $a = ?$, $b = 4$, $c = ?$

WANT: (which one?) pick the eq. you have the info for.

TOOL: equations

$c^2 = a^2 + b^2$

$a = ?$ $c = ?$

step #2

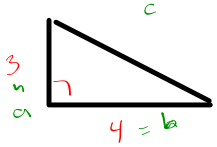
$\therefore A = \frac{b \times h}{2}$

step #1

INFO $A = 6\,cm^2$, $b = 4$, $h = ?$

(work from bottom up)

step #2



$c^2 = a^2 + b^2$

$\sqrt{c^2} = \sqrt{3^2 + 4^2}$

$c = 5$

step #3

$P = a + b + c$

$P = 3 + 4 + 5$

$P = 12\,cm$

You do: The area of a square is 25 cm^2. What's its perimeter?

HMWK:

~~pg 121 "use an eq "~~

pg 125 # 3.12

pg 126

pg 127 #3.16 a)-b) only