Lesson 3: Radicals

March 27th 2024

Scientific Notation

↳ a way to rewrite v. large/small numbers

$$a \times 10^{n}$$

$$a \in [0, 10[$$

e.x. Decimal Notation v.s. Scientific Notation

$623\,000$ $\quad$ 6.23×10^{5}

0.00478 $\quad$ 4.78×10^{-3} (same value) $= 47.8 \times 10^{-4}$ → not S.N. cuz 2 digits

Convert to Decimal Notation

i. 5.89×10^{2}

5.89×10^{2}

589

ii. 5.89×10^{4}

5.8900×10^{4}

58 900

iii. 5.890000×10^{6}

5 890 000

iv. 5.89×10^{-2}

$0.05.89 \times 10^{-2}$

0.0589

v. 5.89×10^{-4}

$0.0005.89 \times 10^{-4}$

0.000589

nota bene

- when the exponent is positive, the decimal notation is a large #.

when exp. is (−) # is v. small

You do:

Convert to Decimal Notation

i. 5.89×10^{3} $= 5890$

ii. 5.89×10^{5} $= 589\,000$

iii. 5.89×10^{-3} $= 0.00589$

iv. 5.89×10^{-5} $= 0.0000589$

Convert to Scientific Notation

ex. 730 000

730 000.

7.3×10^{5}

ex 1 764 320

1.764 320.

1.76×10^{6}

ex. 0.0000987 mL

0.0000987 mL

9.87×10^{-5} mL

nota bene

- when the exponent is positive, the decimal notation is a large #.

when exp. is (-) # is v. small

You do:

Convert to S.N.

i. 890 000 000 cm^2 = 8.9×10^8 cm^2

ii. 0.00017 mm = 1.7×10^{-4} mm

iii. 9.87 890 daL = 9.88×10^5 daL

9.8789

9.88

iv. 0.0000 9.027 mm = 9.03×10^{-5} mm

Performing Operations w S.N.

ex. Evaluate

law 3 w coefficient: $r \cdot a^m \times s\, a^n = (r \times s)\, a^{m+n}$

$(3.45 \times 10^3) \times (9.7 \times 10^{-1})$

$3.45 \times 9.7 \times 10^3 \times 10^{-1}$

calculator — law 3

$33.465 \times 10^{3+(-1)}$

33.465×10^2 ← not scientific n.

convert

$3.35 \times 10^1 \times 10^2$

law 3

3.35×10^3

You do Question 2 a) (after lunch) and then try 2b) Division

2. b) Evaluate

$$(0.23 \times 10^{4}) \div (6.8 \times 10^{-2})$$

$$\frac{0.23 \times 10^{4}}{6.8 \times 10^{-2}}$$

$$0.0338 \times 10^{4-(-2)}$$

$$0.0338 \times 10^{6}$$

$$3.38 \times 10^{-2} \times 10^{6}$$

$$3.38 \times 10^{4}$$

Understanding Radicals

$\sqrt{4}$

nota bene: square rooting is the opposite operation of squaring

a) equation

$(2 \times 3)^2 = 6^2$

$4 \times 9 = 36$

$(\sqrt{x})^2 = (3)^2$

$x = 9$

b) expression

$\sqrt{4}$

$\sqrt{2^2}$

$\sqrt{2^2}$

2

$+ \longleftrightarrow -$

$\times \longleftrightarrow \div$

$x^2 \longleftrightarrow \sqrt{x}$

} useful for solving

can't do anything; must only rewrite

put in calculator

$\sqrt{}$ x^2

Simplifying Radicals w/out Calculator

ex. Rewrite the base/radicands as exponential numbers w/ the indicated base.
(trial and error)

i. w/ base 3

$\sqrt{9}$

$\sqrt{3^2}$

3

ii. base 3

$\sqrt[3]{27}$

$\sqrt[3]{3^3}$

3

iii. base 3

$\sqrt[4]{81}$

$\sqrt[4]{3^4}$

3

y^x

$\frown$

iv. base 3

$\sqrt[6]{729}$

$\sqrt[6]{3^6}$

Converting Radicals to Exponents

(using laws #10 - 12)

$\sqrt{a} = a^{\frac{1}{2}}$ ⟵ (useful)

$\sqrt[n]{a} = a^{\frac{1}{n}}$ ⟵ (general)

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$ (when base is exp #)

$\sqrt[3]{x} = x^{\frac{1}{3}}$

$\sqrt[4]{x} = x^{\frac{1}{4}}$

$\sqrt[2]{x} = x^{\frac{1}{2}}$

$\sqrt[4]{a^8} = a^{\frac{8}{4}} = a^2$

$\sqrt[3]{a^9} = a^{9/3} = a^3$

$\sqrt[2]{x^2} = x^{\frac{2}{2}} = x^1 = x$

$\sqrt[2]{4} = 4^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2^{\frac{2}{2}} = 2$

$\sqrt[2]{2^2} = 2$

$\sqrt{9} = 9^{1/2}$

$\sqrt[2]{3^2} =$

You do:

Convert to exponential form (law 10 – 12)

i. $\sqrt{5^4} = 5^2$

ii. $\sqrt[3]{2^6} = 2^2$

iii. $\sqrt[3]{\left(\frac{x}{y}\right)^1} = \left(\frac{x}{y}\right)^{1/3} \neq \frac{x}{y}^{1/3}$

iv. $\frac{\sqrt[3]{x^1}}{y} = \frac{x^{\frac{1}{3}}}{y}$

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Converting Exp #'s to Radicals

ex. Convert to Radicals

i. $a^{\frac{1}{2}} = \sqrt{a}$

ii. $b^{\frac{1}{3}} = \sqrt[3]{b}$

iii $c^{6/5} = \sqrt[5]{c^6}$, the 5th root ....

$\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt[n]{a^m} = a^{m/n}$ law 12

iv. $3 \cdot x^{2/3} = 3 \cdot \sqrt[3]{x^2}$ law 12

iv
exp: 2/3
basi x
coefficient 3

v. $1 \cdot (3 \cdot x)^{2/3} = \sqrt[3]{(3x)^2} \neq \sqrt[3]{3x^2}$

exp: 2/3
base: 3.x
coefficient 1

vi. $(x^{1/2})^{3/4} = x^{\frac{1}{2} \times \frac{3}{4}} = x^{\frac{3}{8}} = \sqrt[8]{x^3}$ rewrite law 5

You do:

v.
exp: 2/3
base: x
coeffi: 2

i. $2x^{2/3} = 2 \cdot \sqrt[3]{x^2}$

ii. law 5 $(a^{3/4})^{1/5} = a^{\frac{3}{4} \times \frac{1}{5}} = a^{\frac{3}{20}} = \sqrt[20]{a^3}$

iii law 3 $x^{\frac{1}{4}} \times x^{2/4} = x^{\frac{1}{4} + \frac{2}{4}} = x^{3/4} = \sqrt[4]{x^3}$

iv. $2x^{3/5} \times 3x^{-2/5} = 6x^{\frac{3-2}{5}} = 6x^{\frac{1}{5}} = 6\sqrt[5]{x}$

exp $\frac{1}{5}$
base x
coeffi 6

Using BEDMAS and Rewriting to Simplify expressions.

ex. Rewrite as exp. in base 7 and simplify:

$\sqrt[4]{49} \times 7^{\frac{1}{2}}$

$\sqrt[4]{7^2} \times 7^{\frac{1}{2}}$

$7^{\frac{2}{4}} \times 7^{\frac{1}{2}}$

$7^{\frac{1}{2}} \times 7^{\frac{1}{2}}$ law 3

$7^{\frac{1}{2}+\frac{1}{2}}$

$7^{\frac{2}{2}} = 7^1 = 7$

Prime Numbers

2, 3, 5, 7, 11, 13, 17

4

2^2

step i. Rewrite in base 7

step ii. B E D M A S / Radical ✓

step iii Convert Rad → Exp

$\sqrt[n]{a^m} = a^{m/n}$

You do: Simplify:

i. $3^{1/2} \cdot \frac{1}{\sqrt[3]{729}} = 3^{5/2}$

ii. $\frac{1}{5^3} \times \sqrt[4]{25^{-5}} = \frac{1}{5^{11/2}}$

iii. $\sqrt[3]{\frac{27}{8}} \cdot \left(\frac{3}{2}\right)^2 = \frac{2}{3}$ ← Bonus cuz difficult

You do:
Exit ticket
HMWK:
handout 1 and 2

Prove that $\sqrt{a} = a^{\frac{1}{2}}$

$(\sqrt{a})^2 = a$ by definition

$\sqrt{a} \times \sqrt{a} = a$

$a^{\square} \times a^{\square} = a^1$

$\square + \square = 1$ law #3

$2\square = 1$

$\frac{2\square}{2} = \frac{1}{2}$

$\square = \frac{1}{2}$

$\therefore \sqrt{a} = a^{1/2}$ Q.E.D

$3^2 = 3 \times 3$

Prove $\sqrt[n]{a} = a^{1/n}$