Exit Ticket

Simplify. Given $y = -2x^2$

$x^2 + 3(-2 \times (-5x^2 - y))$

$x^2 + 3(-2(-5x^2 - (-2x^2)))$

$x^2 + 3(-2(-5x^2 + 2x^2))$

$x^2 + 3(-2(-3x^2))$

$x^2 + 3(6x^2)$

$1 \cdot x^2 + 18x^2$

$19x^2$

Lesson 2 : Laws of Exponents and Simplifying Alg. Expressions

March 26th 2024

Recall:

$2 + 2 + 2 = 3 \times 2$

$2 + 2 + 2 + 2 = 4 \times 2$

$2 + 2 + 2 + 2 + 2 = 5 \times 2$

Note:

$2 \times 2 = 2^2$

$2 \times 2 \times 2 = 2^3$

$2 \times 2 \times 2 \times 2 = 2^4$

$\underbrace{2 \times 2 \times \cdots \times 2}_{10 \text{ times}} = 2^{10}$

$\underbrace{2 \times 2 \times \cdots \times 2}_{n \text{ times}} = 2^n$

ex. Rewrite in Exp. Notation

$-3 \times -3 \times -3 \times -3 = (-3)^4$

(4 ← exp; -3 ← base; coefficient 1)

$-3 \times 3 \times 3 \times 3 = -3^4$

(base 3; coefficient (-1))

$-1 \cdot 3^4$

ex. Rewrite in Exp. Notation

$\frac{4 \times 4}{5 \times 5 \times 5} = \frac{4^2}{5^3}$

$\frac{-2 \times 2}{-3 \times -3 \times -3} = \frac{-2^2}{(-3)^3} = \frac{-1 \cdot 2^2}{(-3)^3}$

$m \times m \times m = m^3$

$x \cdot x \cdot x \cdot y \cdot y = x^3 \cdot y^2$

$(2 + y)(2 + y) = (2 + y)^2$

ex Rewrite by Expanding

$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$(-2)^3 = -2 \times -2 \times -2$

$-2^3 = -2 \times 2 \times 2 = -1 \cdot 2^3 = -1 \cdot 2 \times 2 \times 2$

ex You do

Expand

i. $\dfrac{(-6)^2}{-5^4}$

ii. $\left(\frac{2}{3}\right)^2$

iii. $-x^2$

iv. $(-y)^3$

v. $(x+4)^2$

vi. $(-x+y)^3$

Reading Laws 1 – 3

#1

$a^1 = a$

#2

$a^0 = 1$

ex. Simplify

$10^0 = 1$

^

y^x

$10^0 + 10^1 = 1 + 10 = 11$

$r a^n + s \cdot a^n = (r+s) a^n$

$(-2)^0 = 1$

$-2^0 = -1 \cdot 2^0 = -1 \cdot 1$

law #2

$= -1$

ex. Simplify

$(x+3)^0 = 1$

ex. Simplify

$-x^0$

$-1 \cdot x^0$

$-1 \cdot (1)$

-1

#3

$$a^m \times a^n = a^{m+n}$$

ex. Simplify

$3^2 \times 3^4$

3^{2+4}

3^6

ex $10^2 \times 10^1$

10^{2+1}

10^3

ex. $4^{-2} \times 4^2$

4^{-2+2}

4^0

1

ex. Simplify: Given $b = a^2$

$a^3 \times b$

$a^3 \times a^2$

a^{3+2}

a^5

You do:

i. Simplify

$4^2 \times 4 = 4^3$

ii. $-10^5 \times 10^{-5}$

$-1 \cdot \underbrace{10^5 \times 10^{-5}}_{\text{law 3}}$

$-1 \cdot 10^{5+(-5)}$

$-1 \cdot \underbrace{10^0}_{\text{law 2}}$

$-1 \cdot 1$

-1

iii. Simplify.

Given $x = -2$

$x^2 \times (-2)^4$

$= (-2)^6$

iv. $(x^3 y^{-1})(x y^5 z)$

$x^3 \cdot x \cdot y^{-1} \cdot y^5 \cdot z$

$x^4 \cdot y^4 \cdot z$

4

$$\frac{a^m}{a^n} = a^{m-n}$$

e.x. Simplify

$$\frac{4^3}{4^2} = 4^{3-2} = 4^1 = 4$$

e.x.

$$\frac{y^3}{y^{-3}} = y^{3-(-3)} = y^{3+3} = y^6$$

e.x.

$$\frac{x^4}{x^4} = x^{4-4} = x^0 = 1$$

$$\frac{x^4}{x^4} = 1$$

8

$$a^{-n} = \frac{1}{a^n} \quad \text{- or -} \quad a^{-n} = \left(\frac{1}{a}\right)^n$$

ex. Simplify. Final answer must not contain negative exponents.

$\frac{1}{2}a^{-2}$ (exp: -2, base: a)

$\frac{1}{2}a^{-2}$

$\frac{1}{2} \cdot \frac{1}{a^2}$

$\frac{1}{2 \cdot a^2}$

ex. Simplify

$2^1 \cdot 2^{-4}$

$2^{1+(-4)}$

2^{-3}

$\frac{1}{2^3}$

ex.

$-3^{-3} \cdot 3$

-3^{-3+1}

-3^{-2}

$\frac{1}{-3^2}$

8

$$a^{-n} = \frac{1}{a^n}$$

ex.

$-2 \cdot 3^{-4}$

$\frac{-2}{1} \cdot \frac{1}{3^4}$

$\frac{-2}{3^4}$

You do (last step)

ex. Simply. No negative exp. in final answer.

i. $(-3)^4 \times (-3)^3 = (-3)^7$

ii. $-3^4 \times 3^3 = -3^7$

iii. $(4x^{-2})(5x^6) = 20x^4$

iv. $(3a^2)(4b^4) = 12a^2b^4$

v. $3^2 \div 3^4 = \frac{1}{3^2}$

vi. $\frac{5^{10}}{5^4} = 5^6$

vii. $(a^3b^6) \div (a^3b^8) = \frac{1}{b^2}$

viii. $\frac{4x^3}{8x^5} = \frac{1}{2 \cdot x^2}$

ix. $(2.x^3)(4x^{-5}y) = \frac{8y}{x}$

vii. $(a^3 b^6) \div (a^3 b^8)$

$$\frac{a^3 b^6}{a^3 b^8}$$

$$a^{3-3} \; b^{6-8}$$

$$a^0 \; b^{-2}$$

$$1 \cdot b^{-2}$$

$$\frac{1}{b^2}$$

viii. $\frac{4x^3}{8x^5}$

ix. $(2x^3)(4x^{-5}y)$

$$2 \cdot 4 \cdot \underbrace{x^3 \cdot x^{-5}}_{\text{law 3}} \cdot y$$

$$8 \, x^{3+(-5)} \cdot y$$

$$8 \underbrace{x^{-2}}_{\text{law 8}} \cdot y$$

$$\frac{8y}{x^2}$$

$3^2 \div 3^4$

$\frac{3^2}{3^4}$

3^{2-4}

3^{-2}

$\frac{1}{3^2}$

#4 $\frac{a^m}{a^n} = a^{m-n}$

$a^m \div a^n = a^{m-n}$

5

$$(a^m)^n = a^{m \times n}$$

ex.

$$(2^3)^4 = 2^{3 \times 4} = 2^{12}$$

$$-(4^2)^5 = -1 \cdot (4^2)^5 = -1 \cdot 4^{2 \times 5} = -4^{10}$$

law 5

$$((-5)^3)^2 = (-5)^{3 \times 2} = (-5)^6 \neq -5^6$$

base -5 base 5

$15625 \neq -15625$

6

$$(ab)^n = a^n b^n$$

ex.

$$(2x^2y^6)^2$$

$$2^2(x^2)^2(y^6)^2$$

law 5 law 5

$$4x^{2\times 2}\cdot y^{6\times 2}$$

$$4\cdot x^4\cdot y^{12}$$

BEWARE!

$$(a+b)^n \neq a^n + b^n$$

$$(a+b)^2 \neq a^2+b^2$$

$$(a+b)(a+b)$$

$$a^2+ab+ab+b^2$$

$$a^2+2ab+b^2$$

7

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

ex.

$$\left(\frac{x^2}{y^{-3}}\right)^3$$

$$\frac{\left(x^2\right)^3}{\left(y^{-3}\right)^3} \quad \text{law 5}$$

$$\frac{x^{2\times 3}}{y^{-3\times 3}}$$

law #8 = law #9

$$\frac{x^6}{y^{-9}} \qquad -1\% \qquad \frac{1}{a^{-n}} = a^n$$

$$x^6 y^9$$

Simplify

Refer to Order of Operations
1. B
2. E
3. D, M
4. A, S

i. $\left(\frac{x^5}{x^2 y^3}\right)^{-1}$

$\left(\frac{x^{5-2}}{y^3}\right)^{-1}$

$\left(\frac{x^3}{y^3}\right)^{-1}$

$\frac{(x^3)^{-1}}{(y^3)^{-1}}$

$\frac{x^{-3}}{y^{-3}}$

$\frac{y^3}{x^3}$

$\frac{a^m}{a^n} = a^{m-n}$

You do:

iii. $(a^{-2} \div b^3)^2 \overset{\#7}{=} (a^{-2})^2 \div (b^3)^2 = a^{-4} \div b^6 = \frac{a^{-4}}{b^6} \overset{\#8}{=} \frac{1}{a^4 b^6}$

vii. $\left(\frac{x^4}{x^{-2}}\right)^3 = (x^{4-(-2)})^3 = (x^6)^3 = x^{18}$

homework: handout 1 and page 240 of the textbook

note to teacher: exit ticket was a bit difficult --- consider changing with a base that is a variable

BEDMAS

ex. Simplify. Given $y = -4 \cdot 3^2$

$-2 \cdot 3^4 + 5(3^2(-2 \cdot 3^2 - y))$

$-2 \cdot 3^4 + 5(3^2(-2 \cdot 3^2 - (-4 \cdot 3^2)))$

$-2 \cdot 3^4 + 5(3^2(-2 \cdot 3^2 + 4 \cdot 3^2))$

$-2 \cdot 3^4 + 5(3^2(2 \cdot 3^2))$

$-2 \cdot 3^4 + 5(2 \cdot 3^2 \cdot 3^2)$

law 3

$-2 \cdot 3^4 + 5(2 \cdot 3^4)$

$-2 \cdot 3^4 + 10 \cdot 3^4$

$(-2+10)3^4$

$8 \cdot 3^4$

$r \cdot a^n + s a^n = (r+s)a^n$

law # 3

$a^m \times a^n = a^{m+n}$

$-2x^2 + 4x^2 = 2x^2$

Prove : $a^m \times a^n = a^{m+n}$ L.S. R.S.

(Rewriting) abstractly

L.S.

$a^m \times a^n$

$= a \times a \times \cdots \times a \times a \times a \cdots \times a$

m times, n times, m + n times

$= a^{m+n}$

R.S. ▨

$a^m \times a^n = a^{m+n}$ ✓

concretely

$a \times a \times a \times a \times a$

3, 2

5 times

3 + 2

Prove #2: $a^0 = 1$

start

$a^0 \cdot a^1 = a^{0+1}$ law 3

$a^0 \cdot a^1 = a^1$ law 1

$\frac{a^0 \cdot a}{a} = \frac{a}{a}$

$a^0 = 1$ ∎