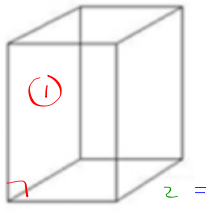
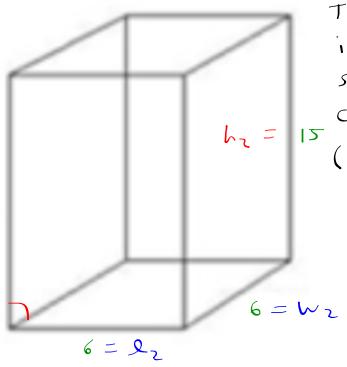
Lesson 14: Similar Figures Cont'd

April 18 2024

(focus on 3D solids + task)

ex The 2 prisms are similar:





$\frac{h_2}{h_1} = \frac{15}{5} = 3$

$\frac{l_2}{l_1} = \frac{6}{2} = 3$

$\frac{w_2}{w_1} = \frac{6}{2} = 3$

same k!

1

- scale factor
- ratio of similitude

Recall: The angles in similar solids are congruent (same "=")

- if sides are $3x$, then the areas are $9x$, the volumes are $27x$
- " " $4x$, " " $16x$, " " $64x$
- " " $5x$, " " $25x$, " " $125x$
- " " $6x$, " " $36x$, " " $216x$
- " " kx, " " k^2x, " " k^3x

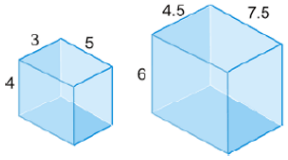
You do:

1.2.2 Example

Determine whether the following solids are similar:

How? find each ratio (k) and verify they're all equal.

a)



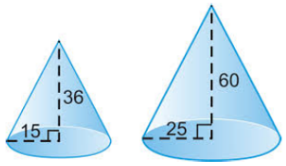
yes ✓

$\frac{2.5}{5} = \frac{3}{2} = 1.5$

$\frac{6}{4} = \frac{3}{2} = 1.5$

$\frac{4.5}{3} = \frac{3}{2} = 1.5$

b)



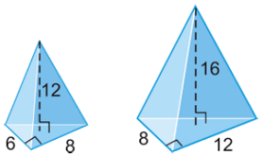
yes.

$\frac{h}{h} = \frac{60}{36} = 1.6$

$= 1.7$

$\frac{r}{r} \quad \frac{25}{15} = 1.7$

c)

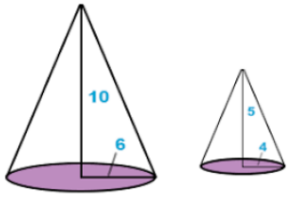


not similar ✗

$\frac{16}{12} = 1.3 \quad \frac{8}{6} = 1.3$

$\frac{12}{9} = 1.5$

d)



$\frac{h}{h} \quad \frac{10}{5} = 2$ ∴ not

$\frac{r}{r} = \frac{6}{4} = 1.5$ similar

How to find k:

$\frac{\text{side}}{\text{side}} = k$

$\frac{\text{area}}{\text{area}} = k^2$

$\frac{\text{volume}}{\text{volume}} = k^3$

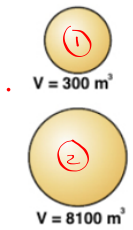
1 unknown → 1 equation (tool)

- 3 possible equations
- use 1 based on given info.

1.2.3 Similarity Ratio (Scale Factor) k

1.2.4 Example

Determine the similarity ratio (scale factor) k for the following *similar* solids:



$\frac{V_1}{V_2} = k^3$

$\sqrt[3]{\left(\frac{300}{8100}\right)} = \sqrt[3]{k^3}$

$k = \frac{1}{3}$

$k = 0.3$

if $k < 1$, then small / big

solve for k w/ 0.0.

$k^3 = \frac{V_2}{V_1}$

$\sqrt[3]{k^3} = \sqrt[3]{\frac{3100}{300}}$

$k = 3$

if $k > 1$, then big / small

You do: 1.2.5 a) b)

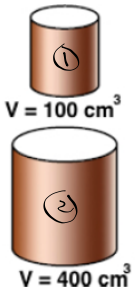
start: Bonus: 1.2.5 c)

Bonus: 1.2.6

1.2.5 Practice

Determine the similarity ratio (scale factor) k for each of the following *similar* solids:

a)



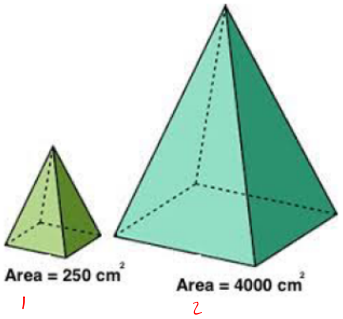
$k = 1.59$ ✓

$k^3 = \frac{V_2}{V_1}$

$\sqrt[3]{k^3} = \sqrt[3]{\frac{400}{100}}$

$k = 0.6^3$ ✓

b)



tool

$\frac{A_②}{A_①} = k^2$

$\sqrt{\left(\frac{4000}{250}\right)} = \sqrt{k^2}$

$k = 4$

sub

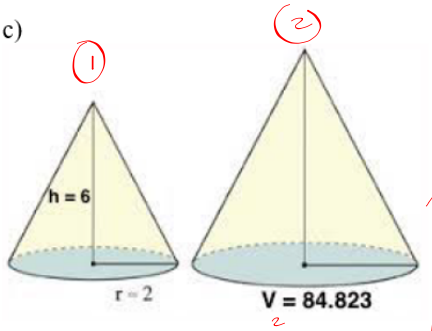
sim

solve

1.2.5 Practice

Determine the similarity ratio (scale factor) k for each of the following *similar* solids:

c)



#1

$V_P = \frac{\pi r^2 \times h}{3}$

$V = \frac{\pi (2)^2 \times 6}{3}$

$V = 8\pi$

$V \approx 25.13$

$V_2 = 84.823$

#2

$\frac{V_2}{V_1} = k^3$

$\sqrt[3]{\frac{84.823}{25.13}} = \sqrt[3]{k^3}$

$k = 1.5$

WANT: k

TOOL: $\frac{V_2}{V_1} = k^3$

INFO: $V_1 = ?$ $k = ?$ $V_2 = 84.823$

WANT: V_1

TOOL: $V_P = \frac{A_B \times h}{3}$

$V_P = \frac{\pi r^2 \times h}{3}$

INFO $r = 2$ $h = 6$

Dead end:

$\frac{\text{side}}{\text{side}} = k$

$\frac{h_2}{h_1} = k$

INFO: $h_2 = ?$ $h_1 = 6$

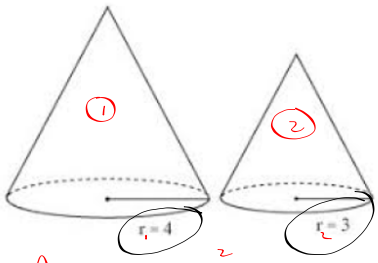
$V = \frac{\pi r^2 \times h}{2}$

$r = ?$

Solving (emphasis on cross-multiplication)

1.2.6 Example

The following Cones are *similar*. If the total area of the first cone is $23m^2$ and its volume is $12m^3$, determine the area and volume of the second cone.



$k^2 = \frac{A_{(1)}}{A_{(2)}}$

1st find k using a tool based on given info.

sub → $A_1 = 23\ m^2$

$V_{(1)} = 12\ m^3$

side / side $= k$

$\frac{r_1}{r_2} = k$

$k = \frac{4}{3}$ ← sub

find $A_{(2)}$

$\frac{A_{(1)}}{A_{(2)}} = k^2$. sub

. simplify

$\frac{23}{A_2} = \left(\frac{4}{3}\right)^2$

$\frac{23}{A_2} = \frac{4^2}{3^2}$

$\frac{23}{A_2} = \frac{16}{9}$

. when unknown in bottom cross multiply

$(23 \times 9) = 16 \cdot A_2$

solve

$\frac{(23 \times 9)}{16} = \frac{16 \cdot A_2}{16}$

$A_2 = 12.9375\ m^2$

find $V_{(2)}$

$\frac{V_{(1)}}{V_{(2)}} = k^3$

$\frac{12}{V_2} = \left(\frac{4}{3}\right)^3$

$\frac{12}{V_2} = \frac{64}{27}$

$\frac{(27 \times 12)}{64} = \frac{64 \cdot V_{(2)}}{64}$

$V_{(2)} = 5.06\ m^3$

+ −

× ÷

x^2 $\sqrt{x}$

You do:

1.2.7

1.2.8

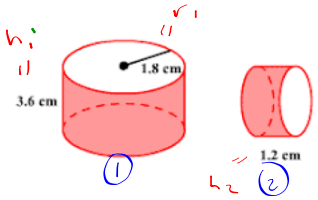
Bonus 1.3.1

You do: 1.2.8

1.2.7 Bonus 1.3.1 : $h = 6$ cm

1.2.7 Practice

The following cylinders are similar. Determine the area and volume of the smaller cylinder.



total area.

$k = \frac{\text{side}}{\text{side}}$

$k = \frac{h_1}{h_2}$

$k = \frac{3.6}{1.2}$

$k = 3$

find V_2:

i. $V_2 = \pi r^2 \times h$

ii $\frac{V_1}{V_2} = k^3$

$V_2 = \pi (0.6)^2 \times 1.2$

$V_2 = 1.357\ cm^3$

find r_2

$k = \frac{r_1}{r_2}$

sub

$k = 3$

$r_1 = 1.8$

$\frac{3}{1} = \frac{1.8}{r_2}$

$\frac{3 \cdot r_2}{3} = \frac{1.8}{3}$

$r_2 = 0.6$ cm

$A_2 = A_L + 2A_B$

$A_2 = P_B \times h + 2\pi r^2$

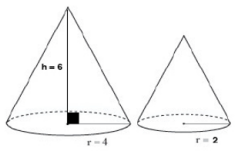
$A_2 = 2\pi r \times h + 2\pi r^2$

$= 2\pi(0.6) \times 1.2 + 2\pi(0.6)^2$

$A_2 = 6.786\ cm^2$

1.2.8 Practice

The following cones are similar. Determine the area and volume of the smaller cone.

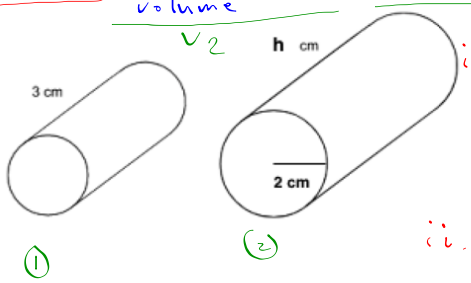


medium - task (not so wordy)

TIPS:
- LABEL
- translate sentences into equations. (directly)

1.3 Similarity, Solids and Missing Measurements

1.3.1 Example: The following two solids are similar. The volume of the smaller solid is 8 times smaller than the area of the larger solid. Determine the value of h.



volume

$V_1 = \frac{V_2}{8}$ ✓

$V_1 = 8 \times V_2$ x (wrong, cm2 word smaller!)

attention

$V_1 = \frac{1}{8} \times V_2$ ✓

- Key words: similar

want find h:

tool: $\frac{\text{side}}{\text{side}} = k$

info: $h_1 = 3$ cm, $h_2 = ?$, $k = ?$

want: k

$V_1 = \frac{V_2}{8}$

make a ratio of $\frac{V}{V}$

$8 \cdot V_1 = V_2$

$\frac{V_2}{V_1} = 8$

$\frac{V}{V} = k^3$

$\sqrt[3]{k^3} = \sqrt[3]{8}$

$k = 2$

$\frac{\text{side}}{\text{side}} = k$

$\frac{\text{area}}{\text{area}} = k^2$

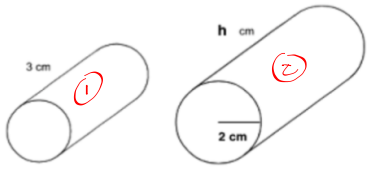
$\frac{\text{vol}}{\text{vol}} = k^3$

cat = chat

cat = قطة

∴ chat = قطة

since ∵



find h_2

$\frac{h_2}{h_1} = k$

$\frac{3 \times h_2}{3} = 2 \times 3$

$h_2 = 6$ cm

solve for h

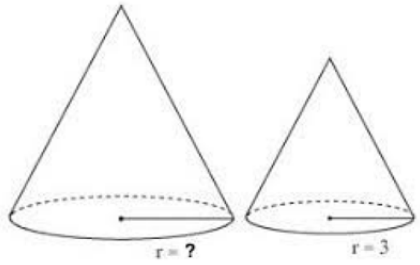
You do:

1.3.2

1.3.2 Practice

You do:

The following two cones are similar. The area of the smaller cone is 9 times smaller than the area of the larger cone. Determine the radius of the larger cone.



HMWK:

pg 194 #4.20 and #4.21 a) and b)

pg 195 #4.22

pg 200 #4.31

BONUS: pg 196 #3.24

} task like!

You do:

1.3.2 ans: $r = 9$

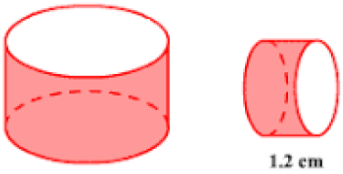
1.3.3 ans: $h = 3.6$ cm

1.3.4 ans: $A = 108$ cm^2 (Full)

start the task:

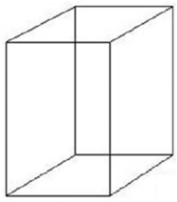
1.3.5

1.3.3 Practice: The following cylinders are similar. The volume of the larger cylinder is 27 times greater than the volume of the smaller cylinder. Find the height of the larger cylinder.



1.3.4 Practice

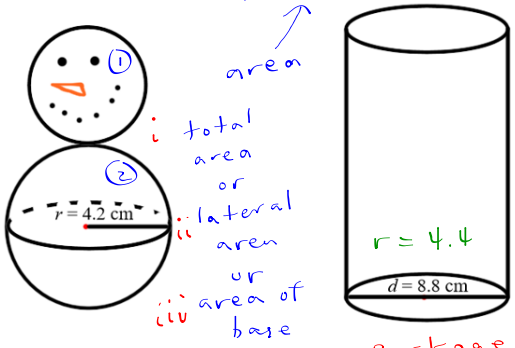
The following two rectangular prisms are similar. The volume of the smaller prism is one eighth times the volume of the larger prism. If the total area of the larger prism is 432 cm^2, what is the total area of the smaller prism?



1.3.5 Practice

You make three-dimensional handmade snowmen that you sell on Etsy. You need to find a cylindrical package to place the snowmen in before mailing them to customers. Below is an image of one of your snowmen and a possible cylindrical package. Your snowman is composed of two similar spheres. The volume of the smaller sphere is $\frac{125}{343}$ times the volume of the larger sphere. If the cylindrical packaging uses $5.22\,dm^2$ of cardboard, will the snowman fit?

area



i. total area or
ii. lateral area or
iii. area of base

package

yes or no, will the snowman fit in the box/package?

Want:
✓ i. diameter of sphere (2)
$d = 2r$
$d = 2(4.2)$
$d = 8.4$ cm ← lengthwise yes!

✓ ii. diameter of cylinder = 8.8 cm

Based on given info:

WANT:
i. height of snowman
ii. height of cylinder

Equation / TOOL:

$A_T = A_L + 2A_B$

$A_T = P_B \times h + 2\pi r^2$

$A_T = 2\pi r h + 2\pi r^2$

info to sub
- $r = 4.4$ cm
- $A = 5.22\,dm^2 \times 10^2$
- $A = 522\,cm^2$

Key words:
i. packaging : total area
ii. pipe : lateral area
iii. plastic cup : $A_L + A_B$
iv. water in cup : volume
v. parisian clay chimney. lateral area

$522 = 2\pi(4.4)h + 2\pi(4.4)^2$ · sub

$522 = 27.646h + 121.642$ · simplify

$-121.642 \quad -121.642$ · solve for h w/ 0.0.

BEDMAS

$\frac{400.358}{27.646} = \frac{27.646h}{27.646}$

$h_{cy} = 14.48$ cm

find $h_{sm} = r_1 + r_1 + r_2 + r_2$

$h_{sm} = 2r_1 + 2r_2$

want : r_1

tool : $\frac{r_1}{r_2} = k$

find k

$\sqrt[3]{k^3} = \sqrt[3]{\frac{125}{343}}$

$k = \frac{5}{7} = 0.7$

$\frac{r_1}{4.2} = \frac{5}{7}$ ×4.2

$r_1 = 3$

+ −
× $\frac{0}{0}$

$h_{sm} = 2(3) + 2(4.2)$

$h_{sm} = 14.4$ cm

$d_{sm} = 8.4$ cm

$h_{cylin} = 14.48$ cm

$d_{cylin} = 8.8$ cm

∴ yes the snowman will fit since its dimensions are smaller than the cylinder's.