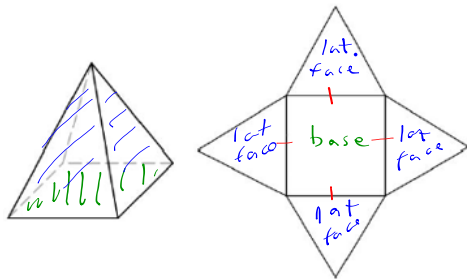Lesson 10: Area and Volumes of Regular Pyramids + medium tasks　　April 10th 2024

1   The Surface Area of Pyramids

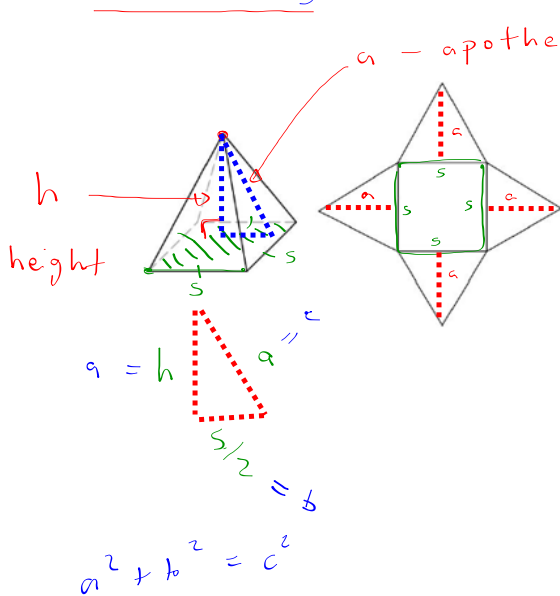## 1.1   How do we determine the Total Surface Area of a Pyramid?   pyramid

$$A_T = A_L + A_B$$

lat. face

lat. face — base — lat. face

lat. face

1.2   How do we determine the Lateral Area of a Pyramid?

Let's consider the net of a pyramid again. How would we go about determining the lateral area?

Terminology.

a — apothem (slanted height)

h — height

$a = h$

$\dfrac{s}{2} = b$

$a^2 + b^2 = c^2$

$$A_L = A_{D_1} + A_{D_2} + A_{D_3} + A_{D_4}$$

$$A_L = \frac{b \times h}{2} + \frac{b \times h}{2} + \frac{b \times h}{2} + \frac{b \times h}{2}$$

$$A_L = 1 \cdot \left(\frac{s \times a}{2}\right) + \left(\frac{s \times a}{2}\right) + 1 \cdot \left(\frac{s \times a}{2}\right) + \left(\frac{s \times a}{2}\right)$$

$$A_L = \frac{\boxed{4 \cdot s} \times a}{2}$$

$$A_L = 4 \cdot s \cdot \frac{a}{2}$$

Pyramid $\longrightarrow$ $\boxed{A_L = P_B \times \dfrac{a}{2}}$

prism $\longrightarrow$ $A_L = P_B \times h$

Pyramid

$$A_T = A_L + A_B$$
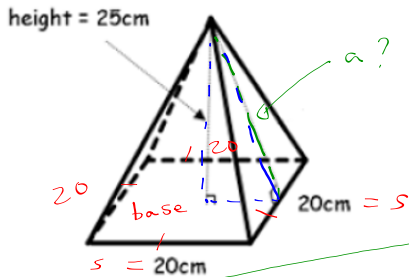
$$A_T = P_B \cdot \frac{a}{2} + A_B$$

prism

$$A_T = A_L + 2A_B$$

$$A_T = P_B \times h + 2A_B$$

In notebook, write : refer to L10, ho1, Pg 2, ex 1.3.1 total area.

1.3.1 Example: Robin wants to make a model of a pyramid tent in the shape of a square pyramid. He buys 2.5 bags of canvas. It cost him $5.60 per bag and each one contains 600 cm^2 of canvas. Will the 2.5 bags be enough to make his tent model?

WANT:

(i) amount of canvas we need

(ii) amount of canvas we have

$5.60 \frac{\$}{bag} \times 2.5 \ bag$

unit   unit

wrong → $\$$

$2.5 \ bags \times 600 \frac{cm^2}{bag}$

$1500 \ cm^2$ of canvas we have

height = 25cm

a ?

20 base $20cm = S$

$s = 20cm$

subir

$A_L = P_B \times \dfrac{a}{2}$

$A_B = S^2$

label the diagram a and S

$$A_T = A_L + A_B$$
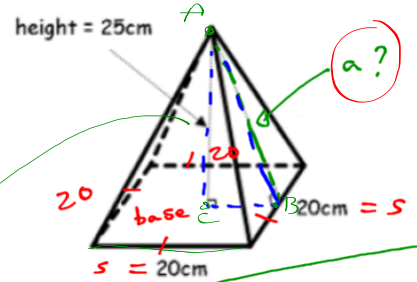
$$A_T = P_B \times \frac{a}{2} + S^2$$

$P = s + s + s + s$

$P = 4 \cdot s$

$$A_T = 4 \cdot s \times \frac{a}{2} + S^2$$

height = 25cm

A

a ?

20 base C B $20cm = S$

$s = 20cm$

use pythagorus theorem to find a!

A

$h = 25$ $= b$

$a = c$

C B

$10 = \frac{s}{2}$ $= a$

$c^2 = a^2 + b^2$ .sub

$\sqrt{c^2} = \sqrt{(10^2 + 25^2)}$ .simplify

.solve for c

$c = 26.9 \ cm$
$c = 5\sqrt{29} \ cm$
$a = 26.9 \ cm$
↳ apothem.

$$A_T = 4 \ (\cdot 20) \cdot \frac{26.9}{2} + 20^2$$
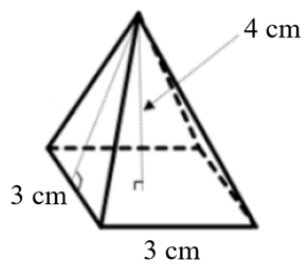
$$A_T = 1476 \ cm^2 \text{ of canvas we need.}$$

we have: $1500 \ cm^2$

∴ Yes, we do have enough since $1500 cm^2 > 1476 cm^2$.
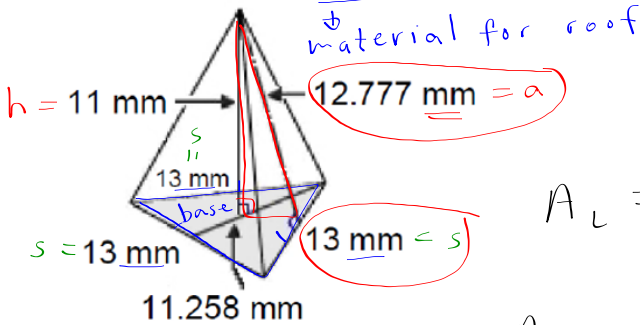
You do: Practice 1.3.2

1.3.2 Practice: This time Robin wants to be thrifty when making his tent model, so he reduces the dimensions of the square pyramid as shown below. He buys 3.5 bags of plastic wrap and 2 highlighters. If each bag contains $9cm^2$ of plastic, will he have enough to make his tent model?

4 cm

3 cm

3 cm

we have:

31.5 cm² of plastic

we need:

34.63 cm² of plastic

1.3.3 Example: An architect buys 2 packs of shingles to use on the roof of the model of a triangular pyramid with the dimensions indicated in the drawing below. If each pack covers 120cm^2 and contains ⑤⑤ shingles, will the architect have enough to shingle the roof?

$h = 11$ mm   12.777 mm $= a$

13 mm
base

$s = 13$ mm   13 mm $\leftarrow s$

11.258 mm

You do
_1.3.4_

Want

ⓘ Lateral area?
we need
material for roof

$$A_L = P_B \times \frac{a}{2}$$

$$A_L = 3 \cdot s \cdot \frac{a}{2}$$

$$A_L = 3 \cdot 13 \cdot \frac{12.777}{2}$$

$$A_L = 249.15 \text{ mm}^2$$

∴ we don't
have enough!

ⓘⓘ we have

55 shingles 2 pack shingles

$\frac{120 \text{ mm}^2}{\text{pack}} \times 2$ packs

$240 \text{ mm}^2 \leftarrow$ we have

sub in:

$P_B = s + s + s$

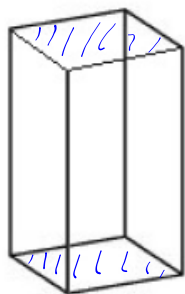$P_B = 3s$

label the diagram
s a.

← we need

1.3.4 Practice: The architect makes a different model of a triangular pyramid and wants to protect it with a waterproof tarp. He will use half a tarp (that) measures $700mm^2$ and that costs \$2. Will he have enough to cover the pyramid?
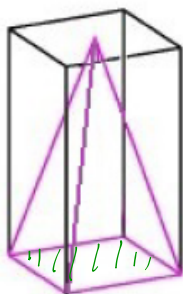
10 mm ← → 12.207 mm

14 mm

14 mm   14 mm

12.124 mm

You do

. tarp

. bâche

. no base

we have :   we need :

$350 mm^2$ $256.38 mm^2$

of tarp   of tarp.

∴ yes, he will have enough

Volume of a Pyramid

- Prism
- 2 bases

- pyramid (inside a prism)
- 1 base

$$V = A_B \times h \qquad V = \frac{A_B \times h}{3}$$

write on memory aid:

Prisms: (2 bases)

$A_L = P_B \times h$

$A_T = A_L + 2A_B$

$A_T = P_B \times h + 2A_B$

$V = A_B \times h$

Pyramids: (1 base)

$A_L = \dfrac{P_B \times a}{2}$

$A_T = A_L + A_B$

$A_T = \dfrac{P_B \times a}{2} + A_B$

$V = \dfrac{A_B \times h}{3}$

Where,

A_L = lateral area (3D)

P_B = Perimeter of base (2D)
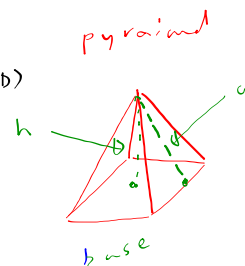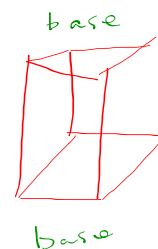
h = height

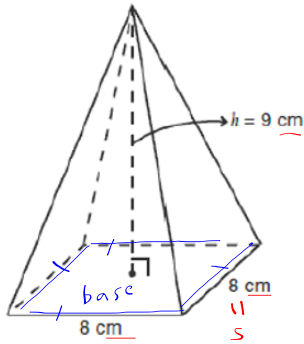A_T = total area (3D)

A_B = area of base (2D)

V = Volume (3D)

a = apothem

base

base

pyramid

base

Finding the Volume and Converting — memory aid

2.2.1 Example: Find the volume of the following pyramid. Give final answer is dL.

h = 9 cm
base
8 cm
8 cm   s

square

$$V = \frac{A_B \times h}{3} \qquad A_B = s^2$$

$$V = \frac{s^2 \times h}{3}$$

triangle

$$A_B = \frac{b \times h}{2}$$

$$V = \frac{8^2 \times 9}{3}$$

$$V = 192 \, cm^3 \quad \leftarrow \text{convert to } dL$$

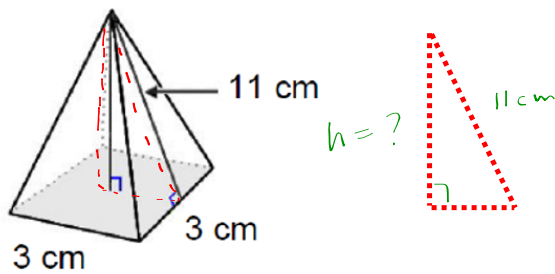$$V = 192 \, mL \qquad 1 \, cm^3 = 1 \, mL$$

$$192 \div 10 \div 10$$

$$V = 1.92 \, dL$$

You do :

2.2.2

2.2.3

2.2.2 Practice: Find the volume of the following pyramid. Give the final answer in *cL*.

11 cm

h=?

11 cm

3 cm

3 cm

$h = ?$

use

$a^2 + b^2 = c^2$

$h = 10.897$

$V = 32.7 cm^3$

$= 32.6 mL$

$= 3.27 cL$

2.2.3 Practice

Find the volume of the following pyramid. Give the final answer in mL.

h_{3D}
11

11 mm   12.777 mm

13 mm

$b = 13$ mm   13 mm

$h_{2D} = 11.258$ mm

$$V = \frac{\left(\left(\frac{b \times h_{2D}}{2}\right) \times h_{3D}\right)}{3}$$

$$V = \frac{\left(\left(\frac{13 \times 11.258}{2}\right) \times 11\right)}{3}$$

$$V = 268.32 \frac{mm^3}{\circ 10^3}$$

$$V = 0.2683156 \; cm^3$$

$$V = 0.268 \; mL$$

$$V = \frac{A_B \times h_{3D}}{3}$$

$$A_B = \frac{b \times h_{2D}}{2}$$

$$A_B = \frac{13 \times 11.258}{2}$$

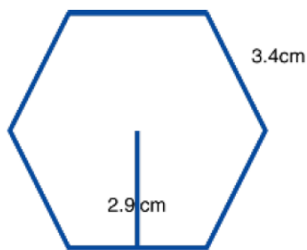$$V = \frac{73.177 \times 11}{3}$$

$$A_B = 73.177 \; mm^2$$

$$\boxed{V = 268.3156 \; mm^3}$$

$$1 \; cm^3 = 1 \; mL$$

$$10 \; cm^3 = 10 \; mL$$
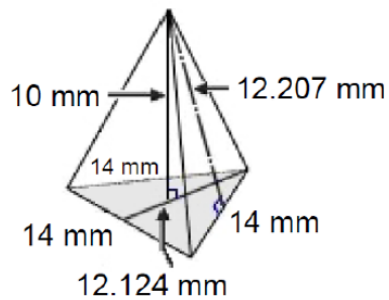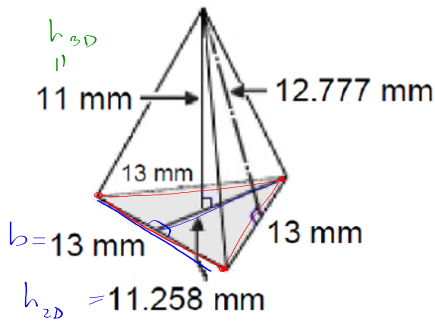
2.2.4 Practice

Calculate the volume of a pyramid whose height is 17 cm and whose base is the regular hexagon as shown below. How much water (in L) could this pyramid contain?

3.4cm

2.9 cm

medium - level task

strategies for task:
→ take notes and label.

we have:

3.1.1 Example: Robin is now making the actual tent that is in the shape of a square pyramid whose volume is 180 000 daL. He buys 2.5 bags of canvas. It cost him $50.60 per bag and each one contains 600 m^2 of canvas.
↳ If the height of the pyramid is 8 m, will the 2.5 bags be enough to make his tent?

$V = 180\ 000\ daL$

$2.5\ bags \times 600\ m^2/bag$

$= 1500\ m^2$ of canvas

(equation)

h = 8 m   a = ?
s = 25.98 m
s = 25.98

Which tool to find the unknown measurements?

It's based on given info?

$V = \dfrac{A_B \times h}{3}$ $A_B = s^2$

must convert to m^3

$V = \dfrac{s^2 \times h}{3}$

sub in info:

$h = 8\ m$

$V = 180\ 000\ daL$
$\div 10 \div 10$
$V = 1800\ kL$

$V = 1800\ m^3$

solving
B
E
D
M
A
S

· sub
· simply
· solve

$\overline{w}$ $\dfrac{o.o.}{=}$
$+$ $-$
$\times$ $\dfrac{o}{o}$
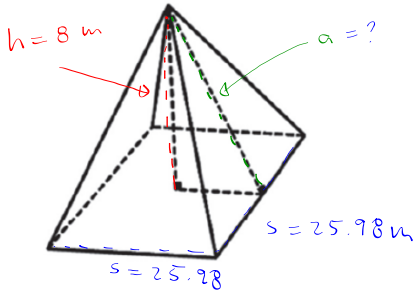x^2 $\sqrt{x}$

$3 \times 1800 = \left(\dfrac{s^2 \times 8}{3}\right) \times 3$

$\dfrac{5400}{8} = \dfrac{s^2 \times 8}{8}$

$\sqrt{675} = \sqrt{s^2}$

$s = 25.98\ m$ ✓

3.1.1 Example: Robin is now making the actual tent that is in the shape of a square pyramid whose volume is 180 000 daL. He buys 2.5 bags of canvas. It cost him $50.60 per bag and each one contains 600 m^2 of canvas.
↳ If the height of the pyramid is 8 m, will the 2.5 bags be enough to make his tent?

h = 8 m   a = ?
s = 25.98 m
s = 25.98
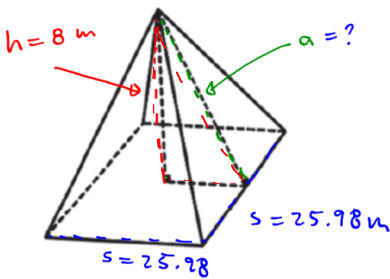
we have
1500 m^2 of canvas ✓

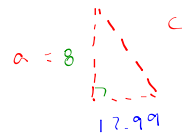WANT: $A_B = s^2$

$A_T = A_L + A_B$ $A_L = P_B \times \dfrac{a}{2}$

$A_T = P_B \times \dfrac{a}{2} + s^2$

find apothem:
$c^2 = a^2 + b^2$

$A_T = 4 \cdot s \cdot \dfrac{a}{2} + s^2$ a = 8

$P_B = s + s + s + s$

$P_B = 4s$

$A_T = 4(25.98) \cdot \dfrac{15.26}{2} + 25.98^2$

$A_T = 1467.87\ m^2$
canvas we need!

$\sqrt{c^2} = \sqrt{8^2 + 12.99^2}$
$c = 15.26\ m.$
apothem

12.99

1500 m^2 - canvas we have.

∴ yes we will have enough
since 1500 > 1467

Exit Ticket:

3.1.2 Practice: Robin decides to make a thriftier tent still in the shape of a square pyramid with reduced measurements. The tent has a capacity of 16 000 hL. He buys 3.5 bags of plastic tent material and 2 highlighters. If each bag contains $9m^2$ of plastic and one side of the base is $18\,m$, will he have enough plastic to make his tent?
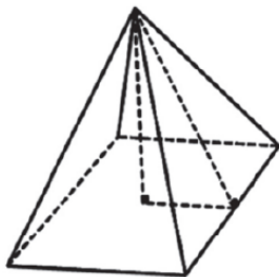
You do

• 3.1.2 on exit ticket

• do 2nd exit ticket from yesterday

hand in for feedback

• do in-class assignment

• do. 3.1.3

homework
p 115   #3.9 b)
        # 3.10 b)
p 134   #3.20
p 142 #3.24
p 143 #3.26

$\longrightarrow h = 14.814\,m$ apothem $= 17.33\,m$

$A_T = 948.04\,m^2$ (we need)

$3.5\,bags \times 9\,m^2/bag$

$= 31.5\,m^2$ (we have)

∴ no he won't have enough

3.1.3 Example: The architect is now making the actual triangular pyramid whose roof he will shingle with 3.5 bags of shingles. The total surface area of the pyramid is $96.588m^2$. Each bag covers $25m^2$ and contains 28 shingles. If the apothem of the pyramid is $9\,m$ and the height of the base is $3\sqrt{3}\,m$ ($or\ 5.196m$), will the 3.5 bags be enough to shingle the roof?
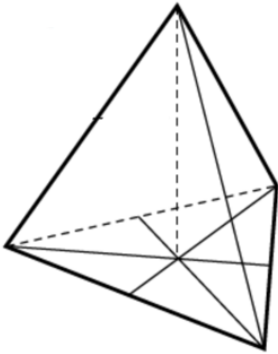
side $= 6\,m$
base

$A_L = 81\,m^2$
(we need)

He bought
$87.5\,m^2$ of shingles (we have)

$\therefore$ yes he does have enough