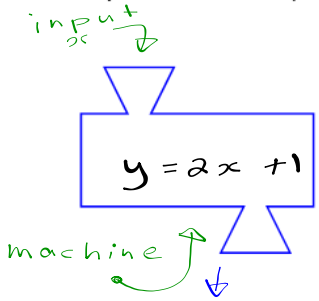
Lesson 7: Functions and Relations April 4th, 2023

Characteristics / Properties of a Function, that is Reading the Story of the Graph

1 An Introduction to Functions

context

Let's try to understand what a *function* is by thinking about it as a machine:



Recall:
- independent variable is x (sthg we input)
- dependent variable is y (sthg the equation outputs)

** y depends on x
y is a function of x
- or -
[slightly different: there's a relation between x and y]

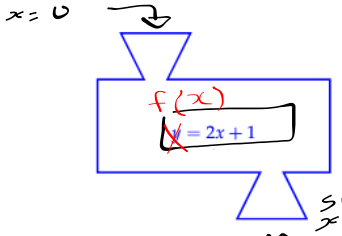
1.1 Example

Complete the associated table of values for the following function and then plot the points on a Cartesian Graph and complete the graph of the function:

(points)

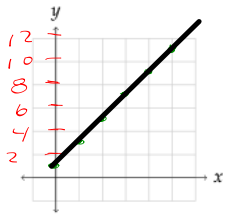
TOV - a list of ordered pairs (x, y)

x-coordinate, y-coordinate



sub x into equation

x	y
0	1
1	3
2	5
3	7
4	9
5	11



- plot points
- draw line or curve!

* if axes are not numbered, we assume you're going up by 1.

x | y

0 | $f(x) = 2(0) + 1$, $y = 1$

1 | $f(x) = 2(1) + 1$, $y = 3$

2 | $y = 2x + 1$, $y = 2(2) + 1$, $y = 5$

3 | $y = 2x + 1$, $y = 2(3) + 1$, $y = 7$

4 | $y = 2x + 1$, $y = 2(4) + 1$, $y = 9$

5 | $y = 2x + 1$, $y = 2(5) + 1$, $y = 11$

- sub in x's value
- evaluate

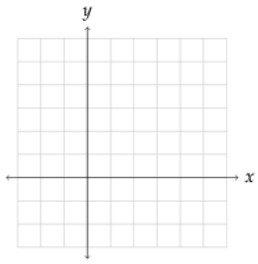
1.2 Practice

you do pg 2 a) and b)

(a) Complete the associated table of values for the following function and then plot the points on a Cartesian Graph and complete the graph of the function:

$f(x) = -x + 3$

x	$f(x)$
0	
1	
2	
3	
4	
5	

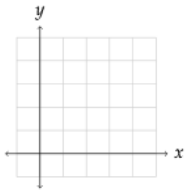


these ones

(b) Complete the associated table of values for the following function and then plot the points on a Cartesian Graph and complete the graph of the function:

$f(x) = \frac{10}{x}$

x	y
1	
2	
3	
4	
5	



1.3 Practice

- MHS Workbook p. 41-42

this rectangle is NOT a square.

$A = s_2^2$

$y = x$

2 Functions and Relations

A *relation* is a "link" between two variables (usually x and y). *Functions* are special kinds of relations. Indeed, functions are sometimes called: functional relations

In a function,

Q: What is so special about *functional* relations?

For any given input (x) there is one and only one output (y).

Q: Are all relations functions?

In a relation, a given input (x) could have multiple outputs (y).

In function/relation the outputs (y) can have multiple inputs (x)

- a square is a rectangle.
- every square is a rectangle, but not every rectangle is a square.

Similarly, every function is a relation, but not every relation is a function.

Question: Is the square function the or the rectangle?

Page 4

2.2 Determining whether a Relation is a Function from a Table of Values

R call the d fin tion f a funct n: Proximity

Functions and Relations

A *function* is a relation between an independent variable (input - x) and a dependent variable (output - y) such that:

- for any given input value (x) there is **one and only one** output value (y)

If a relation is *not* a function, then we simply call it is a *relation*. If a relation is a functional relation, then we simply call it a *function*.

Now consider the following relations. Determine whether they are functions or not.

x	y
0	10
1	9
2	8
3	7
4	6
5	5

· no repeats for x.
∴ ✓ a function

x	y
0	10
4	9
5	8
3	7
4	6
6	5

· there are repeats for x
∴ ✗ NOT a function

x	y
0	10
2	10
4	8
3	7
5	10
7	5

· no repeats for x
∴ ✓ a function
· there are repeats y and that's okay!

2.2.1 Practice

Consider the following relations. Determine whether they are functions or not.

x	y
0	-3
10	-3
20	-3
30	-3
40	-3
50	-3

x	y
-1	10
2	9
5	8
8	7
11	6
14	5

x	y
a	10
b	10
c	8
a	7
e	10
f	5

2.3 Determining whether a Relation is a Function from a Diagram

Pg 5

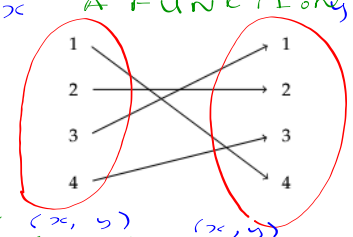
A *function* is a relation between an independent variable (input - x) and a dependent variable (output - y) such that:

for any given input value (x) there is **one and only one** output value (y)

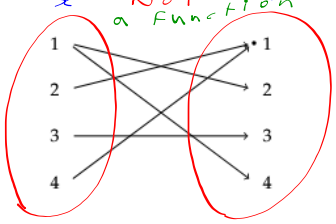
If a relation is *not* a function, then we simply call it is a *relation*. If a relation is a functional relation, then we simply call it a *function*.

Consider the following relations and determine which are functions and which are not.

x A FUNCTION y



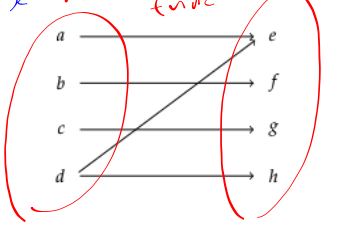
x NOT a Function y



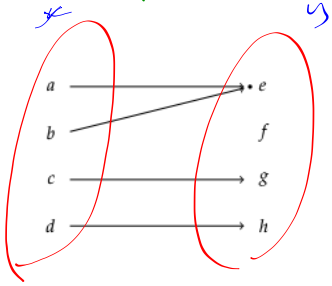
x	y
1	2
1	4

{ (x, y) (x, y) (1, 4), (2, 2), (3, 1) (4, 3) }

x NOT a function. y



x A FUNCTION y



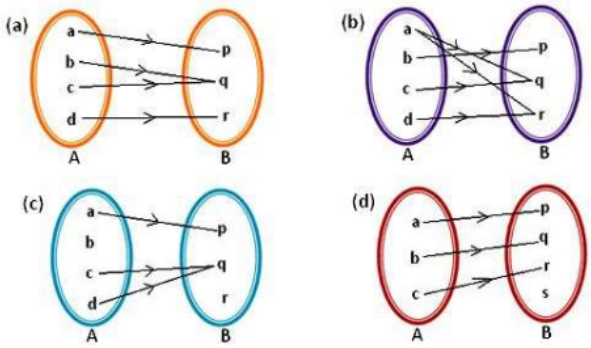
You do
Page 4
Q. 2.2.1
AND
Pg 6
Q 2.3.1
(check answers)

You do

from the textbook:
Pg 53 "consider" part
Pg 65 # 2.5 a) only

2.3.1 Practice

Consider the following relations. Determine which are functions and which are not:



2.4 Determining whether a Relation is a Function from a Set of Points (Ordered Pairs)

Recall the definition of a function:

Functions and Relations

A *function* is a relation between an independent variable (input - x) and a dependent variable (output - y) such that:

- for any given input value (x) there is **one and only one** output value (y)

If a relation is *not* a function, then we simply call it is a *relation*. If a relation is a functional relation, then we simply call it a *function*.

curly brackets a set

Consider the following relations. Determine which are functions and which are not:

1. $\{(1,1),(2,3),(4,5),(6,7)\}$ Function
2. $\{(1,1),(2,1),(3,1),(4,1)\}$ Function
3. $\{(1,1),(1,3),(4,7),(5,8)\}$ NOT a function cuz there's repeats for x.

(,), (,), (,) round brackets for ordered pairs (x, y)

x, y x, y x, y x, y

x x x x

2.4.1 Practice

Consider the following relations. Determine which are functions and which are not:

1. $\{(1,1),(2,3),(4,5),(6,7)\}$
2. $\{(0,0),(2,1),(3,0),(4,1)\}$
3. $\{(5,1),(-6,3),(5,7),(5,8)\}$

2.5 Determining whether a Relation is a Function from a Graph

R call the d fin tion f a funct n

Functions and Relations

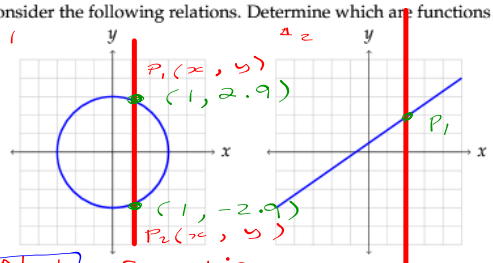
A *function* is a relation between an independent variable (input - x) and a dependent variable (output - y) such that:

- for any given input value (x) there is **one and only one** output value (y)

If a relation is *not* a function, then we simply call it is a *relation*. If a relation is a functional relation, then we simply call it a *function*.

Consider the following relations. Determine which are functions and which are not.

#1

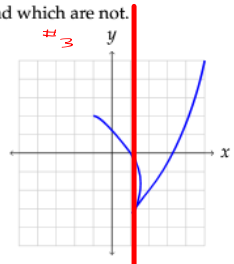


Not a function. cuz a vertical line drawn touches the relation @ 2 points.

#2

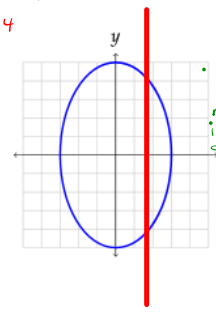
A function! 1 point.

#3



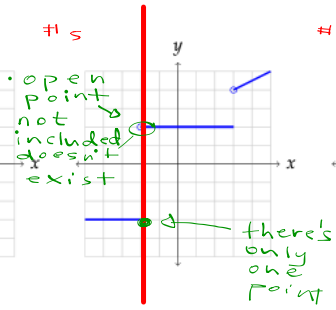
not a function

#4



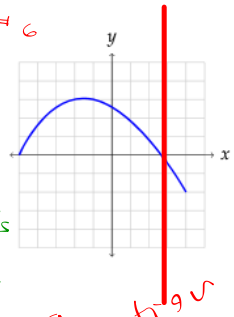
not a function

#5



a function

#6



a function

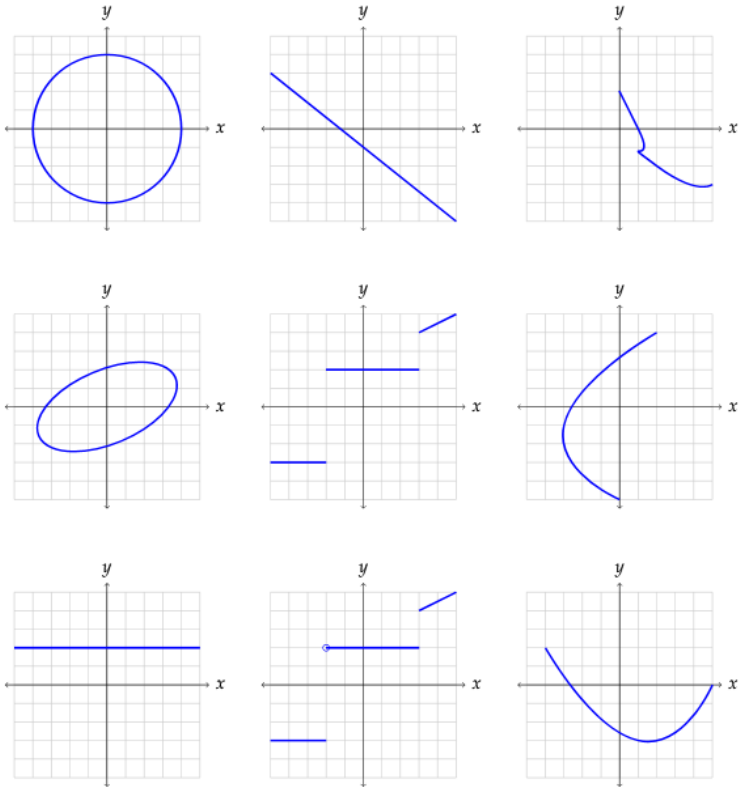
Do the vertical (up/down) Line, and check line goes through only one point.

You do:

pg 7
2.4.1
pg 9
2.5.1
and from textbook pg 54 "In a graph" part

2.5.1 Practice

Consider the following relations. Determine which are functions and which are not.



3 Function Notation

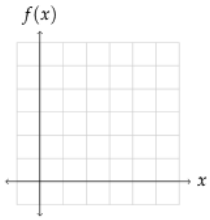
Let's now add a new piece of vocabulary in our mathematical language: *function notation*. We will see why this is actually quite useful and makes certain things easier.

3.1 Example

Complete the associated table of values for the following function and then plot the points on a Cartesian Graph and complete the graph of the function:

$f(x) = 2x + 1$

x	$f(x)$
0	
1	
2	
3	
4	
5	

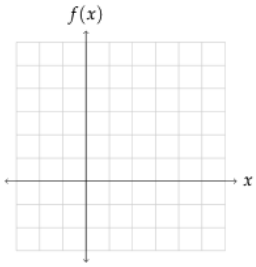


3.2 Practice

(a) Complete the associated table of values for the following function and then plot the points on a Cartesian Graph and complete the graph of the function:

$f(x) = -x + 3$

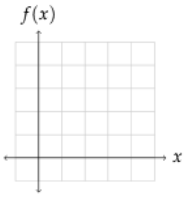
x	$f(x)$
0	
1	
2	
3	
4	
5	



(b) Complete the associated table of values for the following function and then plot the points on a Cartesian Graph and complete the graph of the function:

$f(x) = \frac{10}{x}$

x	$f(x)$
1	
2	
3	
4	
5	



3.3 Example: Determining Values in Function Notation

Now let's discover some reasons why function notation is algebraically useful:

Consider the following function:

$$y = 3x - 2$$

1. What is the value of y when $x = 2$?
2. What is the value of x when $y = 7$?

Consider the following function:

$$f(x) = 3x - 2$$

1. What is $f(2)$?
2. What is the value of x if $f(x) = 7$

3.4 Example: Determining Values in Function Notation

Now let's discover some reasons why function notation is algebraically useful:

Consider the following function:

$$y = x^2 - 2x + 1$$

1. What is the value of y when $x = 2$?
2. What is the value of y when $x = -3$?

Consider the following function:

$$f(x) = x^2 - 2x + 1$$

1. What is $f(2)$?
2. What is $f(-3)$?

3.5 Practice

Given the following function:

$$f(x) = \frac{100}{x}$$

(a) What is $f(25)$?

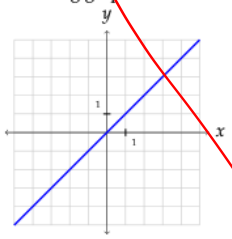
(b) What is $f(-2)$?

(c) What is the value of x when $f(x) = 1000$

(d) What is the value of x when $f(x) = -2$?

3.6 Example: Function Notation and Graphs

Consider the following graph and determine which claims are *true* and which are *false*



$f(1) = 1$

$f(0) = -1$

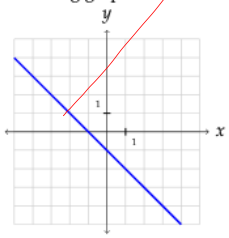
- $f(3) = 3$

$f(\ \ 2) = \ \ 2$

$f(2+3) = -5$

3.7 Practice

Consider the following graph and determine which claims are *true* and which are *false*



$f(1) = 1$

- $f(0) = -1$

$f(3) = -3$

$f(-1) = 0$

$f(3-3) = 0$

3.8 Practice

MHS Workbook p. 44-45

P 15

4 Properties of a Function

(characteristics)

In this section we will cover the following properties of functions:

Reading the Graph of Function Left → Right

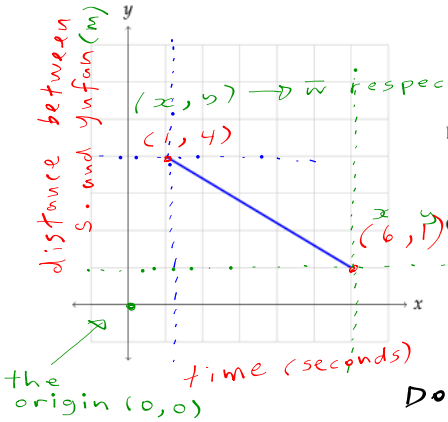
Domain and Range

- x and y intercepts, Maximum and Minimum values

Variation of a Function: Increasing, Decreasing, Strictly Increasing, Strictly Decreasing

4.1 Domain and Range of a Function

Consider the following function:



(x, y) → with respect to (0,0) — the origin [min, max]

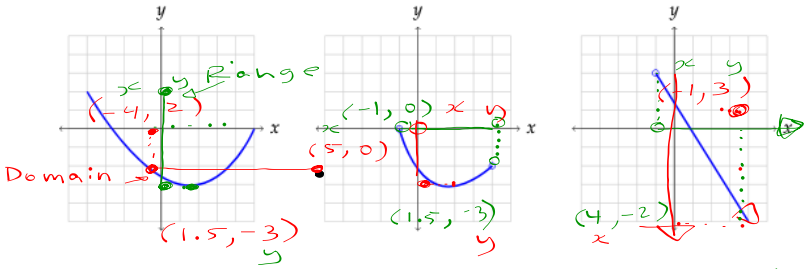
Domain: → the set / interval of x-values over which the function is defined / exists : [smallest x-value, biggest x-value]

Co-domain (Range): → the interval of y-values " " " [smallest y-value, biggest y-value]

Domain : [1, 6] Range : [1, 4]

4.1.1 Example

Determine the domain and co-domain (range) of the following functions (assume each grid is 1 unit):



TIPS For Properties

- Label points (x, y)
- Highlight the x-axis (the line called x) for Domain
- Highlight the y-axis for Range

D: [-4, 5] D]-1, 4[D]-1, ∞[(∞, -∞)

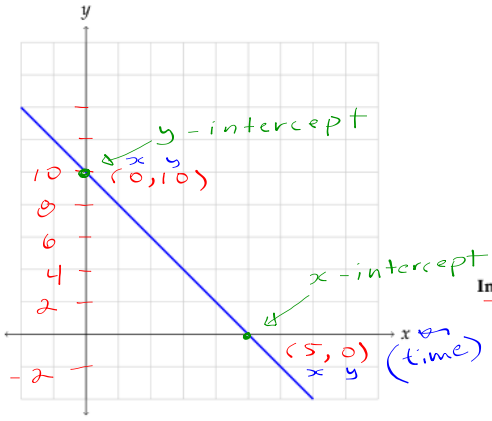
R: [-3, 2] R[-3, 0[R]-∞, 3[

4.1.2 Practice

MHS Workbook p. 46-47

4.2 The Zeros (x-intercepts) and Initial Value (y-intercept) of a Function

Consider the following function:

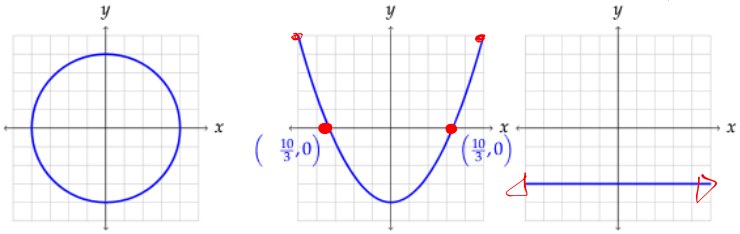


Zeros (x-intercepts): (x, y)
→ the point(s) where the function touches the x-axis. (5, 0)
→ the y-coordinate is always zero.
ANS: (5, 0) or 5
x y
(answer)

Initial Value (y-intercept):
→ the point where the function touches the y-axis
→ the x-coordinate is always zero.

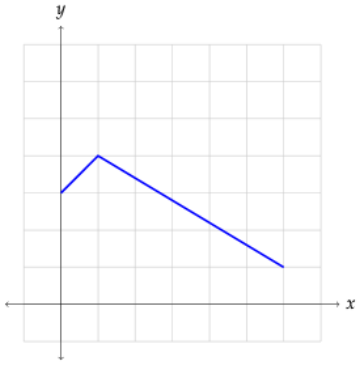
4.2.1 Practice

Determine the x and y intercepts of the following relations (assume each grid is 1 unit): AND Domain / Range for each.



4.3 Maximum and Minimum of a Function

Consider the following function:

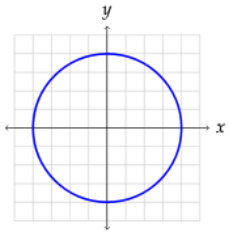


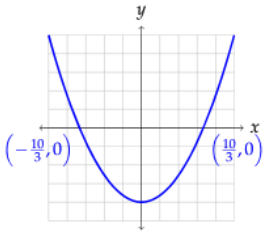
Maximum:

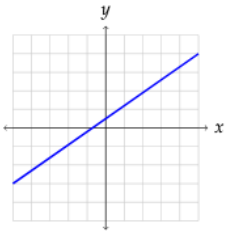
Minimum:

4.3.1 Practice

Determine the maximum and minimum of the following relations (assume each grid is 1 unit):

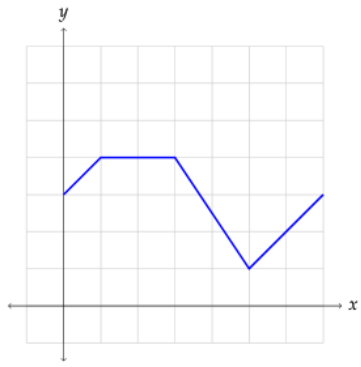






4.4 Variation of a Function

Consider the following function:



Increasing:

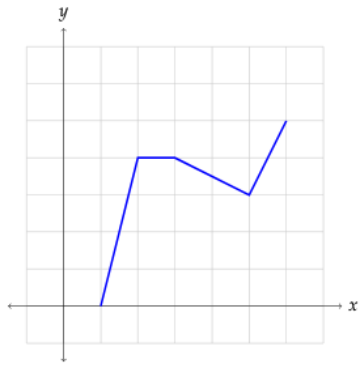
Strictly Increasing:

Decreasing:

Strictly Decreasing:

4.4.1 Practice

Determine the variation of the following function:



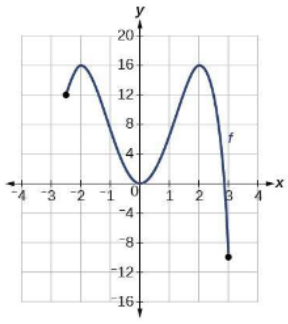
Increasing:

Strictly Increasing:

Decreasing:

Strictly Decreasing:

Determine the variation of the following function:



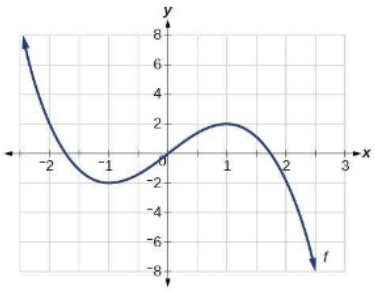
Increasing:

Strictly Increasing:

Decreasing:

Strictly Decreasing:

Determine the variation of the following function:



Increasing:

Strictly Increasing:

Decreasing:

Strictly Decreasing: