Lesson 8 : Properties ^of the function → y Cont'd and Intro to Linear Functions

April 8th, 2023

y depends on x

y is a function of x

↳ dependent variable
→ the function

↳ independent variable

Definition : Always read graph left to right.

Pg 79 :

Change/Variation of the function $f = f(x) = y$

[lowest x, highest x]

The Increasing Interval of the function (y) :

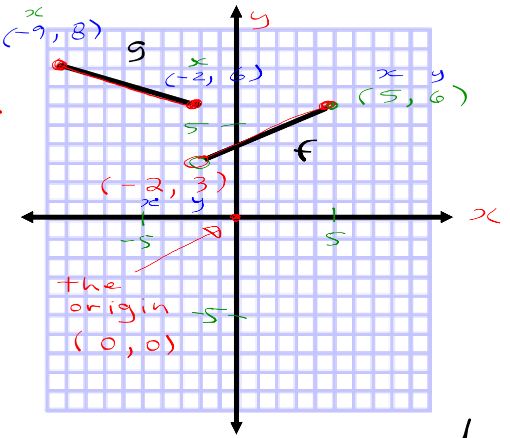
→ where the function goes up
→ where the y's increase as the x's increase.
→ give answer in terms of x

ex : f :]-2, 5]   g : ∅

The Decreasing Interval of the Function (y) :

→ where the function goes down
→ where the y's decrease as the x's increases
→ give answer in terms of x

ex f : ∅   g : [-9, -2]



plot (-9, 8)
plot (-2, 6)

closed dots

1st-Degree Functions of Linear Functions

$y = mx + b$

$f(x) = mx + b$

1 Graphing Linear Relations to $\boxed{\text{Model Real Life Situations}}$

Let's try to understand a linear function by seeing how it can *model* a real-life situation.

1.1 Example

Create a model for the following situation:

- Alex opens up a new savings account with nothing saved initially. Alex sets up a savings plan and begins saving $110 per month. (n.b. ~~Because of low interest rates~~, the ~~bank does not offer Alex any interest on his deposits~~.)

step i
Define variables

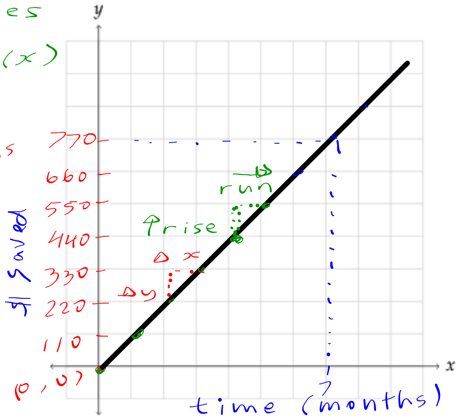
Dependent Variable: y / f(x)
$ Alex saves

Independent Variable: x
time : # of months

step ii: Fill out TOV

x	$f(x)$
0	0
1	110
2	220
3	330
4	440
5	550

+110 +110 +110 +110



LABEL x/y axes
step iii : Plot points in TOV

Answer Q1 and Q2
check w/ partner

blue points : Extrapolating

y-int = 0$

Q1: Use the graph and extrapolate to determine how much Alex will have saved after (at x) 7 months.

a) $770

Q1 b) use the graph and interpolate to determine how much $ Alex has at 3.5 months.

Q2: What is the rate of change (m) (slope) (m) of the linear function that models this situation?

$m = 110$ $/month
→ units of rate of change

• y / f(x)
• amount saved.

$m = \frac{\Delta y}{\Delta x} \Rightarrow m = \frac{\text{change in } y}{\text{change in } x} \Rightarrow m = \frac{\text{rise}}{\text{run}}$

(f of 0)

Find f(0), that is, find the y when x = 0
f(x)

$f(0) = 0$, $f(1) = 0 + 110$, $f(2) = 0 + \underbrace{110 + 110}_{2\,m} = 0 + 2(110)$

$f(3) = 0 + \underbrace{110 + 110 + 110}_{3\,m} = 0 + 3(110)$

$f(4) = 0 + \underbrace{110 + 110 + 110 + 110}_{4 \text{ months}} = 0 + 4(110)$

$\vdots$

$f(x) = 0 + \underbrace{110 + 110 + \cdots + 110}_{x \text{ months}} = 0 + x \cdot 110$

$y = 110x + 0$

110: (slope) rate of change
0: initial value (y-int)

1.2 Practice

You do.

Create a model for the following situation:

> Francesca loves amusement parks. During a recent vacation, Francesca went to *LoLoLand*, which doesn't charge admission, but charges $1.50 for each ride.

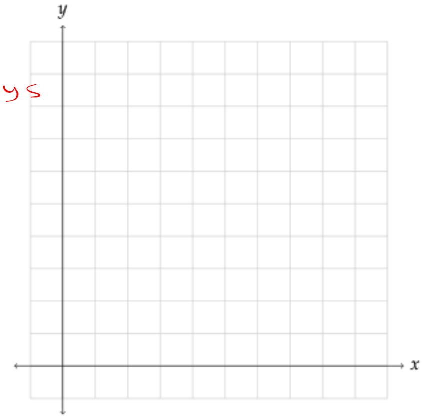
Dependent Variable:
Cost ($) she pays

Independent Variable: x
# of rides she goes.

$\boxed{m}$

x	$f(x)$
0	0
1	1.5

+1.5
+1.5
+1.5
+1.5



When done start pg 3 and pg 4

Q₁: Use the graph to determine how much it will cost Francesca if she plans on going on 8 rides.
a)

Q1 b) how much will she pay if she goes on "2.5" rides.

Q₂: What is the rate of change (slope) of the linear function that models this situation?

$m = 1.50$ \$/ride

1.50 \$ for each ride

Q2 b) what are the $\boxed{\text{units}}$ of the rate of change.

what are units

$m = \frac{\Delta y}{\Delta x}$

$m = \boxed{\$/\text{ride}}$

1.3 Example

Create a model for the following situation:

> Feng opens up a new savings account and initially deposits \$350. Feng sets up a savings plan and begins saving \$100 per month. (n.b. ~~Because of low interest~~ rates, the bank ~~does not offer Feng any interest on his deposits~~).

Dependent Variable: y

\# saved (#) (units)

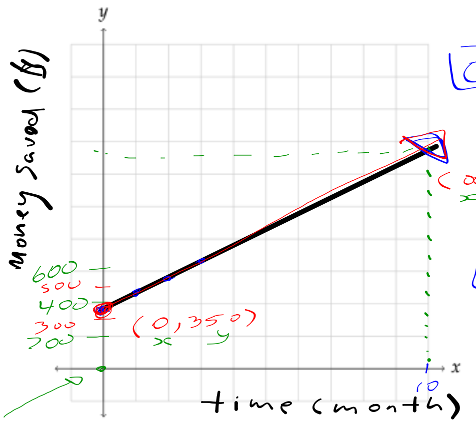
Independent Variable: x

time (months) (units)

$\boxed{m}$

x	$f(x)$
0	350
1	450
2	550
3	650
4	750
5	850

+100, +100, +100, +100



$\boxed{Q4}$ What's the increasing interval? of f/(amount #)

[lowest x, highest x]

[0, ∞[

$\boxed{Q5}$ What's the decreasing interval? ∅

$\boxed{Q3}$: what are the units of $\boxed{m}$

$m = \frac{\Delta y}{\Delta x} = \frac{\#}{\text{month}}$

$\boxed{Q6}$ What's the y-int? $y = 350$ or $(0,350)$

find y when $x = 5$

f of 5

find $f(5)$

$f(x) = 100x + 350$

$f(5) = 100(5) + 350$

$= 850$ \$

Q$_1$: Use the graph to determine how much Feng will have saved after 10 months.

\$1350 (exactly)

Q$_2$: What is the rate of change (slope) of the linear function that models this situation?

$m = 100$ \$/month

$y = mx + b$

slope — initial value of f

$f(x) = 100x + 350$

1.4 Practice

You do

Create a model for the following situation:

Dae loves amusement parks. During a recent vacation, Dae went to *Fantasy Circuit*, which charges \$10 for admission and \$1 for each ride.

$\boxed{Q3}$ What's the decreasing interval?

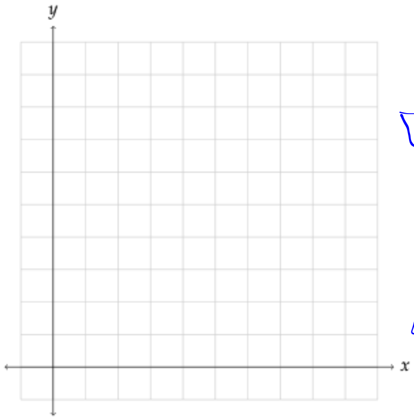
$\boxed{Q4}$ What's the increasing interval?

$\boxed{Q5}$ What's the initial value?

Dependent Variable:

Independent Variable:

x	$f(x)$



Q₁: Use the graph to determine how much it will cost Dae to go on 9 rides.

Q₂: What is the rate of change (slope) of the linear function that models this situation?

1.5 Practice

- MHS Workbook p. 48-49

2 An Introduction to Linear Functions

2.1 The Rule of a Linear Function

Let's go back and check the rules (equations) for the linear functions we used in Section 1 and list them here. Can w find a patt rn?

$y \quad\quad m \cdot x + b$

$f(x) = 110x + 0$

$f(x) = 1.5x + 0$

$f(x) = 100x + 350$

$f(x) = 1 \cdot x + 10$

m = slope (rate of change)

b = y-int (initial value)

~~2.2 The Slope and Initial Value (y-intercept)~~

~~Given that the rule (equation) of a line if $f(x) = mx + b$ or $y = mx + b$, let's try to explain, intuitively, why m is the slope of the function and b is the initial value.~~

2.2.1 Practice

properties

For each of the following functions, determine the *slope* (m) and *initial value* (y-intercept) (b) of the function:

$y = mx + b$

1. $f(x) = 3x + 1$ $\quad m = 3 \quad b = 1$

$y = mx + b$

2. $y = -3x - 4$ $\quad m = -3 \quad b = -4$

3. $f(x) = \frac{1}{3}x + 10$ $\quad m = \frac{1}{3} \quad b = 10$

4. $f(x) = -\frac{1}{4}x - 1$ $\quad m = -\frac{1}{4} \quad b = -1$

$f(x) = -\frac{1}{4} \cdot x - 1$

$y = mx + b$ $\quad m = -1 \quad b = 4$

6. $y = -x + 4$

$y = -1 \cdot x + 4$

7. $f(x) = 9x +$ $\quad m = 9 \quad b = 0$

$= mx + b$

8. $y = -13x + 0$ $\quad m = -13 \quad b = 0$

9. $f(x) = 9$

10. $y = 13$

11. $f(x) = 0$

2.3 "Types" of Linear Functions

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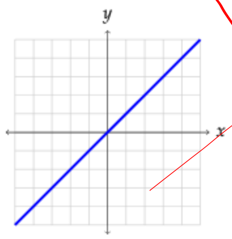
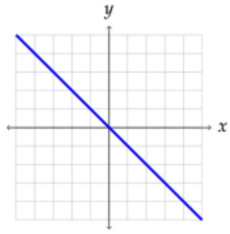
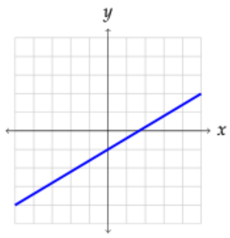
All linear functions have the following form:

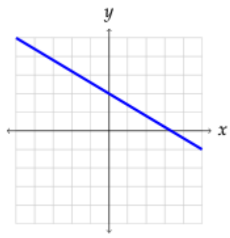
$$f(x) = mx + b \text{ or } y = mx + b$$

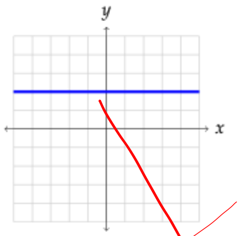
But we often give certain lines special names because they are typically used to model certain kinds of real-life situations:



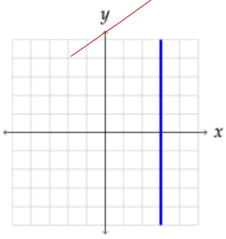






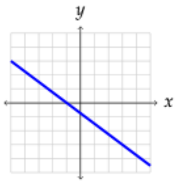


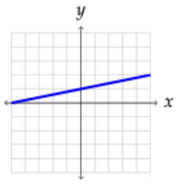
2.3.1 Special Case: The Vertical Line

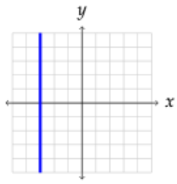


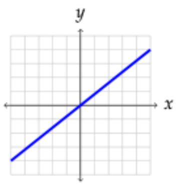
2.3.2 Practice

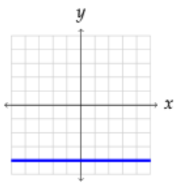
For each of the following graphs, indicate whether we have a direct variation linear function, partial variation linear function, constant linear function or a vertical line.

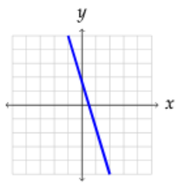












2.3.3 Practice

For each of the following graphs, indicate whether we have a direct variation linear function, partial variation linear function, constant linear function or a vertical line.

1. $f(x) = 3x + 1$
2. $y = \quad 3x \quad 4$
3. $f(x) = \frac{1}{3}x + 10$
4. $f(x) = -\frac{x}{4} - 1$
5. $y = x + 4$
6. $y = \quad x + 4$
7. $f(x) = 9x$
8. $x = 70$
9. $y = -13x$
10. $f(x) = 9$
11. $y = \quad 13$
12. $f(x) = 0$
13. $x = \quad 900$

2.3.4 Example: Determining the rule of a Constant Linear Function and a Vertical Line

Consider the point $(-3, 4)$. Determine:

(a) The rule of the horizontal line passing through the point;

(b) The rule of the vertical line passing through the point.

2.3.5 Example: Determining the rule of a Constant Linear Function and a Vertical Line

Consider the point $(6, -10)$. Determine:

(a) The rule of the horizontal line passing through the point;

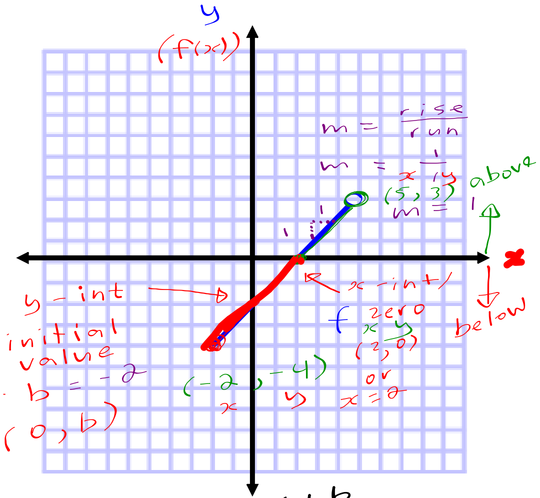
(b) The rule of the vertical line passing through the point.

Properties of Functions

- **maximum of function** (y)
  (f = f(x) = y)
  → highest y value
  max = 3 or (5, 3)

- **minimum of a function** (y)
  → smallest y value
  min = −4 or (−2, −4)

Plot: open dot (5, 3)
closed dot (−2, −4)



$f(x) = m \cdot x + b$

$f(x) = 1 \cdot x + (-2)$

$\boxed{f(x) = x - 2}$

Positive interval of function (y)

↳ where the y's are positive.

↳ where the function is above the x-axis

↳ interval answer in terms of x-value

[lowest x, highest x] f: [2, 5[

Negative interval of f

↳ where the y's are negative

↳ where the function is below the x-axis

→ ans in x'values

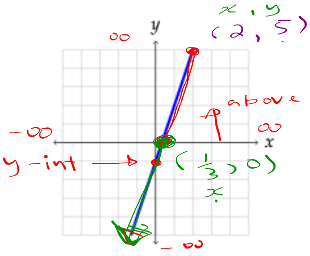
[−2, 2]

TIP
· LABEL points

2.4 Properties of Linear Functions p 79

Let's consider the following linear function and determine its properties:

$f(x) = 3x - 1$



$(-\infty, -\infty)$
x y

[min , max]

Domain: $]-\infty, 2]$ ← x-values

Range: $]-\infty, 5]$ ← y-values.

y-intercept (initial value): $y = -1$ or $(0, -1)$

- x-intercept (zero): $x = \frac{1}{3}$ or $(\frac{1}{3}, 0)$

Variation: / change

→ f is increasing over it's entire domain : $]-\infty, 2]$ ← x-values

→ f is NOT decreasing

sign

→ f is positive ← x-values
$[\frac{1}{3}, 2]$

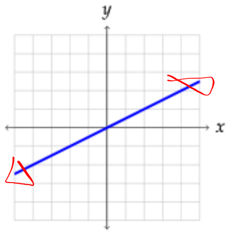
→ f is negative ← x-values
$]-\infty, \frac{1}{3}]$

put arrows @ end of lines

HmWK
→ finish handouts
textbook p 116 #3.4
p 120 "what are... units of m"
p 128 #3.5
p 130 #3.8

Let's consider the following linear function and determine its properties:

$f(x) = \frac{1}{2}x$



Domain:

- Range:

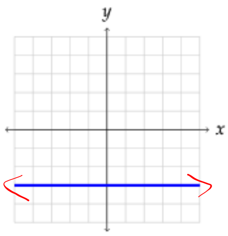
y-intercept (initial value):

- x-intercept (zero):

Variation:

Let's consider the following linear function and determine its properties:

$f(x) = -3$



Domain:

Range:

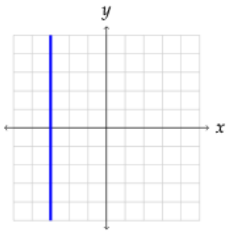
- y-intercept (initial value):

x-intercept (zero):

Variation:

Let's consider the following linear function and determine its properties:

$x = -3$



Domain:

Range:

y-intercept (initial value):

x-intercept (zero):

Variation:

2.5 Practice

MHS Workbook p. 50-51