Lesson 6: Modelling Word Questions w/ Inequalities

April 3rd, 2023

$\leq$ $\geq$ + − × ÷

Inequality Symbol	Key Words / Meaning (when reading LEFT to RIGHT) +2 ×2
$<$	is less than, is smaller than is fewer than, is lower than under, below
$\leq$	is less than or equal to, is smaller than " " is fewer/lower than or equal to at most (or less than), a maximum of no more than not/no greater than, does not exceed/go over up
$>$	is greater than, is more than is larger/bigger than, taller than over, exceeds
$\geq$	is greater than or equal to at least (or more than), a minimum of more than or equal, not under not below not/no less than

Handout 1 : Question Context.

MODELLING LANGUAGE WITH INEQUALITIES

Part A: Translate each statement into an inequality:

step 1 (done for us)

Define variables/unknowns

1. "I am having a small party, so bring no more than 3 friends", your friend says to you. Let f represent the number of friends you can bring to the party.

$f \leq 3$ ✓ $f > 3$ ✗

2. Deelan should sleep more than 8 hours per night. Let s represent the number of hours Deelan should sleep per night.

same

$8 > s$ ✓ (8 is more than # of hours Deelan)

$s < 8$ ✗ $s \leq 8$ ✗ $s > 8$

(1) > (2) <

(3) ≥ (4) ≤

3. The length of a soccer field is more than 100 meters. Let l represent the length of the soccer field.

$l > 100$

4. "I want to have a big party, so bring no less than 10 friends", your friend says to you. Let f represent the number of friends you can bring to the party.

$f \geq 10$ ✓

5. Ms. Short should eat less than 2 zeppole next St. Patrick's Day. Let z represent the number of zeppole Ms. Short should eat next year.

$z < 2$

You do until #4 (could start part B)

Context where Variable is in Between

Part B: Translate each statement into two separate inequalities:

Two Endpoints

$1 \leq x < 4$

$\min \leq x \leq \max$

TRUE!

make write 2 separate inequality for tasks.

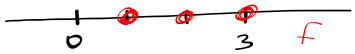
$x \geq 1$ AND $x < 4$

1. "I am having a small party, so bring no more than 3 friends but at least 1 friend", your friend says to you. Let *f* represent the number of friends you can bring to the party.

$f \leq 3$

$f \geq 1$

solution set



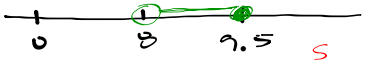
set-builder notation

$\{f \in \mathbb{N} \mid 1 \leq f \leq 3\}$

2. Deelan should sleep more than 8 hours per night but at most 9.5 hours. Let *s* represent the number of hours Deelan should sleep per night.

$s > 8$

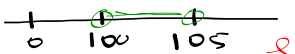
$s \leq 9.5$



3. The length of a soccer field is more than 100 meters but less than 105 meters. Let *l* represent the length of the soccer field.

$l < 105$

$l > 100$



4. "I want to have a big party, so bring no less than 10 friends but don't exceed 25 friends", your friend says to you. Let *f* represent the number of friends you can bring to the party.

5. Ms. Short should eat less than 3 zeppole next St. Patrick's Day but at least one. Let *z* represent the number of zeppole Ms. Short should eat next year.

(1) (2)

(3) $>$ $<$

(4) $\leq$ $\geq$

You finish ho 1 and represent and the solution set on the # line // set builder notation

You do ho 2 and start ho 3

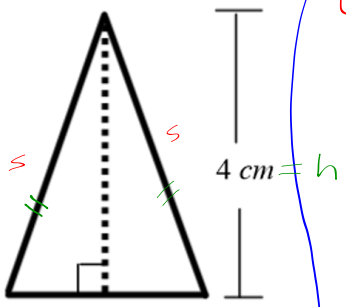
Inequalities w/ multiple unknowns and using Substitution Method to Get one unknown

Practice:

Translate the following and use substitution method to attain two separate inequalities with only one unknown:

1. You're building an isosceles triangular window. You must respect the following constraints:

- One of the congruent sides corresponds to $\frac{3}{2}$ the base. ($s = \frac{3}{2} \times b$)
- 2 windowpanes require at least 12 cm^2 of glass. (these units ^{2} indicate area)
- The frame is at most 15 *cm*.
- The measure of the base is a natural number.



$s = \frac{3}{2} \cdot b$ $\quad P = s + s + b$

$A = \frac{b \times h}{2}$ $\quad P = 2s + b$

① $2 \cdot A \geq 12$ } 2 unknowns

② $P \leq 15$

sub in

① $2 \cdot \left(\frac{b \times h}{2}\right) \geq 12$

② $2s + b \leq 15$

sub in

① $2 \cdot \left(\frac{b \times 4}{2}\right) \geq 12$

② $2\left(\frac{3}{2}b\right) + b \leq 15$

① $2 \cdot (2b) \geq 12$

① $4b \geq 12$ ✓

② $3b + b \leq 15$

② $(3+1)b \leq 15$

② $4b \leq 15$ ✓

optional

TASK:

step i. Read / take notes / make drawing and Define your variables / unknowns

· thing that varies / changes || thing you don't know

let s = measure of congruent sides

let b = base of △

let A = Area of △

let P = Perimeter of △.

step ii: LABEL drawing w/ variables

step iii. Translate sentences into separate equations and inequalities using variables

step iv: Substitute the value of the unknowns into the inequalities.

step v. Ensure you have the value of 1 unknown in terms of the other. Sub that in to make system have 1 unknown.

$h = 4$ (from diagram)

$s = \frac{3}{2}b$

step vi Evaluate / Simplify and ADD like terms

After break you do ho 3

#4 (geometric) answer key in back and start #2

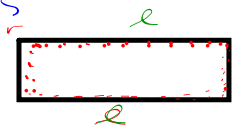
#4 Compare Chart in L3 vs Chart in L6

Do 3 - 5 questions from Review

L3

P = perimeter
w = width
l = length

You're are playing on a rectangular soccer field:



- The width is 5 less than three fifths of the length of the field.

$w = \frac{3}{5}l - 5$

- 2 complete laps around the field is no more than 205 m but exceeds 203 m.

L3 vs L6	
−	<
"5 less than"	"is less than" 120
$x = y - 5$	$h < 120$
where x = # of friend I have, y = # of friends you have	where h = height

① $2P \leq 205$

② $2P > 203$

$P = w + w + l + l$

$P = 2w + 2l$

① $2(2w + 2l) \leq 205$

② $2(2w + 2l) > 203$

$w = \frac{3}{5}l - 5$ sub in again!

① $2(2(\frac{3}{5}l - 5) + 2l) \leq 205$

$2(\frac{6}{5}l - 10 + 2l) \leq 205$

$2((\frac{6}{5} + 2)l - 10) \leq 205$

$2(\frac{16}{5}l - 10) \leq 205$

· evaluate
· simplify

Brackets helps

① $\frac{32}{5}l - 20 \leq 205$

② $2(2(\frac{3}{5}l - 5) + 2l) > 203$

$2(\frac{6}{5}l - 10 + 2l) > 203$

$2(\frac{16}{5}l - 10) > 203$

② $\frac{32}{5}l - 20 > 203$

2. After holding three rounds of a raffle, you organize a final raffle consisting of 1^{st} round winners, 2^{nd} round winners, and 3^{rd} round winners. You must respect the following constraints with regards to the number of winners you include in the final draw:

   - Six 1^{st} round winners must be included.
   - Four 2^{nd} round winners less than double the number of 3^{rd} round winners must be included.
   - Half the number of 3^{rd} round winners included must be less than or equal to 5.
   - More than 31 participants must be included

3. You and your friends spend the weekend at the cabin. Consider the following travel and booking constraints:

   - The guests arriving Friday is 2 people more than triple the number of guests arriving Saturday morning

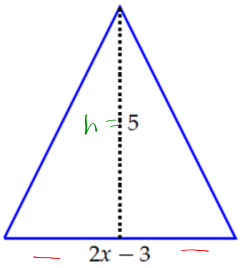
   - The cabin has a maximum capacity of 10, but more than 6 guests are required in order to book

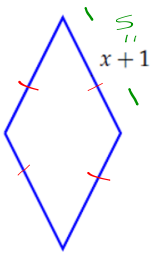
handout 4 : Page 2

3.4 Practice

Consider the following triangle and rhombus.

- The area of the triangle is less than $20m^2$
- The perimeter of the rhombus is more than 11m





What is x if x is a natural number?

step i : / step 2

A_Δ = Area Δ

b = base of Δ

h = height of Δ

P_r = Perimeter r.

s = side of r

step iii

$A_\Delta = \frac{b \times h}{2}$

$P_r = s + s + s + s$

$P_r = 4s$

h = 5

b = 2x − 3

s = x + 1

(1) $A_\Delta < 20$ (2) $P_r > 11$

(1) $\frac{b \times h}{2} < 20$ (2) $4s > 11$

(1) $\frac{(2x-3) \times 5}{2} < 20$

(1) $2 \times \frac{(10x - 15)}{2} < 20 \times 2$

$10x - 15 < 40$ +15

$10x < 55$

$\frac{10x}{10} < \frac{55}{10}$

$x < 5.5$

$4(x+1) > 11$

$4x + 4 > 11$ −4 −4

$\frac{4x}{4} > \frac{7}{4}$

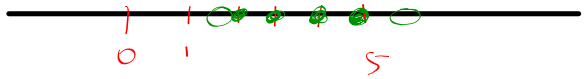
$x > 1.75$

NEW FINAL STEP

solve for x w/ 0.0. to both AND flip ineq. sign if x or 0/0 by negative #

New final final step

- graph solution set on # line and pick answer(s)



$\therefore x = 2$ or 3 or 4

or 5 or 10

HMWK

You do the rest of the handouts and additional questions from textbook:

- pg 200 #4.17 a) and b)
- pg 192, ex 4
- pg 193
- pg 199 #4.16

3.2 Example

The pet store *Cogs and Dats* sells cats and dogs exclusively with the following constraints:

- The number of cats sold is 2 less than the number of dogs.
- The number of animals sold in a day must be more than 6 but less than 13.

What possible combination of cats and dogs can be sold in a day, given these constraints?