Lesson 4: Introduction to Inequalities and Interval Notation

May 2^{nd}, 2024

$\leq$ $\geq$

$[\text{min } \#, \text{max } \#]$

to describe sets/groups of numbers.

ex. I have more than 2 friends.

$f \neq 2$

$f > 2$, where f = # of friends

Consider:

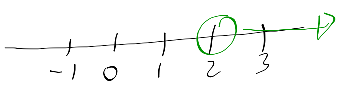
equal sign: $=$

inequality sign: $>$

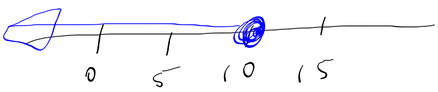
---

note: meaning of inequalities when reading Left to Right

$\geq$	$>$	$\leq$	$<$
· is greater than or equal to (closed dot)	(strictly) · is greater than (open dot)	· is less than or equal to (closed dot)	(strictly) · is less than (open dot)
ex. $4 \geq 4$	$x > 2$	$2 \leq 2$	$4 < 8$



$x \leq 10$

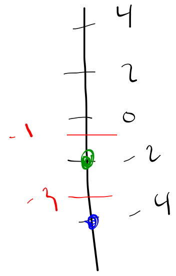


Reading Inequalities

e.x. Read and determine if True or False

a) $4 \leq 4$ True

b) $4 < 4$ False

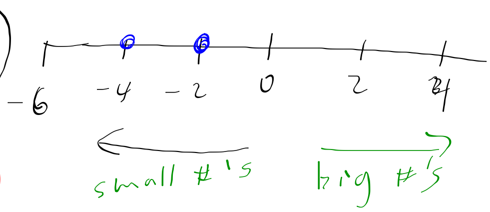


c) $5 \geq 4$ True

d) $5 > 4$ T

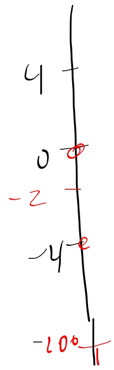
e) $0 \geq 1$ F

f) $-2 < -4$ F



g) $-1 > -3$ T

h) $0 \leq -4$ F



$$\boxed{s \leq 100 km/hr}$$

i) $-100 < -2$ T

You do handout 1

and start handout 2

L 4
no 2

Writing Inequalities
(Algebraic Model)

Difference : more than and a # more than

First translate the following sentences into either equations or inequalities. Then solve for x.

Recall: Solve for x means find the value of x that makes a true statement.

1. The value of x is more than 6.

$x > 6 \longrightarrow$ Solution $= \{7, 8, 9, 10 \ldots\}$

$x \quad = +6$

2. The value of x is 6 more than 10.

$x = 10 + 6$

$x = 16$ Solution $\{16\}$

nota bene: inequalities can have an infinite amount of solutions.

3. The value of x is less than 4.

$x < 4 \longrightarrow$ Solve for $x \longrightarrow \therefore$ Solutions $=$

$\{3, 2, 1, 0, -1, -2, -3 \ldots\}$

$x = -4$

4. The value of x is 4 less than 8.

$x = 8 - 4$

$x = 4 \longleftarrow$ solution.

5. The value of x is less than double x plus 10.

$1 \cdot x < \cancel{2x} + 10$

$-2x \quad -\cancel{2x}$

$\frac{-\cdot x}{-1} < \frac{10}{-1}$

$x > -10$

Solutions: $\{-9, -8, -7, \ldots\}$
(answers)

Solve:
· bring x's together.
· same as w/ equations

ATTENTIONIZE!

flip inequality sign when you $\times$ or $\div$ by a negative #.

Until 12:40

You do #7 #8

$x \quad = \quad -10$

6. The value of x is 10 less than double x.

solve

$1 \cdot x = \cancel{2x} - 10$

$-2x \quad -\cancel{2x}$

$\frac{-x}{-1} = \frac{-10}{-1}$

$x = 10$

$\frac{1}{2} \cdot x \quad > \quad x + 10$

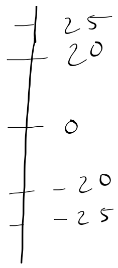
7. Half the value of x is more than x plus 10.

$$\frac{1}{2}x > \cancel{x} + 10$$
$$-x \qquad -x$$

$$\frac{-0.5x}{-0.5} > \frac{10}{-0.5}$$

$$x < -20$$

$\Rightarrow$ Solutions = $\{-21, -22, -40 \ldots\}$



$+10 \qquad \frac{1}{2} \times x$

8. 10 more than half the value of x is equal to x.

$$\frac{1}{2}x + 10 = x$$
$$-x \qquad -x$$

$$-0.5x + 10 = 0$$
$$-10 \qquad -10$$

$$\frac{-0.5x}{-0.5} = \frac{-10}{-0.5}$$

$$x = 20$$

B
E
D
M
A
S

Solve

Writing Inequalities
(Graphical Model)

ex. Graph the inequality. Represent the solution set on the # line

step i: Draw a line and go to given #, and determine if open endpoint ○ > or <

or closed endpoint ● ≥ or ≤

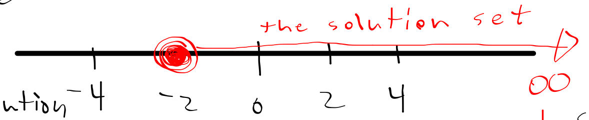
step ii: Determine if you shade to the left < or ≤

or to the right > or ≥

ex. 1 $x \geq -2$

not the solution set

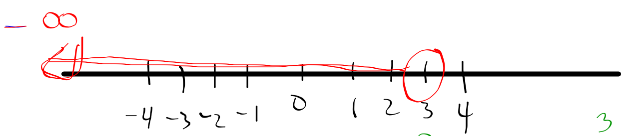
the solution set



-2 is included

symbol for infinity

ex. 2 $x < 3$



3 is excluded since open dot

You do: a couple of questions pg 2

pg 4

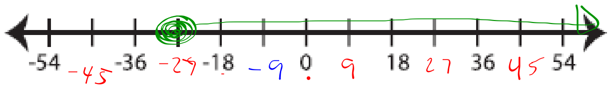
pg 6 pg 8

one endpoint

4) $x \ge -27$

filling out the line

∞



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taking notes

AIM

Expressing constraints as inequalities.

Just like in everyday life, the concept of **inequality** is widely used in mathematics. For example, road signs indicating a minimum speed of 60 km/h and a maximum speed of 100 km/h represent constraints. When the weather forecast calls for a maximum daytime temperature of 21 °C and a minimum nighttime temperature of 10 °C, these are also constraints. In Québec, the minimum age to vote is 18, while the minimum age to drive is 16—yet more constraints.

$s \geq 60$

AND

$s \leq 100$

s = speed km/h

Constraints can be translated into **inequalities**. Thus, to express algebraically the fact that the maximum speed is 100 km/h, you would write $s \leq 100$, where s is the speed (km/h). If the minimum speed is 60 km/h, you would write $s \geq 60$.

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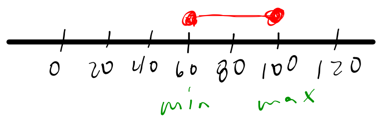
· graphical representation

When a variable is located between two endpoints, it can be represented algebraically in the following way: minimum $\leq x \leq$ maximum.



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CAUTION!

This type of inequality is always read starting with the variable. Thus, $2 < x \leq 5$ means that x is greater than 2 and less than or equal to 5. This method of writing should only be used for sets of numbers between two endpoints.

· algebraic rep.

min $\leq x \leq$ max

$60 \leq s \leq 100$

Caution: can't use min $\leq x \leq$ max

ex.

$b \geq 10$

and

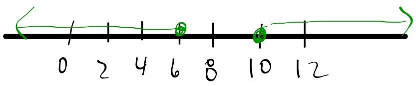
$b \leq 6$

b = the # of books to bring on vacation

Break!

Never False:

$6 \geq b \geq 10$



not overlapping ∴ an impossible situation

2.3 Example:

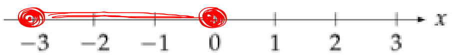
group

i. Use the number line below to represent the solution set to the following systems of inequalities.
ii. Use an algebraic expression/representation to represent the solution set to the same system.

$x \geq -3$ and $x \leq 0$

step i. same steps just don't extend lines.

the



min solution set max

step ii : shade between 2 endpoints and label min and max

$-3 \leq x \leq 0$

step iii $\min \leq x \leq \max$

closed

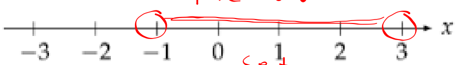
$x > -1$ and $x < 3$

open

$x \leq -1$ and $x \geq 1$

No overlap

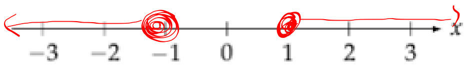
the solution



set

min max

not the



solution set

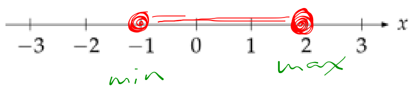
∴ No solution

$-1 < x < 3$

you do Practice 2.4

closed endpoint

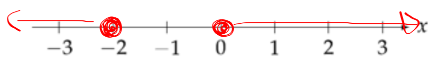
$x \geq -1$ and $x \leq 2$



$-1 \leq x \leq 2$

$x \leq -2$ and $x \geq 0$

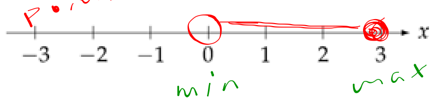
no overlap ∴ no solution



$x > 0$ and $x \leq 3$

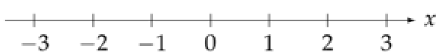
open end point

closed endpoint



$0 < x \leq 3$

$x < 3$ and $x > -1$



step i. same steps just don't extend lines
Left Right

step ii : shade between 2 endpoints and label min and max

step iii $\text{min} \leq x \leq \text{max}$

open dot $<$ $>$

closed dot $\leq$ $\geq$

(Solution Interval)

Interval Notation to Represent a set of #'s

[min , max]

e.x. use a solution interval to rep. the solution set to the inequalities.

a) $x \geq 2$

solution set = { 2, 3, 4, 4.5, 10, ..., ∞ }

min — max

closed bracket — open bracket

$x \in [\, 2 \,,\, \infty \,[$

x belongs to the interval of #'s including 2 to ∞ not included (excluded)

b) $x \leq 1$

Solution set = { 1, 0, -1, -2 .... -∞ }

{ -∞ ..., -2, -1, 0, 1 }

min — max

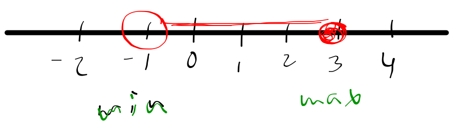
$x \in \,]\, -\infty \,,\, 1 \,]$

[min , max]

Write in interval notation

c) $-1 < x \leq 3$

step i. put solution on # line



step ii. label max + min

· consider if

$\leq \;\; \geq$. including bracket - closed dot

$> \;\; <$. excluding bracket - open dot

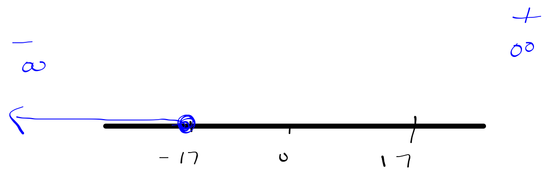
$x \in \,]\, -1 \,,\, 3 \,]$

3.3 Example

infinity

Express the following inequalities as intervals:

- (a) $x \geq 70$ $\quad x \in [70, \infty[$
- (b) $x \leq -17$ $\quad x \in]-\infty, -17]$

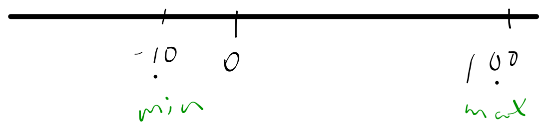


- (c) $x > 0$ $\quad x \in]0, \infty[$
- (d) $x < -12$ $\quad x \in]-\infty, -12[$

open

[min, max]

- (e) $1 \leq x \leq 90$ $\quad x \in [1, 90]$
- (f) $-10 < x \leq 100$ $\quad x \in]-10, 100]$



3 Representing Inequalities Using Interval Notation

3.4 Practice

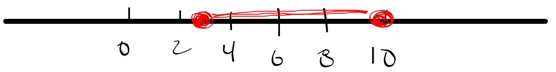
Represent the following intervals on the number line:

$x \in [3, 10]$

min max

$x \in]3, 10]$

You do the rest!



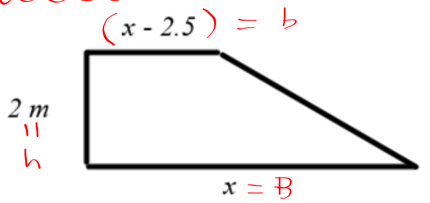
Name: ________________________

Exit Ticket:

You are making a sandbox in the shape of a trapezoid pictured below. You note the following information:

- Three layers of sand results in 1.5 less than 5 times the big base.

3 A = −1.5 5 B



Determine the length of the small base.

Tips:

- Translate sentences into equations.
- equations from properties
- label
- equations from diagram

① $3A = 5B - 1.5$ ← 2 unknowns

- a problem!!

Tip

- write 2nd unknown in terms of 1st.

- sub:

$b = x - 2.5$

$B = x$

$h = 2$

- simplify
- solve

$$A = \frac{(b + B) \times h}{2}$$

$$A = \frac{(x - 2.5 + x) \times 2}{2}$$

$$A = \frac{(2x - 2.5) \times \cancel{2}}{\cancel{2}}$$

B
E
D
M
A
S

simplify

use sub. method to make one equation w one unk.

① $3A = 5B - 1.5$ ② $A = 2x - 2.5$

③ $B = x$

$3(2x - 2.5) = 5x - 1.5$

$6x - 7.5 = 5x - 1.5$

$-5x$ $-5x$

$x - 7.5 = -1.5$

$+7.5$ $+7.5$

$\boxed{x = 6 \text{ units}}$ ← Big base

$b = x - 2.5$

$b = 6 - 2.5$

$b = 3.5$ unit

HMWK:

P 166 # 4.1

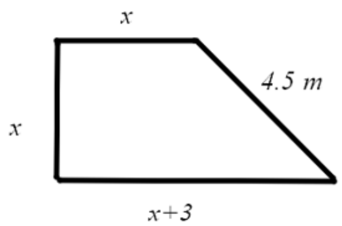
P 167 # 4.3

Name: ___________________________

Exit Ticket:

You are making a sandbox in the shape of a trapezoid pictured below. You note the following information:

Four laps around the sandbox results in 48 meters more than triple the big base.



Determine the length of the big base.