Lesson 12: Solving a System (group) of Equations and Tasks

April 18, 2023

(Comparison Method)

Recall:

Q1: Solve:

$5 = x + 2$

$5 - 2 = x + 2 - 2$ ← isolate the unknown

$3 = x$

$x = 3$

ANS:

$x = 3$

Check

$5 = x + 2$

$5 = 3 + 2$

$5 = 5$ ✓

Q2: Solve → Not really solvable cuz there's infinite solutions

$y = x + 2$

Find the value of the unknown(s) that make a true statement.

ANS: a set $\{(3,5), (0,2), (4,6)$ (x, y)

$(8,10), (1,3), (2,3)$ (x, y)

$(5,7), (6,8), (12,14), (14,16)$ (x, y)

Check: sub $(15,17), \ldots\}$ (x, y)

$y = x + 2$

$17 = 15 + 2$

How many solutions are there?

an infinite amount

BEST ANSWER IS in graph form.

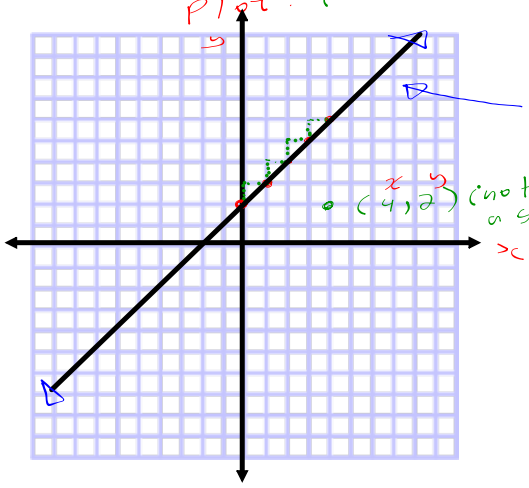
$y = x + 2$

$y = ax + b$

$a = \frac{1}{1} = \frac{\text{rise}}{\text{run}}$ ← + or − ← +

$b = 2$ y-int: (x, y) $(0, b)$

Plot: $(0, 2)$



the solution set

$y = x + 2$

$2 = 4 + 2$

$2 = 6$ ✗ False

WANT 2 unk.

TOOL 2 eq.

1st method: Solving Systems Graphically

Solve

ℓ_1 : $y = x + 2$

ℓ_2 : $y = \frac{3}{2}x - 1$

ℓ_1: $y = x + 2$

$y = ax + b$

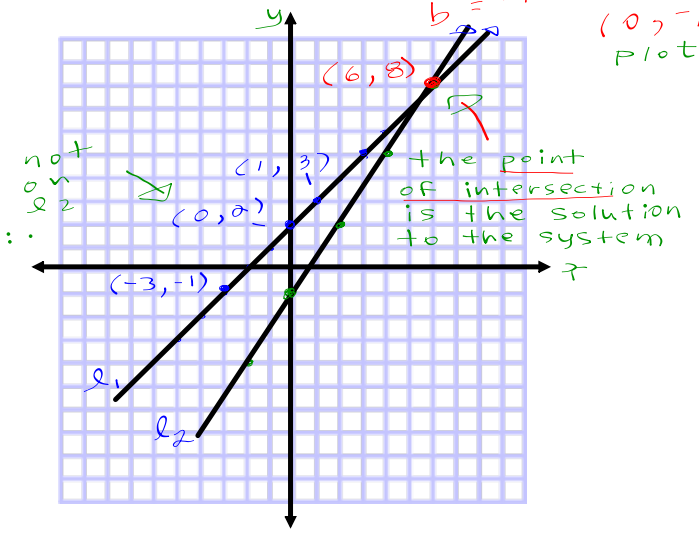
$a = \frac{1}{1}$ $b = 2$ $(0, 2)$

ℓ_2: $y = \frac{3}{2}x - 1$

$y = ax + b$

$a = \frac{3}{2} = \frac{\text{rise}}{\text{run}}$

$b = -1$ $\therefore (0, b)$ $(0, -1)$ plot



To FIND the Solution.

step i. graph each line on same graph
LABEL LINES
• slope / y-int

step ii. Identify the point of intersection between the 2 lines
POI (6, 8)

step iii: check answers

Find a value of x and y that make a true statement for both equations.

solution? $(1, 3)$ (x, y) no cause not on ℓ_2

ℓ_1 : $y = x + 2$

$3 = 1 + 2$

$3 = 3$ ✓

ℓ_2 : $y = \frac{3}{2}x - 1$

$3 = \frac{3}{2}(1) - 1$

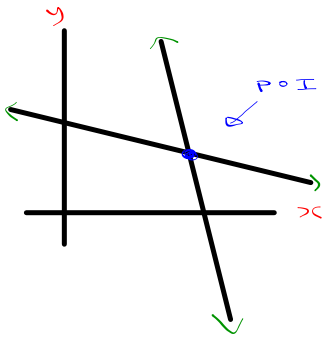
$3 = \frac{1}{2}$ ✗

∴ $(1, 3)$ is not a solution to the system.

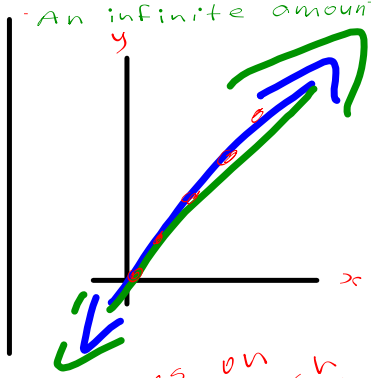
Number of Solutions to a System

3 possibilities:

① 1 solution

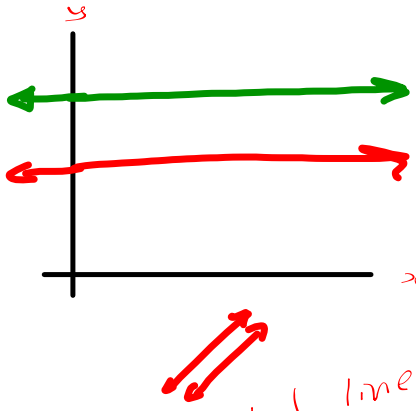


② An infinite amount



2 lines on top of each other an infinite amount of POI

③ No solutions



2 parallel lines no POI

2nd method: Using Comparison method to Solve systems (to Find the POI)

très important !

WANT: · POI (x, y)
TOOL: · 2 unk
· 2 eq. and comparison method

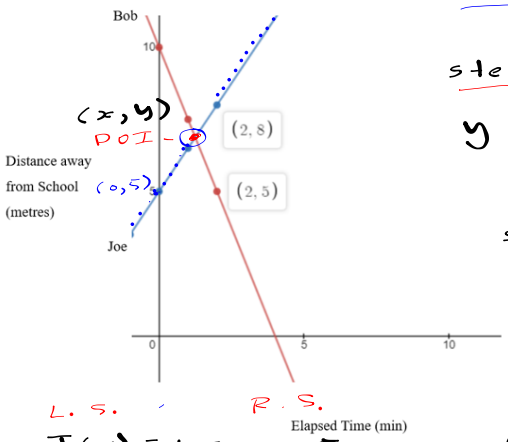
To Find POI:

Question 1: Two friends are walking, one towards school and other away from school. Their distance from school is a function of time in minutes. This distance is described in the graph below and in the following equations:

left side : right side

Bob: $B(x) = -2.5x + 10$
Joe: $J(x) = 1.5x + 5$

L.S. : R.S.



For what value of x is $B(x) = J(x)$? That is, when are the friends at the same distance from school? (Find the coordinates of the point of intersection between the two lines)

step i: Isolate the y in both equations (done already)

step ii: Since the left sides are equal (only @ POI), put the right sides of the equations to each other.

given

L.S. R.S.

$B(x) = -2.5x + 10$

cat = pusa

L.S. R.S.

$J(x) = 1.5x + 5$

chat = yoia

given since:

L.S. L.S.

$B(x) = J(x)$

cat = chat

$-2.5x + 10 = 1.5x + 5$

pusa = yoia

1 eq. and 1 unk. that's solvable.

step iii: Bring the x's together w o.o.

$-1.5x \quad -1.5x$

$-2.5x + 10 = 1.5x + 5$

$-2.5x - 1.5x + 10 = 5$

$(-2.5 - 1.5)x + 10 = 5$

$-4x + 10 = 5$ $\quad -10 \quad -10$

$\frac{-4x}{-4} = \frac{-5}{-4}$

$x = 1.25$

step iv solve for x by isolating w opposite operations which is?

step v. sub x into eq. ① to find y.

① $B(x) = -2.5x + 10$

$y = -2.5(1.25) + 10$ · evaluate

$y = 6.875$ or ~~6.88~~ ← approx ans.

POI (x, y)

ANS: (1.25, 6.875)
x y

② $J(x) = 1.5x + 5$

$6.875 = 1.5(1.25) + 5$

$6.875 = 6.875$ ✓ ∴ true

ANS: (

step vi check by subbing POI into ②

You do Q2. moving on @ 11:05

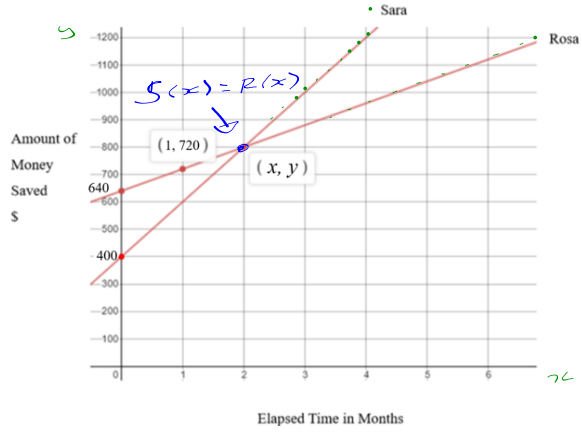
Medium Level Task Still important

"see handout 2. Question 3"

Question 3: Two friends are saving money at different rates. Their amount of money saved is a function of time in months. The money saved over time is described in the below graph.

- The rate of change of function S is 200.

For what value of x is $S(x) = R(x)$? That is, when do the friends have the same amount of money saved? (Find the coordinates of the point of intersection between the two lines)



WANT: x + y (POI)

2 unk

Tool 2 equations and comparison Method

$y = ax + b$ (S) $y = ax + b$ (R)

INFO: S: $a = 200$, $b = 400$; R: $a = ?$, $b = 640$

WANT: a_R

TOOL: 1 eq $a_R = \frac{y_2 - y_1}{x_2 - x_1}$

INFO Rosa points (0, 640) (x_1, y_1) (1, 720) (x_2, y_2)

find a_R

$a = \frac{y_2 - y_1}{x_2 - x_1}$

$a = \frac{720 - 640}{1 - 0}$

$a_R = 80$ \$/m

(1) $S(x) = ax + b$ (L.S.) $S(x) = 200x + 400$ (R.S.)

(2) $R(x) = ax + b$ (L.S.) $R(x) = 80x + 640$ (R.S.)

Comparsion method:

since $S(x) = R(x)$

$200x + 400 = 80x + 640$

$200x - 80x + 400 = 640$

$120x + 400 = 640$

$120x = 240$

$\frac{120x}{120} = \frac{240}{120}$

$x = 2$ months

sub into (1)

$S(x) = 200x + 400$

$y = 200(2) + 400$

$y = 400 + 400$

$y = 800$ \$

ANS POI (2, 800) (x, y)

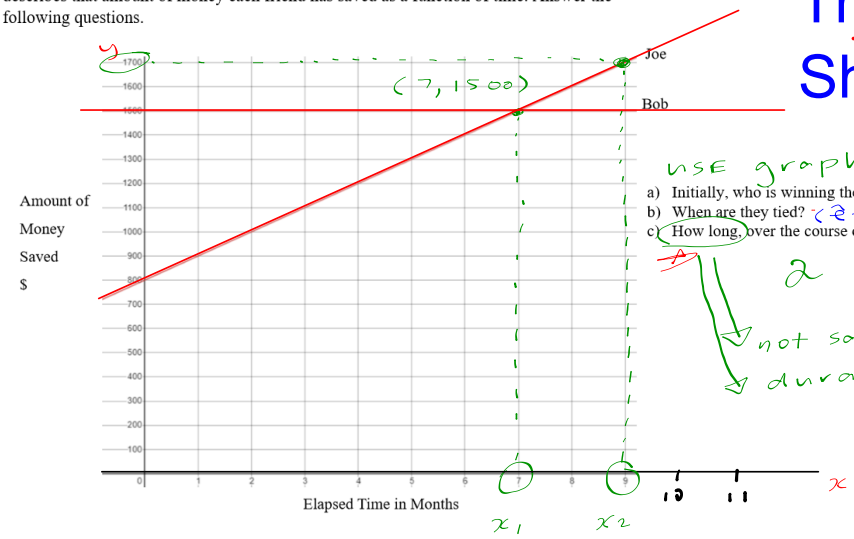
∴ the friends have the same amount of \$ (800\$) at 2 months.

You do #4

Reading the Graph Question that Important for the Hard Tasks

keeping s.o. "on track"

Question 1: Two friends try to keep each other financially accountable by participating in a savings race. They want to see who can save $1700 the quickest. Read the graph below that describes that amount of money each friend has saved as a function of time. Answer the following questions.



Think - Pair - Share

use graph to answer

a) Initially, who is winning the race? BOB (0, 1500) [JOE $800]
b) When are they tied? (être à l'égalité)
c) How long, over the course of the entire race, will the winner stay ahead of the runner-up?

2 months

not same as "when?"

duration

loser in 2nd place.

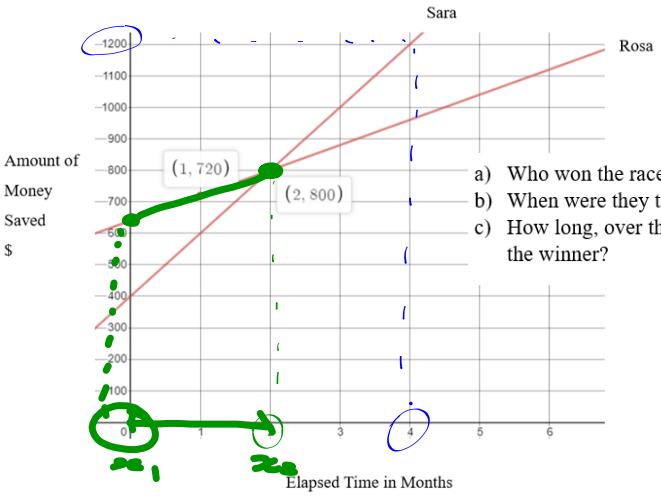
$d = |x_2 - x_1|$

$d = |7 - 9|$

$d = |-2|$

$d = 2$ hr.

Question 2: Two friends try to keep each other financially accountable by participating in a savings race. They want to see who can save $1200 the quickest. Read the graph below that describes that amount of money each friend has saved as a function of time. Answer the following questions.



a) Who won the race? Sara
b) When were they tied? 2 month
c) How long, over the course of the entire race, did the runner-up hold an advantage over the winner? Rosa

- when she was ahead.

$$d = |x_2 - x_1|$$
$$d = 2 - 0$$
$$= 2 \text{ month}$$

FULL-ON TASK. w/ a POI

TIPS:
- Define variables/unknowns?
  - x = elapsed time in month
  - y = amount of $ saved
- make graph, label it and plot points

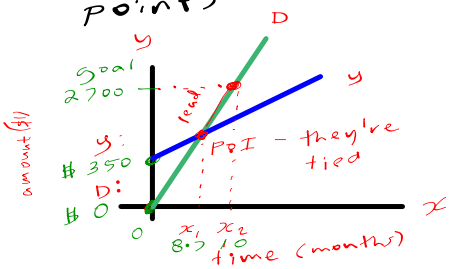
PRACTICE QUESTIONS: WORD QUESTIONS AND POINT OF INTERSECTION

Question 1: Two friends try to keep each other financially accountable by participating in a savings race. They want to see who can save $2700 the quickest. Consider the following information:

- Yufan saves money at a rate of $230 per month.
- Each month, Deelan saves $270.
- Deelan will start saving after Yufan will have saved $350.

slope ✓ $a = \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{\$}{m} \frac{y}{x}$

slope ✓ = 270 $/m → faster rate



How long, over the course of the entire race, will the winner stay ahead of the runner-up?

WANT: duration of the winner's lead

TOOL: $d = |x_2 - x_1|$

INFO: $x_1 = ?$ $x_2 = ?$

WANT: x_1 (in PoI) (x_1, y)

2 unk

TOOL 2 eq

Deelan

① $y = 270x + 0$

$y = 270x$

② $y: y = ax + b$

$y = 230x + 350$

have all info

@ PoI $y = y$

$270x = 230x + 350$

$40x = 350$

$x_1 = 8.75$ month

WANT x_2 when $y = 2700$

TOOL: equation Deelan's eq.

$y = ax + b$

$a = 270$

$b = 0$

have all info

$y = 270x$ sub $y = 2700$

$\frac{2700}{270} = \frac{270x}{270}$

$x_2 = 10$ month

∴ Deelan held the lead for (10 − 8.75) 1.25 months

You do #2

HMWK:
- pg 243 − 244 #5.13 / 5.14
- Top of pg 239 p 252 − 55 #5.21
- pg 232 #5.10
- TASK pg 257 − 259 (not pg 258)

You do

Question 2: Two friends try to keep each other financially on-track by participating in a reimbursement race. They both bought a new $750 cellphone and wants to see who can pay it off the quickest. Consider the following information:

- Marwah will be paying $150 a month to reimburse the amount financed.
- Antonia makes monthly payments of $100 but paid $200 initially in cash.

How long, over the course of the entire race, will the winner stay ahead of the runner-up?