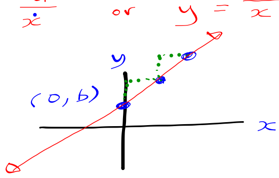
Lesson 10: Word Questions Involving Linear Functions and Rational Functions April 14, 2023

$y = a x^1 + b$ $y = \frac{a}{x}$ or $y = \frac{k}{x}$

$y = mx + b$



8- Graphing Lines and Rational Functions - April 7, 2021 Math-3051-2

$a = \frac{rise}{run}$ y-int : (0, b)

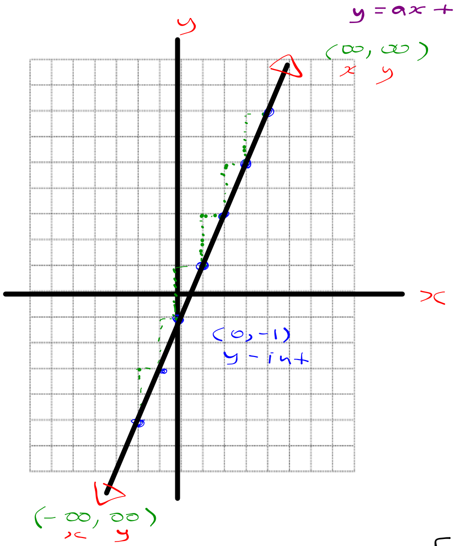
2 Graphing Linear Functions: Using the Slope-Intercept Method

Let's try to devise a method for graphing linear functions using the slope and y-intercept of the function. For example:

Graph the following Linear Function:

$$f(x) = 2x - 1$$

$y = ax + b$



Domain:]-∞, ∞[

Range :]-∞, ∞[

step i : Label equation and determine parameters (slope and y-int)

a = 2 b = -1

step ii : state the coordinates (x, y) of the y-int and state the slope in a fraction

y-int : (0, b)
(0, -1)
(x, y)

$a = \frac{2}{1} = \frac{rise}{run}$

step iii : plot the y-int ·(x, y)

step iv : starting from the y-int, rise y units up or down and run x units always to the right to plot the next points.

step v : Draw straight line through points and put arrows.

2.1 Example

Graph the following linear functions:

$a = -\frac{1}{2} = \frac{rise}{run}$

$b = 5$

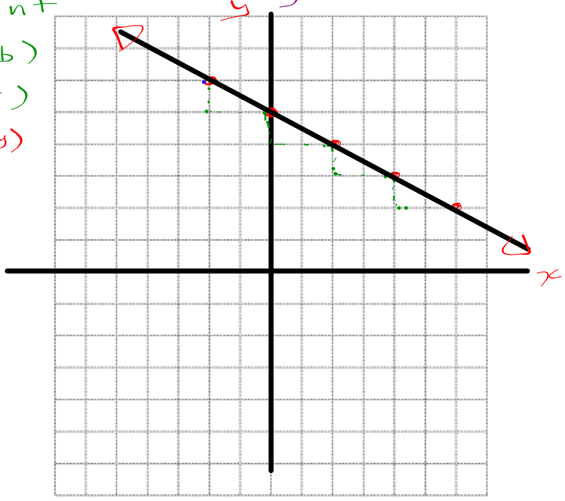
y-int (0, b)

(0, 5)

(x, y)

$$f(x) = -\frac{1}{2}x + 5$$

y = ax + b



$a = \frac{1}{3} = \frac{rise}{run}$

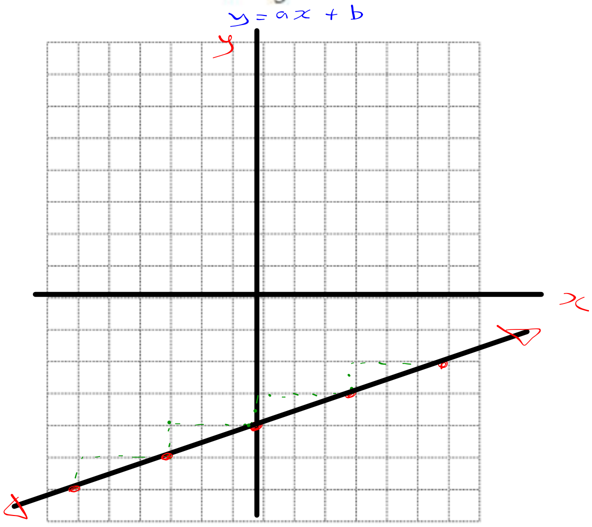
$b = -4$ → y-int (0, b)

(0, -4)

x y

$$y = \frac{1}{3}x - 4$$

y = ax + b

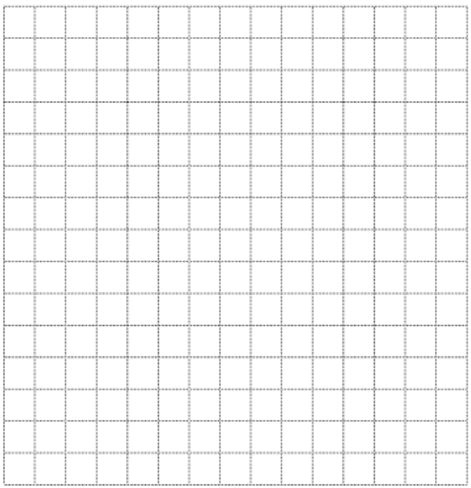


You do the next 4 questions

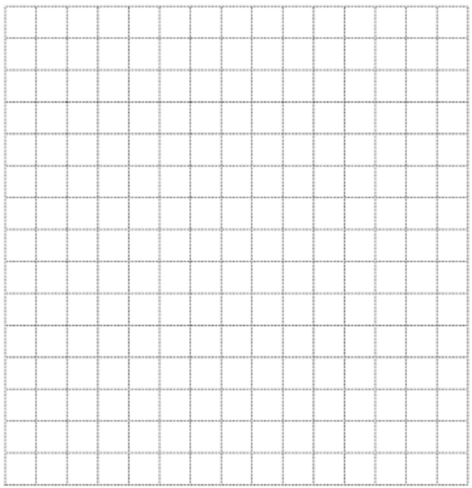
2.2 Practice

Graph the following linear functions:

$$f(x) = -3x + 7$$



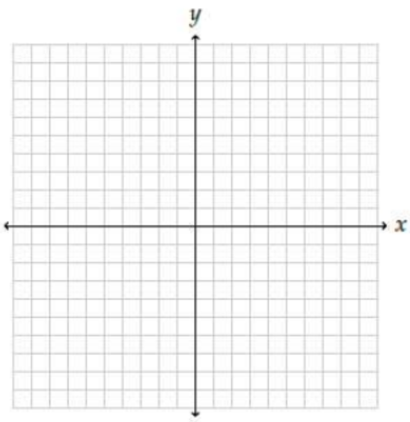
$$y = \frac{4}{5}x - 4$$



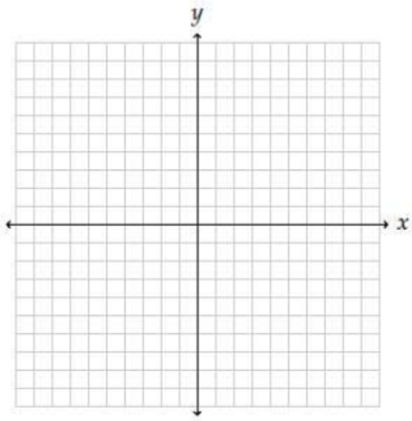
1.1 Practice

Graph the following linear functions:

$$f(x) = -x + 5$$



$$y = \frac{1}{2}x - 4$$



1st method of Determining the Equation of a Linear Function Given 2 points

Find $f(x) = ax + b$ (that is, find a and b)

ex: (x, y)

ex: y-int = 2 → (0, 2), (0, b)

x-int = 3 → (3, 0), (x, 0)

3 Linear Functions and Short Word Problems

$a = \frac{\Delta y}{\Delta x}$ — units for slope: km/hr (y-var / x-var)

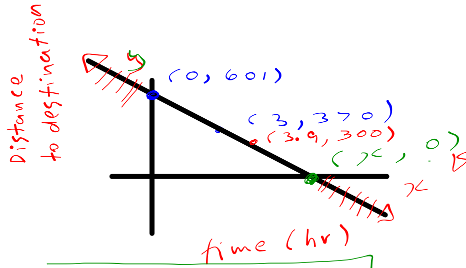
Example: Jimmy is travelling from *Lala Land* to *Imaginary Place* at a constant speed. When he began, he was 601km from *Imaginary Place*. After 3 hours, he was 370km from his destination. When will he be 300km from his destination?

($x = 0$, $y = 601$)

x = Elapsed time (in hours)

y = Distance (km) to Imaginary Place/Destination

Tips
- Define variables/unknowns (what changes) (what you're being asked to find)
- Sketch a graph that's labeled and label and plot points. and plot points say want in a point/or x or y



Find slope/a

$a = \frac{y_2 - y_1}{x_2 - x_1}$

$a = \frac{(370 - 601)}{(3 - 0)}$

$a = \frac{-231}{3}$

$a = -77$

WANT: x when $y = 300$

TOOL: equation $y = ax + b$

INFO: $a = ?$ $b = 601$, $y = 300$

WANT: a

TOOL: equation $a = \frac{y_2 - y_1}{x_2 - x_1}$

INFO: $P(0, 601)$ $P(3, 370)$ (x_1, y_1; x_2, y_2)

Find y-int/b

- sub in slope
- sub in a point (x, y): (0, 601) (x, y)
- evaluate

$y = ax + b$

$y = -77 \cdot x + b$

$601 = -77(0) + b$

$601 = 0 + b$

$601 = b$

$b = 601$

state equation

$a = -77$ $b = 601$

tool: $y = ax + b$

$y = -77x + 601$

find x when $y = 300$

- sub y into
- Solve for x w/ o.o. opposite operation. × → ÷

$300 = -77x + 601$ (-601, -601)

$-301 = -77x$ ($\div -77$, $\div -77$)

$3.9 = x$

$x = 3.9$ hr

∴ it will take Jimmy 3.9 hr to be 300 km from his destination.

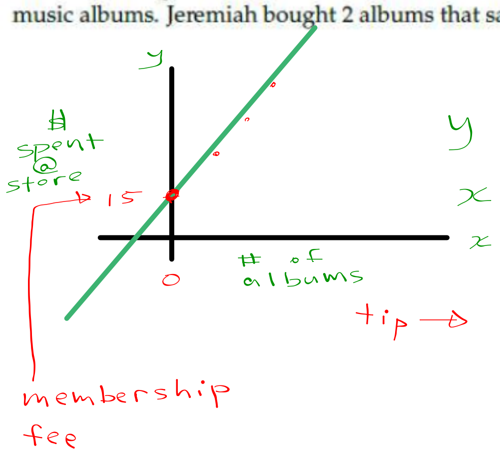
Check if answer makes sense w/ graph.

You do

3.1 example and 3.3 example and check answers.

3.1 Example

In long forgotten ancient times, a company named *Columbia House* would provide music albums at a reduced price, and required members to pay a monthly membership fee. Assuming the price per album was the same, we need to figure out how many albums we would be able to purchase with $265 in a given month. We know that Arelius, an avid music fan, paid $115 in a month for 10 music albums. Jeremiah bought 2 albums that same month and spent $35.



units → $a = \frac{\$}{\text{album}}$ → $\frac{\Delta y}{\Delta x}$

y = $ spent @ store

x = # of albums

tip → y depends on x

reduced rate ↓ membership fee ↓

$y = 10x + 15$

$y = ax + b$

3.3 Example

Slim Jim, a used car salesman, is paid a certain fixed rate for every car he sells in addition to a fixed amount of \$2700 regardless of how many cars he sells. When Jim sold 20 cars, he earned a total income of \$6700. If Jim sells just 12 cars, what will be his total income?

2nd method of Determining the Equation of a Linear Function Given Slope and a point

- $y = ax + b$
- find a + b

Recall: Rate of change of y (per change in x)

Key words: ex # per hr. (#/hr)
a month
each day
km per hr
each hr, km

3.4 Example

a) Roberto is a plumber who is saving money for the purchase of a high-tech tool. The cost of this tool is \$2500. He charges his customers \$45 per hour plus an additional \$15 for travel expenses. How many hours will it take Roberto to save for this tool?

units of slope = #/hr ← Δy / Δx

flat fee, a constant, initial value (b)

200 a month ∴ in 2 months I have # 400 cuz salary/rate is 200 #/month

b) If Roberto increases his hourly rate by \$10, determine the rule of the function that models this new salary situation.

① Define variables
- x = # of hr he works
- y = \$ earn from customer

② find $y = ax + b$ (a: coefficient; b: generally, constant, specifically, y-int/initial value)

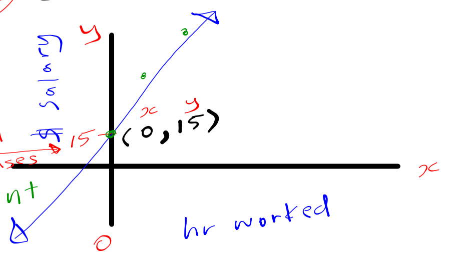
i. find $a = \frac{y_2 - y_1}{x_2 - x_1}$ } don't have info

$a = 45$ \$/hr

ii. find b: travel expenses, y-int

∴ $b = 15$

① graph



WANT x when $y = 2500$ (sub in)

$y = ax + b$

$y = 45x + 15$

$2500 = 45x + 15$ (−15, −15)

$\frac{2485}{45} = \frac{45x}{45}$

$x = 55.22$ hr

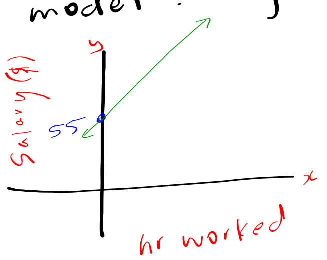
∴ it will take him 55.22 hr to save \$2500

b) If Roberto increases his hourly rate by \$10, determine the rule of the function that models this new salary situation.

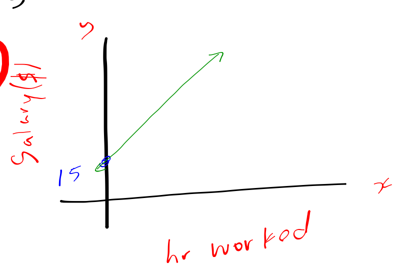
old model: $y = 45x + 15$ (hourly rate; fixed travel expenses.)

new model: $y = 55x + 15$ ✓

a)



b)



You do 3.5 example

3.5 Example

a) Kashana is saving money for a one-week educational trip to Eastern Europe. The cost of the trip is $1200. She already has $300 saved and plans to add $50 per month to her saving. How long will it take Kashana to save for the trip?

b) If Kashana increases her monthly saving by $25, determine the rule of the function that models this new saving situation.

Introduction to the Rational Function — when $x \in \mathbb{R}$

aka, the Inverse - Variation Function — when $x \in \mathbb{N}$ in word questions.

(le changement)

4.1 Discovery: How to Model a Situation Using a Rational Function

A green-tech competition has been announced. ~~The goal is to identify the best idea for reducing greenhouse gases on a global scale.~~ Entrants can be individuals or a team of people (up to a maximum of 50 members). The prize for the best idea is $500 000, split equally among the team (or individual) who win. (the prize)

(a) If I enter the competition with my sibling and we win, how much money do we each get?

2 ppl: $500\ 000 \div 2 = 250\ 000$ $

(b) If a group of five people win the prize, how much does each person get?

$500\ 000 \div 5 = 100\ 000$ $

(c) ~~If the amount of prize money per person is $25 000, how many winners are~~ there?

$\frac{500\ 000}{25\ 000} = 20$ winners.

$f(x) = \frac{a}{x}$; $x \uparrow$, $y \downarrow$

$y = \frac{a}{x}$

$y \cdot x = a$

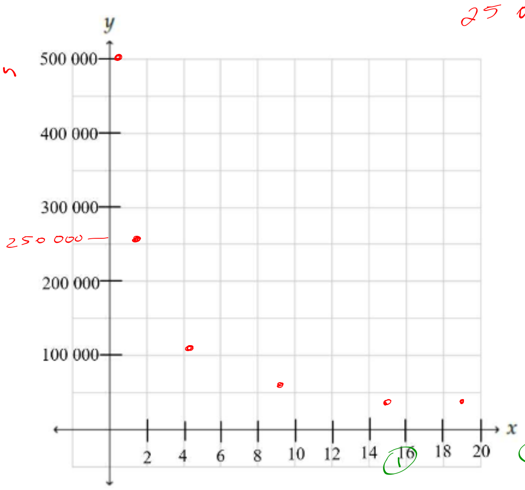
$x = \frac{a}{y}$ — the inverse of

Now let's analyze this situation as we have done in the past with situations that can be modelled using linear functions. Create a table of values to show the relation between the number of winners and the amount of prize money per person.

Dependent Variable: (y) amount of $ each person gets.

Independent Variable: (x) # of ppl spliting the prize money

x	$f(x)$
1	500 000
2	250 000
5	100 000
10	50 000



note: don't draw curve for word questions since $x \in \mathbb{N}$

$500\ 000 \div 1 = 500\ 000$
$500\ 000 \div 2 = 250\ 000$
$500\ 000 \div 5 = 100\ 000$
$500\ 000 \div 10 = 50\ 000$

$500\ 000 \div x = y$

Can we come up with an algebraic rule to model this situation? (using x and y)

$$\therefore y = \frac{500\ 000}{x}$$

Rational Function:

$f(x) = \frac{a}{x}$, where $x \neq 0$

where $y \neq 0$

- Hmwk: you do practice 4.1.1 from handout
- from textbooks:
  - p148 - p150 #3.18 - 3.20
  - p155 - p156 #3.29 - 3.31
  - p139 #3.14
  - p141 #3.17
  - p154 #3.27
  - p140 #3.15 / #3.16

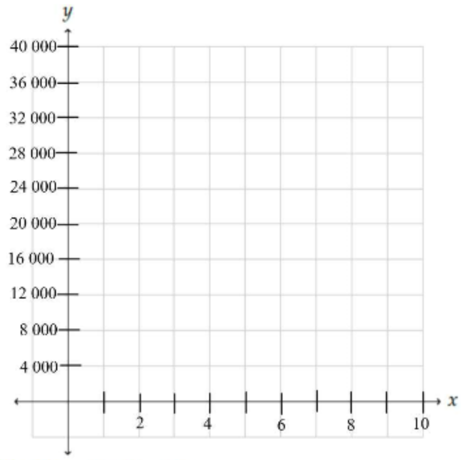
4.1.1 Practice

The prize for a local community lottery is $40 000, split evenly among each of the winners. Model this situation using a rational function.

Dependent Variable:

Independent Variable:

x	$f(x)$



Can you come up with an algebraic rule to model this situation?

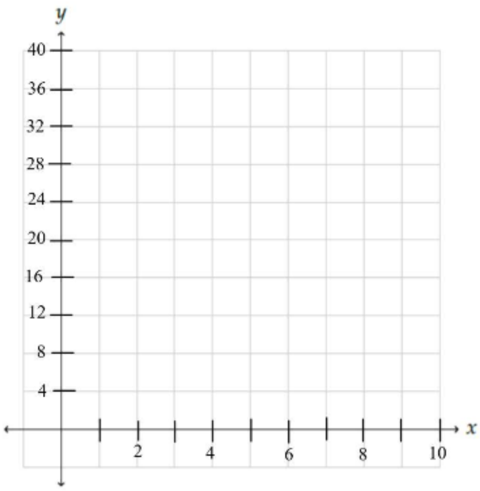
4.2 The Rule of a Rational Function

Le's revisit the rules we discovered from the previous two subsections and write them down. **Can we come up with a general rule for all rational functions?**

4.3 Graphing Rational Functions

Let's try to develop a method for graphing rational functions. How would we graph the following rational function:

$$f(x) = \frac{32}{x}$$



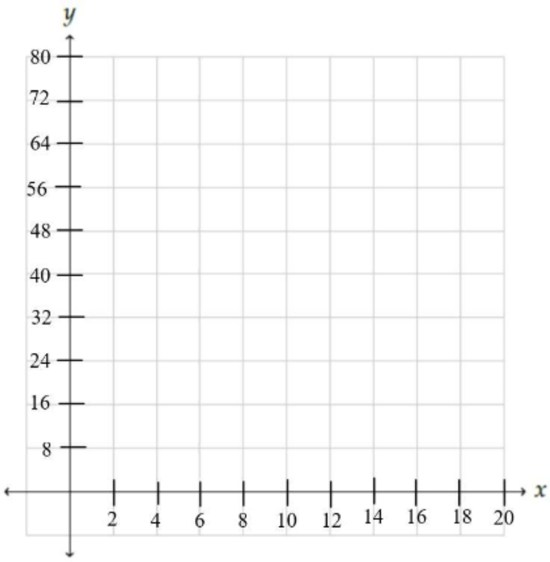
Graphing Rational Functions

1. Create a table of values with a wide sample of x values (at least 5)
2. Find the y value for each x value
3. Plot each point on the graph
4. Draw a curve through the points

4.3.1 Practice

Graph the following rational function:

$$f(x) = \frac{64}{x}$$



4.2.1 Determining Whether A Relation Presented in a Table is a Rational Function

Given the following relations, determine whether they are rational functions:

x	2	5	8
$f(x)$	125	50	$\frac{125}{4}$

x	2	4	6	8
$f(x)$	40	20	0	-20

x	$f(x)$
2	5
3	$\frac{10}{3}$
4	$\frac{5}{2}$
5	2
10	1

4.2.2 Determining Whether A Relation Presented in a Table is a Rational Function

Given the following relations, determine whether they are rational functions:

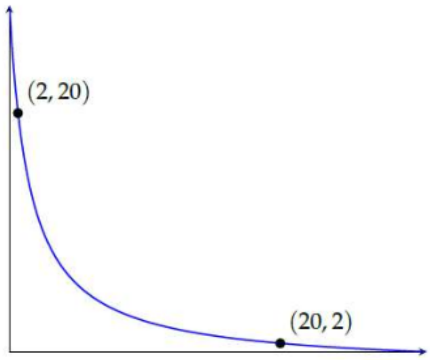
x	$f(x)$
-5	2
-4	5
-3	8
-2	11

x	1	5	10	25
$f(x)$	250	50	25	10

x	1	2	4	8
y	32	16	8	4

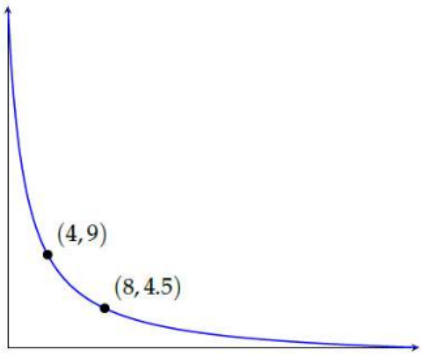
4.4 Determining the Rule of a Rational Function

How can we find the rule of the following function?



4.4.1 Practice

(a) Determine the rule of the following function:



(a) Determine the rule of the following function:

x	1	5	10	25
$f(x)$	250	50	25	10

(b) Determine the rule of the following rational function:

x	1	2	4	8
y	32	16	8	4