Question 1

Given the following functions:

$$f(x) = x^2 + 3$$
$$g(x) = -\sqrt{x+2}$$
$$j(x) = \frac{2}{x+1}$$
$$i(x) = |x+3| - 2$$

Find the following equations:

a) $f \circ g(x) = f(g(x))$
$= f(-\sqrt{x+2})$
$= (-\sqrt{x+2})^2 + 3$
$= x+2+3 = x+5$

b) $g \circ f(x) = g(f(x))$
$= g(x^2+3)$
$= -\sqrt{(x^2+3)+2}$
$= -\sqrt{x^2+5}$

c) $i \circ j(x) = i(j(x))$
$= i\left(\frac{2}{x+1}\right)$
$= \left|\frac{2}{x+1} + 3\right| - 2$

d) $j \circ i(x) = j(i(x))$
$= j(|x+3|-2$
$= \frac{2}{|x+3|-2+1} = \frac{2}{|x+3|-1}$

Question 2

Using the equations from Question 1, evaluate the following compositions:

a) $g \circ f(3) = -\sqrt{(3)^2+5}$

$= -\sqrt{14}$

b) $f \circ g(-1) = (-1)+5$

$= 4$

c) $i \circ j(3) = \left|\frac{2}{3+1}+3\right|-2$

$= |3.5|-2 = 3.5-2 = 1.5$

d) $j \circ i(-2) = \frac{2}{|(-2)+3|-1}$

$= \frac{2}{|1|-1} = \frac{2}{0}$ = undefined!

Question 3

Solve the following inequalities algebraically, and give your solution using interval notation.

a) $-0.2|x-200|+400 \geq -150$

$\frac{-0.2|x-200|}{-0.2} \geq \frac{-550}{-0.2}$

$|x-200| \leq 2750$

$x-200 = 2750$ $\quad -(x-200) = 2750$

$x = 2950$ $\quad -x+200 = 2750$

$\frac{-x}{-1} = \frac{2550}{-1}$

$x = -2550$

F T F

-2550 0 2950

test 0: $|0-200| \leq 2750$

$200 \leq 2750$ ✓

$[-2550, 2950]$

b) $3\sqrt{x-2}+1 < 9$

$\frac{3\sqrt{x-2}}{3} < \frac{8}{3}$

Domain: $x-2 \geq 0$

① $x \geq 2$

$(\sqrt{x-2})^2 < (\frac{8}{3})^2$

$x-2 < \frac{64}{9}$

② $x < \frac{82}{9}$

c) $\left(\frac{x^2}{5} - \frac{7x}{10} + 2 < 0, 2x\ 10\right)$

$2x^2 - 7x + 20 < 2$

$2x^2 - 7x + 18 < 0$

$a=2$ $\quad \Delta = b^2-4ac$

$b=-7$ $\quad = (-7)^2 - 4(2)(18)$

$c=18$ $\quad = 49 - 144$

$= -95$

test $x=0$:

$\frac{0^2}{5} - \frac{7(0)}{10} + 2 < 0.2$

$2 < 0.2$

False!

0

① 2

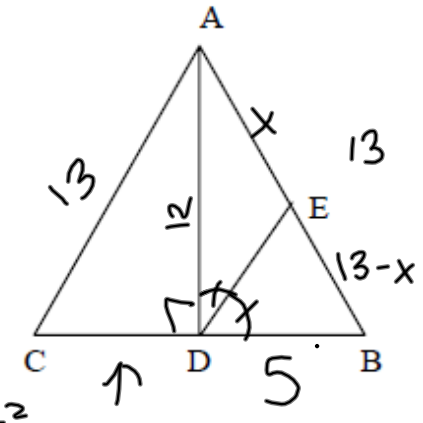
② $\frac{82}{9}$

① ∩ ② 2 $\frac{82}{9}$

$[2, \frac{82}{9}[$

Question 4

Triangle ABC is isosceles. Segment AD is the altitude from A to side BC. AD measures 12 units, and AC measures 13 units. Find the measure of AE if DE is the bisector of angle ADB.



Th 80

$\frac{\overline{AE}}{\overline{AD}} = \frac{\overline{BE}}{\overline{BD}}$

$\frac{x}{12} = \frac{13-x}{5}$

$5x = 12(13-x)$

$5x = 156 - 12x$

$17x = 156$

$x = \frac{156}{17} = 9.18$ units

$\overline{AB}^2 = \overline{AD}^2 + \overline{DB}^2$

$13^2 = 12^2 + \overline{DB}^2$

$169 = 144 + \overline{DB}^2$

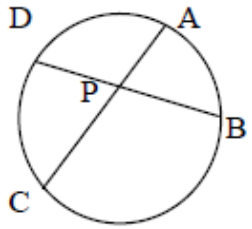
$25 = \overline{DB}^2$

$\overline{DB} = \sqrt{25} = 5$

Question 5

Find the missing measure in each of the circles below. Show all your work, and reference the theorem used to calculate the missing measures.

a)



$m\overline{PD} = 2{,}4\,cm$

$m\overline{PC} = 4\,cm$

$m\overline{AC} = 7\,cm$

$m\overline{BP} = ?$

Th 8.1 $\quad \overline{AP} \times \overline{PC} = \overline{DP} \times \overline{PB}$

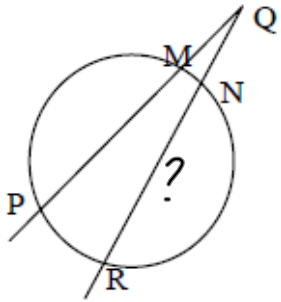
$\overline{AP} = \overline{AC} - \overline{PC}$

$= 7 - 4 = 3$

$\frac{3 \times 4}{2.4} = \frac{2.4 \times \overline{PB}}{2.4}$

$\overline{PB} = \frac{12}{2.4} = 5\ cm$

b)



$m\overline{MQ} = \sqrt{11}$

$m\overline{PM} = 2\sqrt{14}$

$m\overline{NQ} = \sqrt{13}$

$m\overline{NR} = ?$

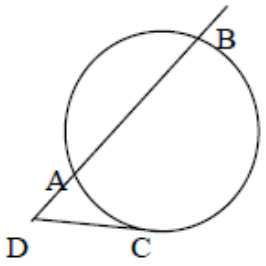
th 8.2 $\quad \overline{QM} \times \overline{QP} = \overline{QN} \times \overline{QR}$

$\frac{\sqrt{11} \times (\sqrt{11} + 2\sqrt{14})}{\sqrt{13}} = \frac{\sqrt{13}\,\overline{QR}}{\sqrt{13}}$

$\overline{QR} = 9.93 \rightarrow \overline{NR} = \overline{QR} - \overline{QN}$

$= 9.93 - \sqrt{13} = 6.32$

c)



$m\overline{AD} = 12{,}5\,m$

$m\overline{AB} = 37{,}5\,m$

$m\overline{DC} = \quad ?$

th 8.3 $\quad \overline{CD}^2 = \overline{DA} \times \overline{DB}$

$= 12.5(12.5 + 37.5)$

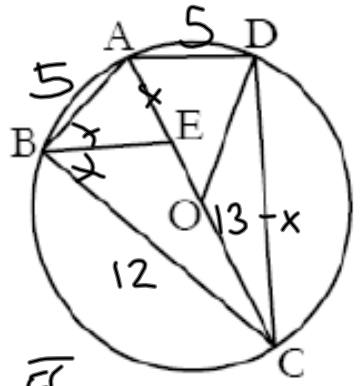
$= 12.5(50) = 625$

$\overline{CD} = \sqrt{625} = 25\ m$

Question 6

In the following figure, AC is a diameter and BE is a bisector. Given the following measures, calculate the measure of the following segments and justify your answer.

$m\overline{AB} = 5mm$
$m\overline{BC} = 12mm$
$m\overline{AB} = mAD$



a) $m\overline{OD}$ (radius)

th 77 $\angle ABC = \frac{1}{2}\overset{\frown}{ADC}$

$= \frac{1}{2}(180^\circ)$

$= 90^\circ$

Pythagorus $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$

$= 5^2 + 12^2$

$= 169$

$\overline{AC} = \sqrt{169} = 13mm$

$\overline{OD} = \frac{1}{2}(\overline{AC}) = \frac{1}{2}(13) = 6.5mm$

b) $m\overline{AE}$

th 80 $\frac{\overline{AE}}{\overline{AB}} = \frac{\overline{EC}}{\overline{BC}}$

$\frac{x}{5} = \frac{13-x}{12}$

$12x = 5(13-x)$

$12x = 65 - 5x$

$\frac{17x}{17} = \frac{65}{17}$ $x = 3.82mm$

Question 7

In triangle ABC, BE is a bisector and BD is an altitude. Determine if the following statements are true or false.

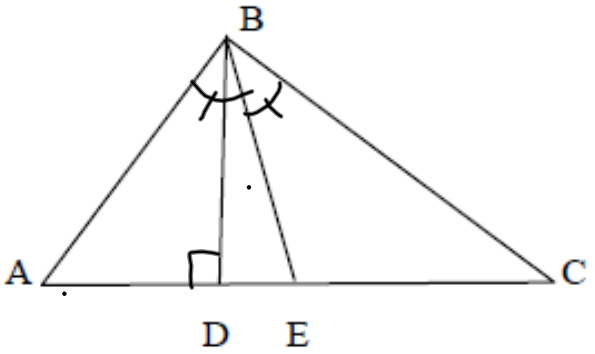
a) $\overline{AB} \cdot \overline{EC} = \overline{AE} \cdot \overline{BC}$ True! Th 8D

b) $\overline{AB}^2 = \overline{AE} \cdot \overline{AC}$ False!

c) $\overline{AD} \cdot \overline{DC} = \overline{AB} \cdot \overline{BC}$ False!

d) $\overline{BD}^2 = \overline{AD} \cdot \overline{DC}$ False!

e) $\overline{AB} \cdot \overline{BC} = \overline{BD} \cdot \overline{AC}$ False!



True if $\angle B = 90°$

$\frac{\overline{AE}}{\overline{AB}} \overset{?}{=} \frac{\overline{EC}}{\overline{BC}}$

$\overline{AE} \cdot \overline{BC} = \overline{AB} \cdot \overline{EC}$

Question 8

During a road test, a car is accelerated and decelerated regularly following the following function:

$$v(t) = 30 - \left|\frac{3t-120}{4}\right|$$

60 sec!

Where $v(t)$ represents the speed (in m/s) reached after t seconds. The test lasts exactly one minute. During what interval, in seconds, is the speed of the car greater than or equal to 10 m/s? Show all your work clearly.

$v(t) \geq 10$

$30 - \left|\frac{3t-120}{4}\right| \geq 10$

$-\left|\frac{3t-120}{4}\right| \geq -20$

$\left|\frac{3t-120}{4}\right| \leq 20$

$+: \frac{3t-120}{4} = 20$

$3t - 120 = 80$

$\frac{3t}{3} = \frac{200}{3} \rightarrow t = 66.\overline{6}$

$-: -\left(\frac{3t-120}{4}\right) = 20$

$-4(20) = 3t - 120$

$\frac{40}{3} = \frac{3t}{3} \rightarrow t = 13.\overline{3}$

test $t=0$:

$\left|\frac{3(0)-120}{4}\right| \leq 20$

$|30| \leq 20$

$30 \leq 20$ False!

F T F

0 $13.\overline{3}$ $66.\overline{6}$

$[13.\overline{3}, 60]$

Question 9

Following a power failure, the interior temperature of a house varies according to the following rule:

$$T(h) = -2\sqrt{h} + 20$$

Where *T(h)* represents the interior temperature after *h* hours in degrees Celsius. What is the minimum length of the power failure for the house to reach freezing (0°C)?

Clearly show all your work.

$T(h) \leq 0$

$-2\sqrt{h} + 20 \leq 0$

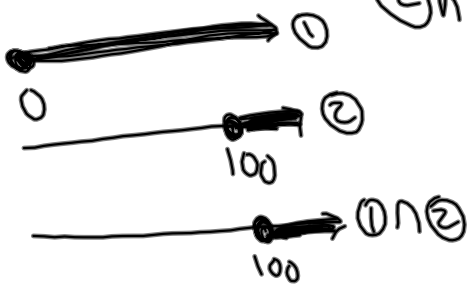
$-2\sqrt{h} \leq -20$

$\frac{-2\sqrt{h}}{-2} \leq \frac{-20}{-2}$

$(\sqrt{h})^2 \geq 10^2$

Domain: ① $h \geq 0$

② $h \geq 100$



It will take at least 100 hours.

Question 10

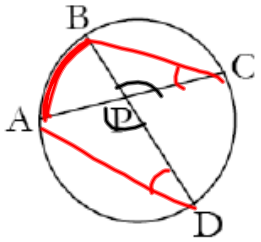
Complete the following proof.

Prove that:

When two chords intersect within a circle, the product of the lengths of the segments of one chord equals the product of the lengths of the segments of the other.

Hypothesis: Given two chords in a circle, AC and BD, intersecting at point P, located within the circle.

Conclusion: $m\overline{PA} \times m\overline{PC} = m\overline{PB} \times m\overline{PD}$



Statement	Justification
1. $m\angle APD = m\angle BPC$	Vertically opposite
2. $m\angle PDA = m\angle PCB$	Th 77 - Both angles share $\overset{\frown}{AB}$
3. Triangles APD and BPC are similar	A - A
4. Therefore, $\frac{m\overline{PA}}{mPB} = \frac{m\overline{PD}}{mPC}$	Similar triangles - ratio of corresponding sides is constant (k)
5. Conclusion: $m\overline{PA} \times m\overline{PC} = m\overline{PB} \times m\overline{PD}$	Cross multiply

Question 11

Given a circle with centre O, point B is the point of tangency of segment AB. A is situated along the extension of chord CF. CF intersects chord BE at point D. Using the following measures, determine the length of segment DE.

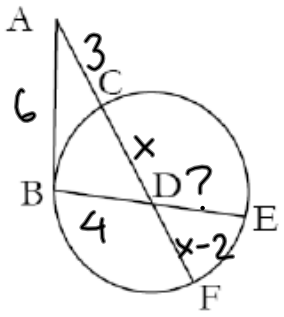
$\overline{AC} = 3$ *unités*

$\overline{AB} = 6$ *unités*

$\overline{CD} = x$

$\overline{BD} = 4$ *unités*

$\overline{DF} = x - 2$



Th 83

$\overline{AB}^2 = \overline{AC} \times \overline{AF}$

$6^2 = 3(3 + x + x - 2)$

$36 = 9 + 6x - 6$

$\frac{33}{6} = \frac{6x}{6}$

$x = 5.5$

Th 80

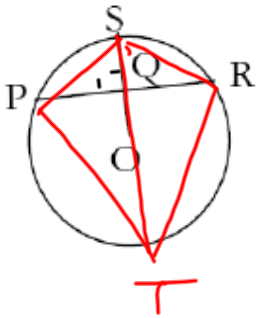
$\overline{BD} \times \overline{DE} = \overline{CD} \times \overline{DF}$

$\frac{4\overline{DE}}{4} = \frac{5.5(3.5)}{4}$

$\overline{DE} = 4.8125$ units

Question 12

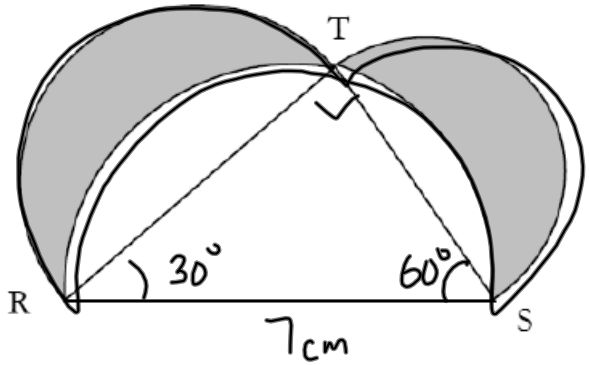
In the following circle, chord PR is perpendicular to radius OS . Show that arc SP is congruent to arc RS.



Statement	Justification
1. $\overline{PQ} \cong \overline{QR}$	Th 71
2. $\angle PQT \cong \angle RQT$	Hypothesis: $\overline{OS} \perp \overline{PR}$
3. $\overline{QT}$ is shared	
4. $\triangle PQT \cong \triangle QRT$	S-A-S
5. $\overset{\frown}{SP} \cong \overset{\frown}{SR}$	(Th 77) Arcs subtended by congruent inscribed angles

Question 13

Below is a representation of Hippocrates' lunules, on the legs of right triangle RST. Calculate the perimeter of these lunules, knowing that the hypotenuse measures 7 cm and one of the acute angles in the triangle is 60°.



$\overline{ST} = \frac{1}{2}(\overline{RS}) = \frac{1}{2}(7) = 3.5$

$\sin 60° = \frac{\overline{RT}}{\overline{RS}}$

$\frac{\sqrt{3}}{2} = \frac{\overline{RT}}{7} \Rightarrow \overline{RT} = \frac{7\sqrt{3}}{2} = 6.06$

$\text{Perimeter} = \frac{1}{2}C_{RS} + \frac{1}{2}C_{RT} + \frac{1}{2}C_{TS}$

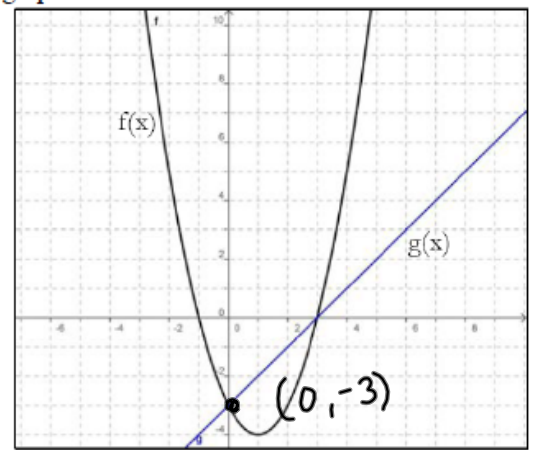
$= \frac{\pi \overline{RS}}{2} + \frac{\pi \overline{RT}}{2} + \frac{\pi \overline{TS}}{2}$

$= \frac{\pi}{2}(\overline{RS} + \overline{RT} + \overline{TS}) = \frac{\pi}{2}(7 + 3.5 + 6.06)$

$= \frac{\pi}{2}(16.56) = 8.28\pi = 26.01$

Question 14

Given functions f and g, graphed below:



Which graph represents $\left(\frac{f}{g}\right)(x)$?

$f(x) \rightarrow 2^{nd}$ degree

$g(x) \rightarrow 1^{st}$ degree

$\frac{2^{nd}}{1^{st}} = 1^{st}$!!

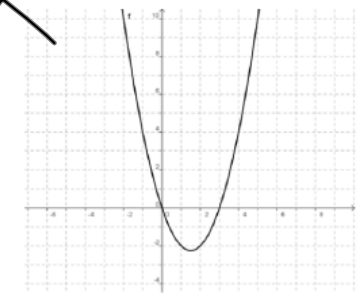
$\frac{f}{g}(0) = \frac{-3}{-3}$

$= 1$

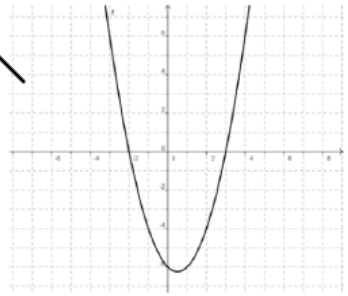
y-int $\frac{f}{g}(x)$

$= (0,1)$

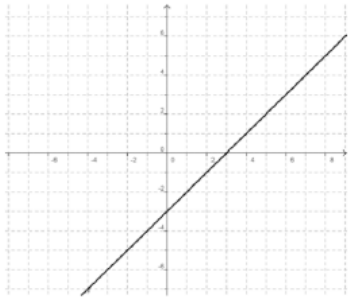
a) ✗



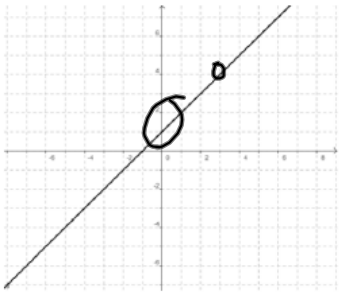
b) ✗



c)



d) ✓



Question 15

Given functions $j(x) = \log_3(2x-3)$ and $k(x) = \frac{1}{2}x + 3$, determine if the following statemtns are true or false. The composition $j \circ k(x)$ is a function.

$(J \circ k)(x) = j(k(x))$

$= j\left(\frac{1}{2}x+3\right)$

$= \log_3\left(2\left(\frac{1}{2}x+3\right)-3\right)$

$y = \log_3(x+3)$

↓ Expo

$3^y = x+3$

$x = 3^y - 3$

a) The composition $j \circ k(x)$ and function $j(x)$ are increasing over $x \in \Re^+$.

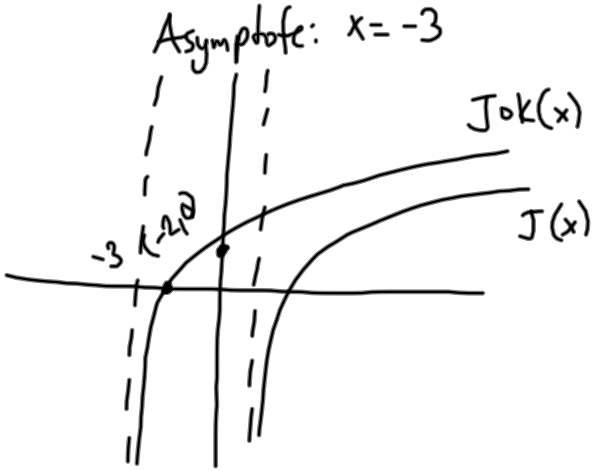
False!

b) The composition $j \circ k(x)$ and function $k(x)$ are negative over $\left]-\frac{3}{2},-1\right[$.

False!

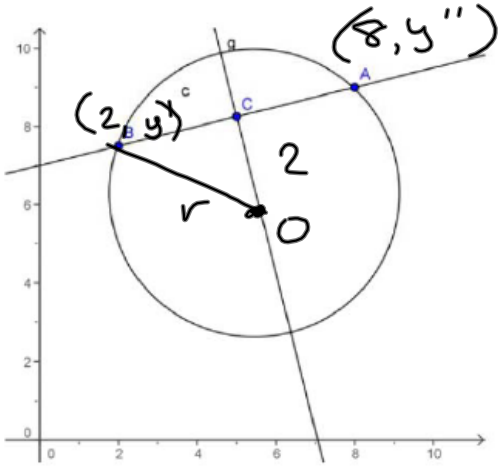
c) The composition $j \circ k(x)$ and function $j(x)$ share the same domain.

False!



Question 16

The linear function $f(x) = \frac{x}{4} + 7$ intersects a circle at x-values 2 and 8. The chord that is coincident on this line is located at a distance of 2 units from the centre of the circle. What is the diameter of the circle? Show all your work.



$f(2) = \frac{2}{4} + 7 = 7.5$

$f(8) = \frac{8}{4} + 7 = 9$

$A(8, 9)$

$B(2, 7.5)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8-2)^2 + (9-7.5)^2}$

$= \sqrt{6^2 + 1.5^2}$

$= \sqrt{36 + 2.25} = \sqrt{38.25}$

$d = 5.81 = \overline{AB}$

$\overline{BC} = \frac{1}{2}\overline{AB} = \frac{1}{2}(5.81) = 2.905$

$r^2 = \overline{BC}^2 + \overline{OC}^2$

$= (2.905)^2 + 2^2$

$= 8.44 + 4 = 12.44$

$r = \sqrt{12.44} = 3.53$

$D = 2r = 2(3.53) = 7.06$

Question 17

Given $f(x) = 3x + 4$ and $g(x) = 5x - 7$. Function h is defined by:

$$h(x) = \frac{f(x)}{g(x)} \text{ où } g(x) \neq 0$$

$g(x) = 0$

$5x - 7 = 0$

$\frac{5x}{5} = \frac{7}{5} \rightarrow x \neq \frac{7}{5}$

Determine if the following statements are true or false.

a) $\text{Dom } f = \mathbb{R}$ and $\text{Dom } h = \mathbb{R} \setminus \left\{\frac{3}{5}\right\}$

False!

b) $\text{Ran } f = \mathbb{R}$ and $\text{Ran } h = \mathbb{R} \setminus \left\{-\frac{7}{5}\right\}$

False

c) $\text{Dom } g = \mathbb{R} \setminus \left\{-\frac{7}{5}\right\}$ and $\text{Dom } h = \mathbb{R}$

False!

d) $\text{Ran } g = \mathbb{R}$ and $\text{Ran } h = \mathbb{R} \setminus \left\{\frac{3}{5}\right\}$

True!

$y = \frac{3x+4}{5x-7}$

$3x + 4 = 5xy - 7y$

$3x - 5xy = -7y - 4$

$\frac{x(3-5y)}{(3-5y)} = \frac{-7y-4}{(3-5y)} \rightarrow x = \frac{-7y-4}{3-5y}$

$0 = 3 - 5y$

$\frac{5y}{5} = \frac{3}{5}$

$y \neq \frac{3}{5}$