Question 1

A) Identify two opposite vectors:

$\vec{d}$ & $\vec{w}$

B) Identify two collinear vectors.

$\vec{e}$ & $\vec{a}$, $\vec{d}$ & $\vec{w}$

C) Identify two orthogonal vectors.

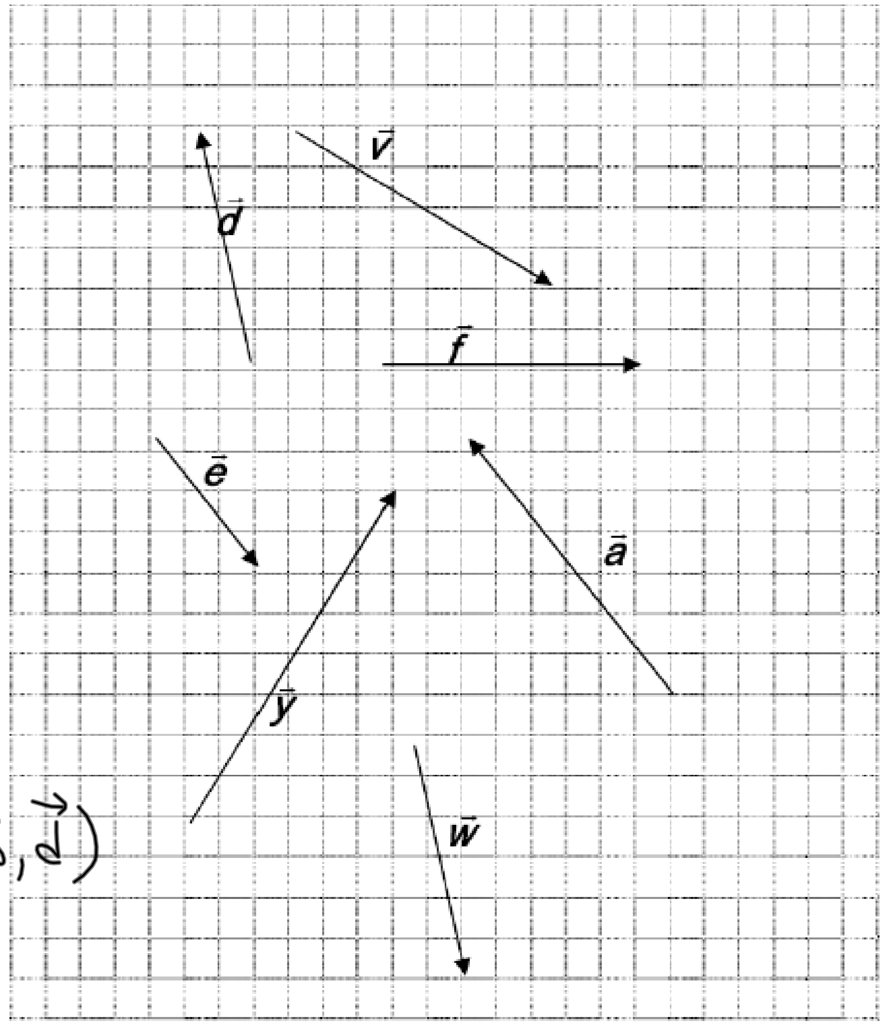
$\vec{v}$ & $\vec{y}$

D) Identify a vector whose magnitude is 7 units.

$\vec{f}$ (maybe $\vec{w}$, $\vec{d}$)

E) Identify two vectors that form a vector basis.

$\vec{v}$ & $\vec{y}$, ... (anything but $\vec{e}$ & $\vec{a}$ and $\vec{d}$ & $\vec{w}$)



Question 2

Given $\vec{u} = (a,b)$, $\vec{v} = (c,d)$ and k is a scalar number, verify if the following equations are true or false, and give the property that explains your answer.

a) $\frac{1}{k}(\vec{u}+\vec{v}) = \frac{\vec{u}}{k}+\frac{\vec{v}}{k}$

Property: True, distributive

b) $\vec{u} \bullet \vec{v} = a \bullet c + b \bullet d$

Property: True, scalar product

$((a-c),(b-d))$

c) $\vec{u}-\vec{v} = (a-c)+(b-d)$

Property: False!

Question 3

By using either the parallelogram or triangle method, determine the ~~direction,~~ the angle and the magnitude (Norm) of the following operations:

a) $\vec{w}+\vec{u}$ $1\text{cm} \hat{=} 1\text{ unit}$

$\|\vec{u}\| = 5$ 30°E of N   N 30° E

$\|\vec{w}\| = 2$ West

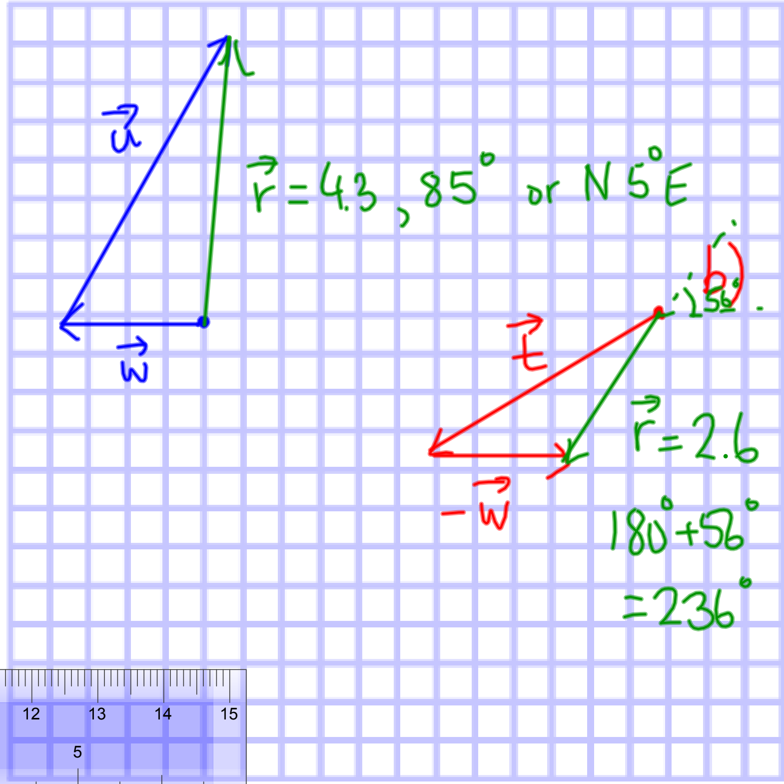
$\|\vec{t}\| = 4$ 60°W of S   → S 60° W

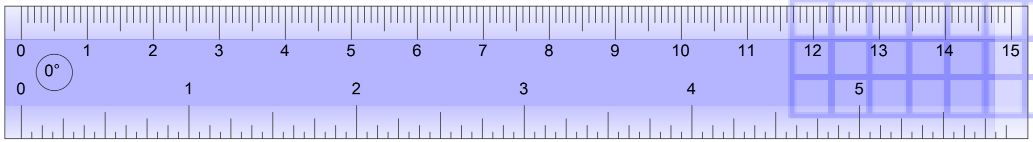
a) $\vec{w}+\vec{u}$

b) $\vec{t}-\vec{w}$

$\vec{t}+(-\vec{w})$

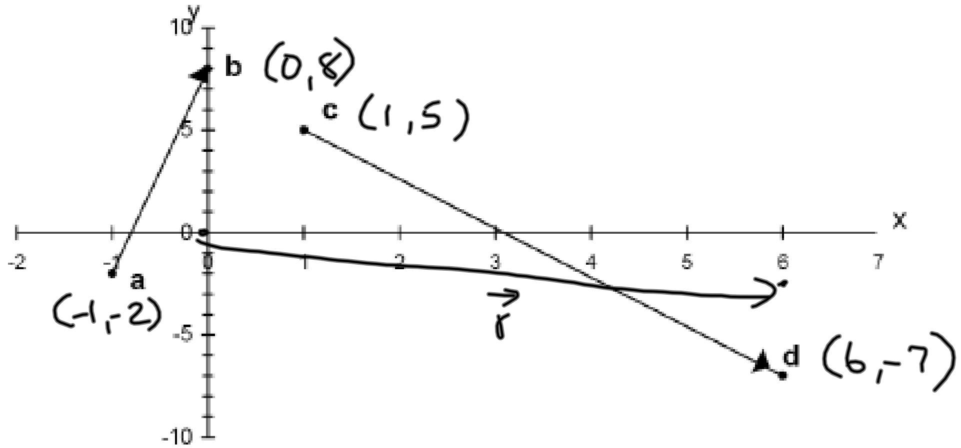
$-\vec{w} = 2,\ E$





Question 4

Calculate the components of the resultant vector $\vec{r} = \vec{ab} + \vec{cd}$ algebraically.



$$\vec{ab} = (x_b - x_a, y_b - y_a)$$
$$= (0-(-1), 8-(-2))$$
$$= (1, 10)$$

$$\vec{cd} = (x_d - x_c, y_d - y_c)$$
$$= (6-1, -7-5)$$
$$= (5, -12)$$

$$\vec{r} = (1,10) + (5,-12)$$
$$= (6, -2)$$

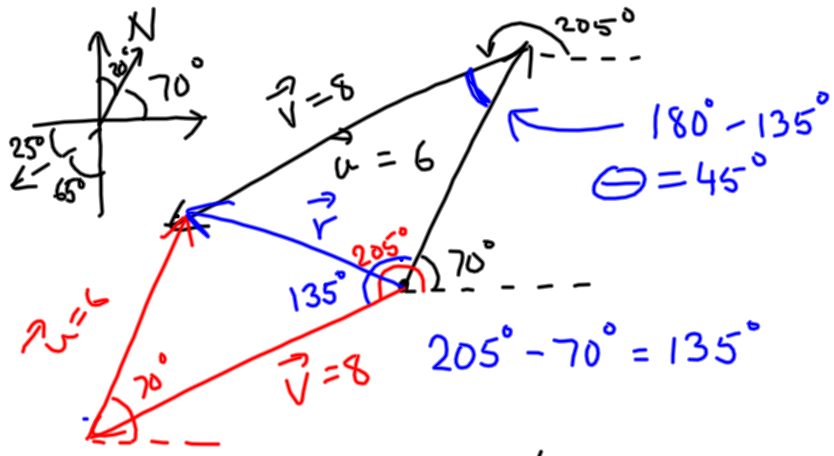
Question 5

(Norm)

Calculate the magnitude of the resultant vector $\vec{r} = \vec{u} + \vec{v}$ given:

$\|\vec{u}\| = 6$ 20°E of N N 20° E = 70°

$\|\vec{v}\| = 8$ 65°W of S S 65° W = 205°



$\vec{r} = \vec{u} + \vec{v}$

Cosine Law

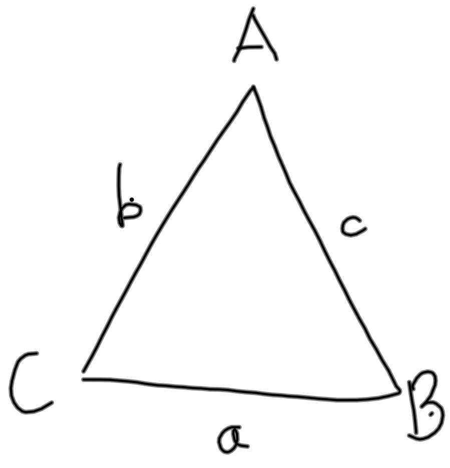
$$\|\vec{r}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\|\vec{u}\| \times \|\vec{v}\| \times \cos\theta$$

$$= (6)^2 + (8)^2 - 2(6)(8)\cos 45°$$

$$= 100 - 96\cos 45°$$

$$= 32.12$$

$$\|\vec{r}\| = \sqrt{32.12} = 5.7$$



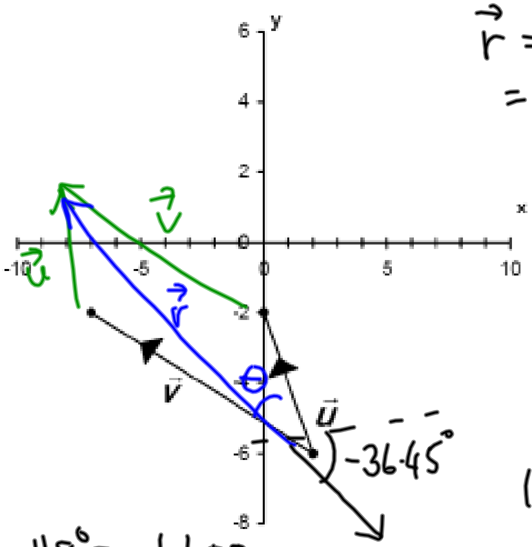
Question 6

Given:

$\|\vec{u}\| = 3$ 28°W of N $= 118°$

$\|\vec{v}\| = 6$ at 156°

Find the angle between the resultant vector $\vec{r} = \vec{u} + \vec{v}$ and vector $\vec{u}$.



$\vec{u}_x = \|\vec{u}\| \cos\theta_{\vec{u}} = 3\cos 118° = -1.408$

$\vec{u}_y = \|\vec{u}\| \sin\theta_{\vec{u}} = 3\sin 118° = 2.649$

$\vec{u} = (-1.408, 2.649)$

$\vec{v}_x = \|\vec{v}\| \cos 156° = 6\cos 156° = -5.481$

$\vec{v}_y = \|\vec{v}\| \sin 156° = 6 \sin 156° = 2.440$

$\vec{v} = (-5.481, 2.440)$

$\vec{r} = \vec{u} + \vec{v}$

$= (-1.408, 2.649) + (-5.481, 2.440)$

$= (-6.889, 5.089)$

$\|\vec{r}\|^2 = \|r_x\|^2 + \|r_y\|^2$

$= (-6.889)^2 + (5.089)^2$

$= 73.356$

$\|\vec{r}\| = \sqrt{73.356} = 8.565$

$\tan^{-1}\left(\frac{5.089}{-6.889}\right) = \theta_{\vec{r}} = -36.45°$

$\theta_{\vec{r}} = 180° + (-36.45°)$

$= 143.55°$

$\theta = \theta_{\vec{r}} - \theta_{\vec{u}}$

$= 143.55° - 118° = 25.55°$

Question 7

Find the angle between vectors $\vec{u}$ and $\vec{w}$ resulting from the following linear combinations of vectors $\vec{r}=(2.5,-1)$ and $\vec{v}=(0,-5)$.

$$\vec{u}=3\vec{v}-\frac{1}{5}\vec{r} \qquad \text{and} \qquad \vec{w}=\frac{1}{10}\vec{v}+4\vec{r}$$

$$\vec{u}=3(0,-5)-\frac{1}{5}(2.5,-1)$$
$$=(0,-15)+(-0.5,+0.2)$$
$$\vec{u}=(-0.5,-14.8)$$

$$\vec{w}=\frac{1}{10}(0,-5)+4(2.5,-1)$$
$$=(0,-0.5)+(10,-4)$$
$$=(10,-4.5)$$

$$\vec{u}\cdot\vec{w}=ac+bd$$
$$=(-0.5)(10)+(-14.8)(-4.5)$$
$$=-5+66.6$$
$$=61.6\ \text{unit}^2$$

$$\vec{u}\cdot\vec{w}=\|\vec{u}\|\times\|\vec{w}\|\cos\theta$$

$$\|\vec{u}\|=\sqrt{\|\vec{u}_x\|^2+\|\vec{u}_y\|^2}=\sqrt{(-0.5)^2+(-14.8)^2}$$
$$=14.81$$

$$\|\vec{w}\|=\sqrt{\|\vec{w}_x\|^2+\|w_y)\|^2}$$
$$=\sqrt{(10)^2+(-4.5)^2}=10.97$$

$$\frac{61.6}{(14.81)(10.97)}=\frac{\cancel{(14.81)(10.97)}\cos\theta}{\cancel{(14.81)(10.97)}}$$

$$\cos\theta=0.379$$

$$\cos^{-1}(0.379)=67.7^\circ$$

Question 8

Find the coefficients of linear combination that allow us to express vector $\vec{w} = (-4,5)$ in terms of the basis vectors $\vec{r} = (6,-15)$ and $\vec{s} = (1,2)$.

$$\vec{w} = k_1\vec{r} + k_2\vec{s}$$

$$(-4,5) = k_1(6,-15) + k_2(1,2)$$

$$(-4,5) = (6k_1, -15k_1) + (k_2, 2k_2)$$

$$x \quad (-4 = 6k_1 + k_2) \times -2$$

$$y \quad 5 = -15k_1 + 2k_2$$

$$8 = -12k_1 - 2k_2$$

$$+ \quad 5 = -15k_1 + 2k_2$$

$$\frac{13}{-27} = \frac{-27k_1}{-27} + \cancel{0k_2}$$

$$k_1 = \frac{-13}{27}$$

$$x \rightarrow -4 = 6\left(\frac{-13}{27}\right) + k_2$$

$$-4 + \frac{26}{9} = k_2$$

$$\frac{-10}{9} = k_2$$

$$\boxed{\vec{w} = \frac{-13}{27}\vec{r} - \frac{10}{9}\vec{s}}$$

Question 9

Find the scalar product of the two following vectors.

$\left\|\vec{u}\right\| = 6$, E 20°S $= -20°$ or $340°$

$\left\|\vec{v}\right\| = 2$, E 70°N $= 70°$

$$\theta = 70 - (-20) = 90°$$

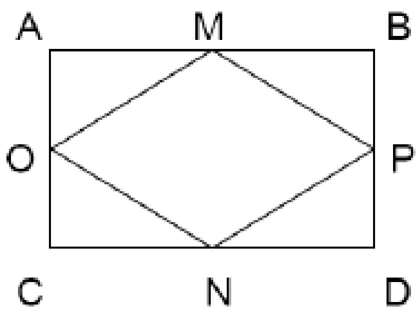
$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \times \|\vec{v}\| \cos 90°$$

$$= (6)(2)(0)$$

$$= 0$$

Question 10

Complete the following proof. Show that if we join the middle point of the adjacent sides of a rectangle, we obtain a Rhombus. Write the affirmations and justifications in the space provided.



Hypothesis	Justification
AB = CD	Rectangle
AC = BD	" "
CN = ND	
AO = OC	N, M, O, P are mid-points
AM = MB	
BP = PD	

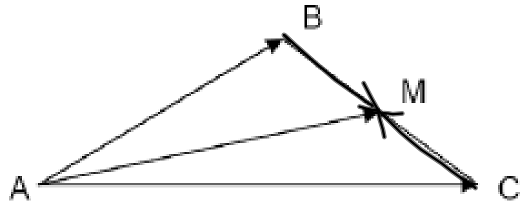
To show that OM = ON = MP = PN

Proof

1. $\vec{AB} = \vec{AM} + \vec{MB}$ Chasles' Relation
2. $\vec{CD} = \vec{CN} + \vec{ND}$
3. $\vec{AM} + \vec{MB} = \vec{CN} + \vec{ND}$ because $\vec{AB} = \vec{CD}$ (Hypothesis)
4. $\frac{\cancel{2}\vec{AM}}{\cancel{2}} = \frac{2\vec{CN}}{2}$ because $\vec{AM} = \vec{MB}$ and $\vec{CN} = \vec{NB}$ (Hyp)
5. $\cancel{\vec{AO}} + \vec{OM} = \cancel{\vec{CO}} + \vec{ON}$ Chasles'
6. OM = ON because $\vec{AO} = \vec{CO}$

Question 11

Given triangle ABC, point M is the mid-point of side BC and $\vec{AM}, \vec{AB}, \vec{AC}$ are vectors.



Using Chasles' principle, prove that:

$\vec{AM} = \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AC}$

$$\left.\begin{aligned}\vec{AM} &= \vec{AB} + \vec{BM}\\ \vec{AM} &= \vec{AC} + \vec{CM}\end{aligned}\right\} \text{Chasles'}$$

$2\vec{AM} = \vec{AB} + \vec{BM} + \vec{AC} + \vec{CM}$ (Addition)

$2\vec{AM} = \vec{AB} + \vec{AC} + (\vec{BM} + \vec{CM})$ Commutativity + Associativity

$\vec{CM} = -\vec{MC}$ opposite vectors

$\vec{BM} = \vec{MC}$ M is mid-point

$\vec{CM} = -\vec{BM}$

$2\vec{AM} = \vec{AB} + \vec{AC}$

$+ (\cancel{\vec{BM} + (-\vec{BM})})$

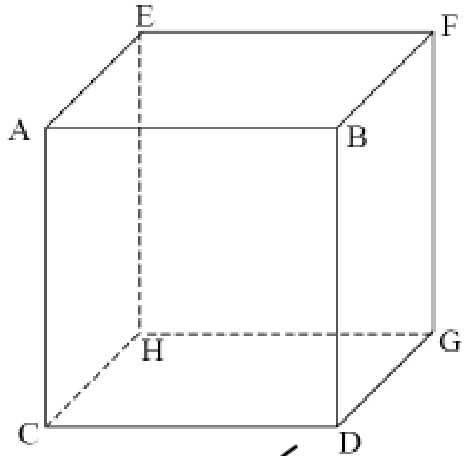
0

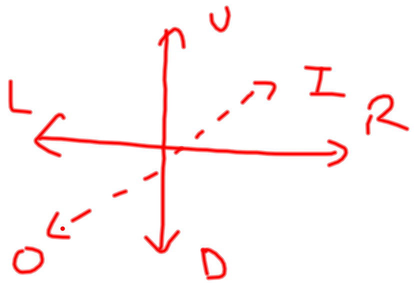
$\frac{\cancel{2}\vec{AM}}{\cancel{2}} = \frac{\vec{AB} + \vec{AC}}{2}$

$\vec{AM} = \frac{\vec{AB}}{2} + \frac{\vec{AC}}{2}$

Question 12

Given the following cube, whose sides measure 1 unit in length:





Determine the false statements below, and correct them.

$R + I = R + I$ ✓

a) $\overrightarrow{AB} + \overrightarrow{CH} = \overrightarrow{AF}$

$U - (L + D)$ False!

$\overrightarrow{DB} + \overrightarrow{FG} = \vec{0}$

c) $\overrightarrow{DB} - \overrightarrow{FH} = \vec{0}$

$= U + R + U$

$R + R = 2R$ ✓

b) $\overrightarrow{AB} + \overrightarrow{EF} = 2\overrightarrow{AB}$

$R + 2(U + I) + R = U + R + I$

d) $\overrightarrow{AB} + 2\overrightarrow{CE} + \overrightarrow{HG} = \overrightarrow{CF}$ False! $\overrightarrow{AB} + 2\overrightarrow{CE} + \overrightarrow{HG} = 2\overrightarrow{CF}$

e) $\overrightarrow{DB} - \overrightarrow{FG} + \overrightarrow{CD} - \overrightarrow{FE} = 2\overrightarrow{CB}$

$U + U + R + R = 2(R + U)$

True

f) $\overrightarrow{FA} + \overrightarrow{FH} + \overrightarrow{FD} = 2\overrightarrow{FC}$

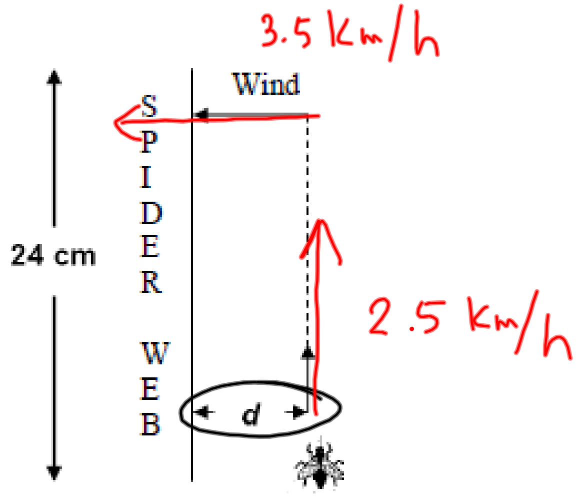
$(L + O) + (L + D) + (D + O) = 2(D + O + L)$

$2L + 2O + 2D = 2D + 2O + 2L$ ✓

Question 13

A) A fly, influenced by extreme sports, moves at a speed of 2.5 km/h. It flies by a spider web 24 cm long. The wind is blowing perpendicularly to its trajectory at a speed of 3.5 km/h, bringing the fly dangerously close to the spider web.

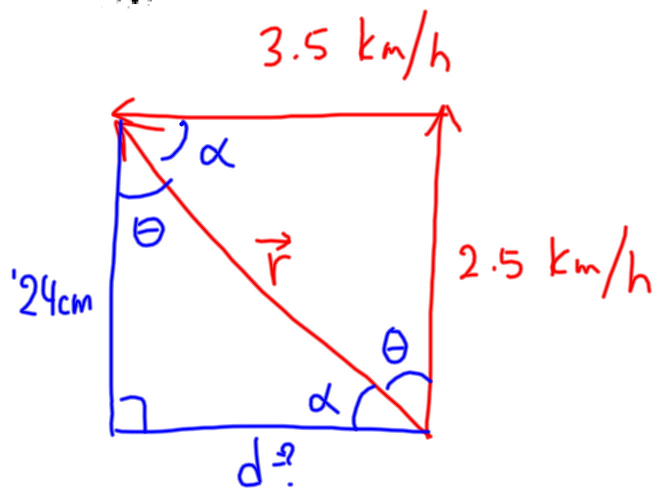
Calculate the distance between the fly's trajectory and the spider web.



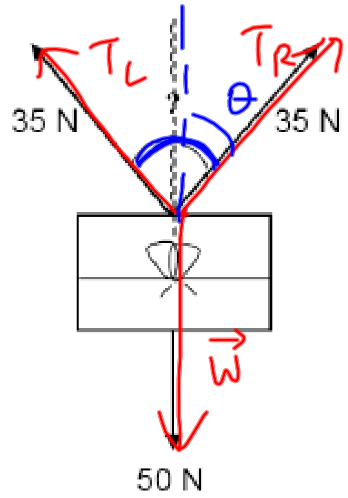
$$\theta = \tan^{-1}\left(\frac{3.5}{2.5}\right)$$
$$= 54.5^\circ$$

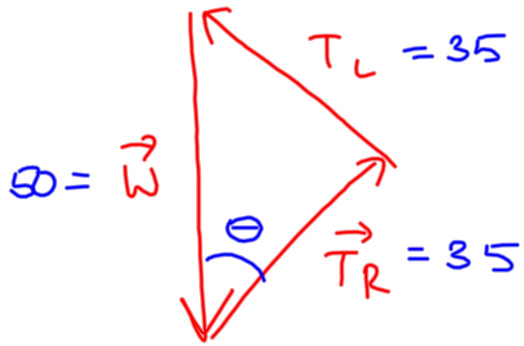
$$\tan 54.5^\circ = \frac{d}{24}$$

$$d = 24 \times \tan 54.5^\circ$$
$$= 33.6 \text{ cm}$$



B) A package of ~~mass~~ weight 50N is suspended using two ropes that can support a maximum force of 35N each. (Force is a vector quantity that takes into account the mass of an object on Earth and it is measured in Newtons (N))





Equilibrium

What is the maximum angle between the two ropes to avoid them breaking? (Make a sketch representing the forces)

Cosine Law

$$\|\vec{T_L}\|^2 = \|\vec{w}\|^2 + \|\vec{T_R}\|^2 - 2\|\vec{w}\| \times \|\vec{T_R}\| \cos\theta$$

$$\cancel{35^2} = 50^2 + \cancel{35^2} - 2(50)(35)\cos\theta$$

$$\frac{-2500}{-3500} = \frac{-3500\cos\theta}{-3500}$$

$$\cos\theta = \frac{5}{7} \longrightarrow \theta = \cos^{-1}\left(\frac{5}{7}\right) = 44.4^\circ$$

The ropes will break if the angle between them exceeds $2 \times 44.4^\circ = 88.8^\circ$.