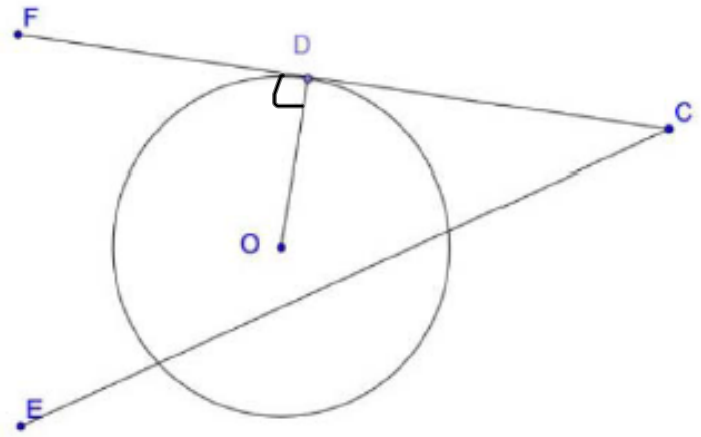
Question 1

Given the following circle O, select from the list of theorems provided the theorem that proves that angle ODF is a right-angle.

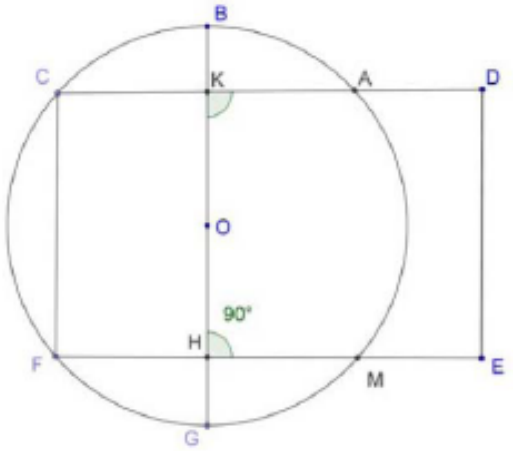


Theorem 8

Question 2

In the figure below, given that:

- O is the centre of the circle
- CDEF is a rectangle
- OK is perpendicular to CD
- OH is perpendicular to EF
- OK and OH are congruent
- BG is a diameter of O

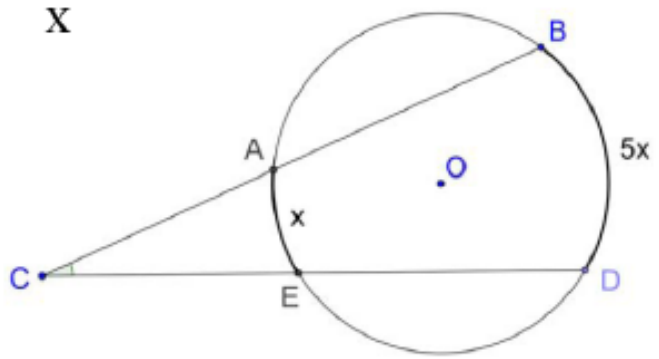


Give the theorem that proves that arcs $\overset{\frown}{CF}$ and $\overset{\frown}{AM}$ are congruent.

Theorem 10 or 7

Question 3

In the following figure, arc BD = 5x and arc AE = x. Calculate the measure of angle ACE. in terms of x.



Th 17 $\angle ACE = \frac{1}{2}(\overset{\frown}{BD} - \overset{\frown}{AE})$

$= \frac{1}{2}(5x - x)$

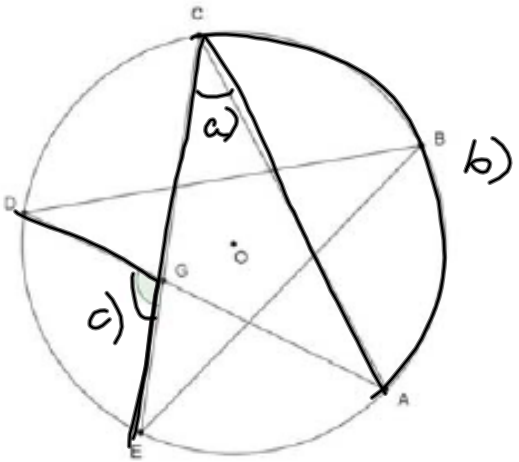
$= \frac{1}{2}(4x)$

$= 2x$

Question 4

5 lights are installed around a 10 m diameter pool. Each light is represented by the letters A, B, C, D and E in the diagram below. One light illuminates exactly one-fifth of the edge of the pool.

Calculate the following values and justify your answer by stating the number of the theorem used.



a. What is the measure of angle ACE?

Th 16 $\angle ACE = \frac{1}{2}\overset{\frown}{AE}$

$m\overset{\frown}{AE} = \frac{360^\circ}{5} = 72^\circ$

$\angle ACE = \frac{1}{2}(72^\circ)$

$= 36^\circ$

b. What is the measure of arc CBA?

$m\overset{\frown}{CBA} = \frac{2}{5}(360^\circ)$

$= 144^\circ$

c. What is the measure of angle DGE?

Th 15 $\angle DGE = \frac{1}{2}(\overset{\frown}{DE} + \overset{\frown}{CBA})$

$= \frac{1}{2}(72^\circ + 144^\circ)$

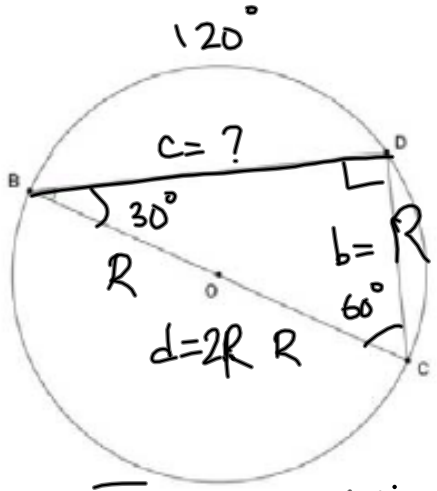
$= \frac{1}{2}(216)$

$= 108^\circ$

Question 5

Given the following circle, whose centre is O and radius is R.

If arc BD measures 120°, what is the measure of segment BD as a fraction of the radius R?



Th 16 $\angle BCD = \frac{1}{2}\overset{\frown}{BD}$

$= \frac{1}{2}(120°)$

$= 60°$

$m\overset{\frown}{BC} = 2R$ (diameter)

th 20 $\overline{CD} = \frac{1}{2}(\overline{BC}) = R$

th 18 ΔBCD is right-angled at $\angle D$.

pythagorus $d^2 = b^2 + c^2$

$(2R)^2 = R^2 + c^2$

$4R^2 - R^2 = c^2 = 3R^2$

$c = \sqrt{3R^2}$

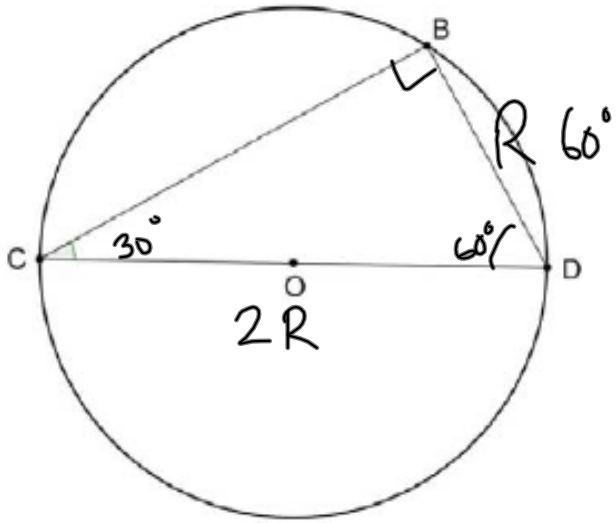
$= \sqrt{3}R$

$\angle DBC = 180° - 90° - 60° = 30°$

Question 6

The following figure illustrates circle O with inscribed triangle BCD below, CD is a diameter and BD is equal to the radius.

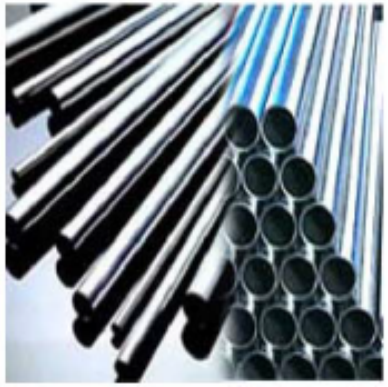
Determine if the following statements are true or false. If the statement is true, give the number of the theorem.

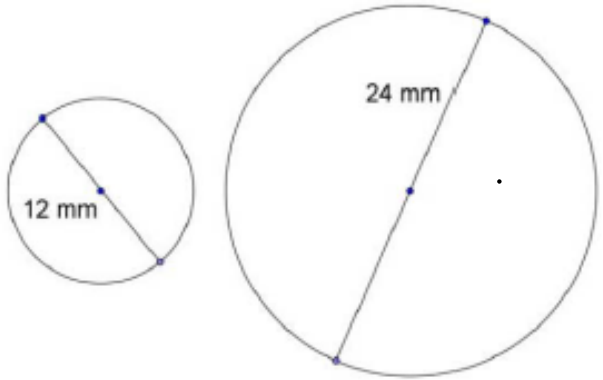


Statement	True or False?	Justification
Angle CBD is a right-angle.	TRUE	18 or 16
Since BD = radius of O, angle BCD = 60°.	False	
The measure of angle CDB is equal to the measure of arc BD.	TRUE	20, 16, 18

Question 7

Copper pipes used in residential plumbing are typically 12 mm or 24 mm in diameter.





th 11 & 12

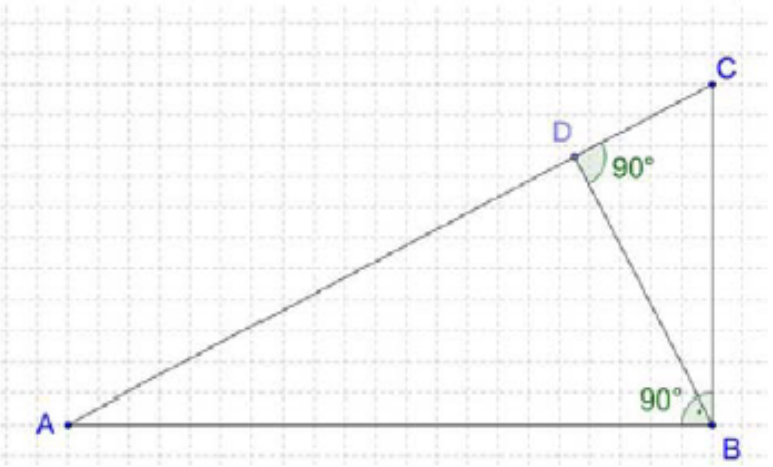
Determine if the following statements are true or false.

Statement	True or False?
The area of the larger pipe is double the area of the smaller one.	False!
The perimeter of the larger pipe is double of the smaller one.	TRUE
The circumference of the larger pipe is twice the diameter of the smaller pipe.	False
4 times as much water can circulate through the larger pipe.	TRUE (Flow ∝ Area)

Question 8

In the following figure, which theorem proves that:

$$DC = \frac{BC^2}{AC}$$



Th 23 or Th 21

Th 23 $\frac{\overline{BC}^2}{\overline{AC}} = \frac{\overline{CD} \times \overline{AC}}{\overline{AC}}$

$\overline{CD} = \frac{\overline{BC}^2}{\overline{AC}}$

Question 9

A new pizzeria just opened up in the neighborhood and they are selling a 12" all dressed pizza for $9.25.

a) Using theorems 11 OR 12, what should the owner charge for a 16" pizza.

COST $\propto$ AREA

th 12 $\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$

$\frac{9.25}{A_2} = \left(\frac{12}{16}\right)^2$

$\frac{9.25}{A_2} = \left(\frac{3}{4}\right)^2$

$\frac{9.25}{A_2} \times \frac{9}{16}$

$A_2 = \frac{9.25 \times 16}{9} = \$16.40$

b) Some people like to spread butter on the crust of their pizza. The owner will give 5 packets of butter for a 12" pizza. Using the same two theorems, how many packets of butter should he give with the 16" pizza?

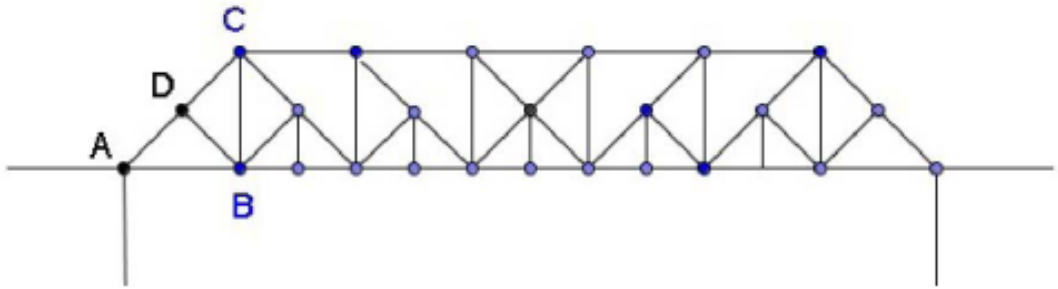
Butter $\propto$ Circumference

th 11 $\frac{C_1}{C_2} = \frac{r_1}{r_2}$ $\frac{5}{C_2} \times \frac{12}{16}$ $\frac{BC_2}{8} = \frac{20}{3}$

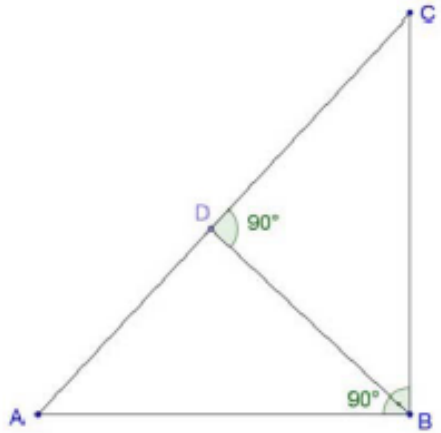
$C_2 = 6.6 \sim 7$ packets

Question 10

Below is a drawing of the Baltimore Bridge:



The engineer in charge of the project would like to re-calculate the left portion of the bridge, enlarged below. Knowing that triangle ABC is right-angled at B and BD is perpendicular to AC.



Which theorem proves that AB x BC = BD x AC?

Th 24

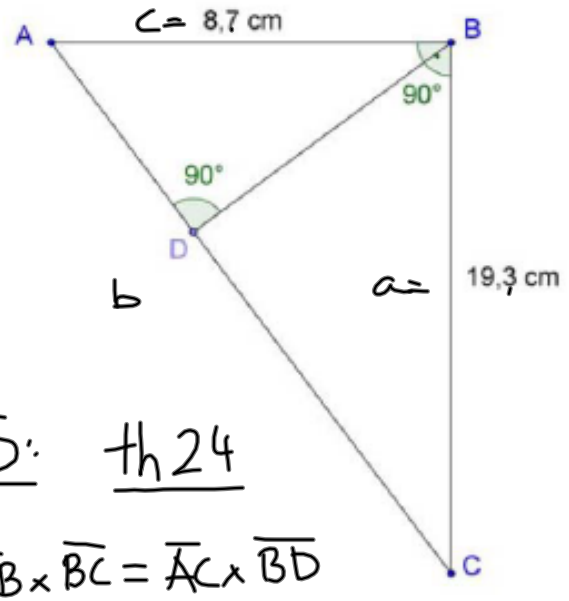
(th21)

Question 11

Triangle ABC is right-angled at B and BD is its altitude.

Find the measure of AC and ~~BC~~ $\overline{BD}$

Support your calculations with the appropriate theorem.



$\overline{AC}$: Pythagorus

$b^2 = a^2 + c^2$

$= 19.3^2 + 8.7^2$

$b = \sqrt{19.3^2 + 8.7^2}$

$= 21.17$ cm

$\overline{BD}$: th 24

$\overline{AB} \times \overline{BC} = \overline{AC} \times \overline{BD}$

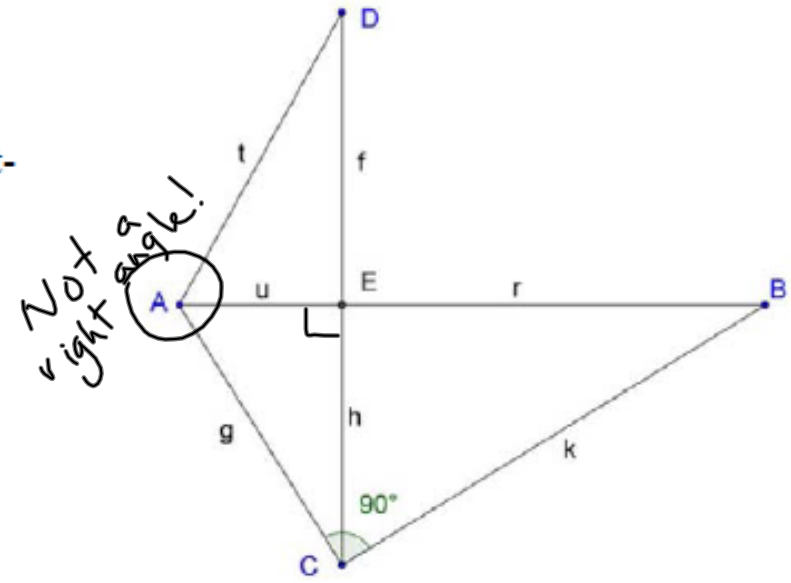
$\frac{8.7 \times 19.3}{21.17} = \frac{21.17 \times \overline{BD}}{21.17}$

$\overline{BD} = 7.93$ cm

Question 12

Given the following:

- Triangles ABC and AEC are right-angled
- t is the length of AD
- f is the length of ED
- r is the length of EB
- k is the length of BC
- g is the length of AC
- h is the length of CE
- u is the length of AE

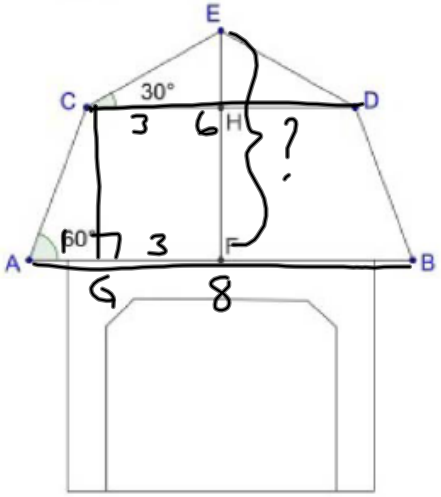


Are the following statements true or false? In the case of a true statement, indicate the theorem that proves it.

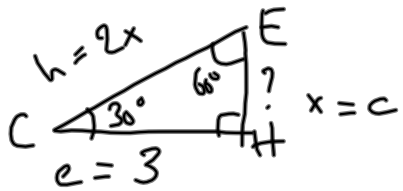
Statement	True or False?	Theorem
1. $u \cdot r = h^2$	TRUE	th 22
2. $g^2 = h \cdot f$	False	
3. $g \cdot k = h\,(u + r)$	TRUE	th 24
4. $u\,(f + h) = t \cdot g$	False	
5. $u^2 = g^2 - h^2$	TRUE	Pythagorus

Question 13

A farmer would like to build a garage for his tractor. Here is a drawing and a photo of what he would like it to look like.



Knowing that the 2 slopes of the roof are symmetrical, that CD = 6m and AB = 8m, what is the length of EF (the height of the loft) ?



th 20 $\Rightarrow \overline{EC} = 2 \times \overline{EH}$

$\overline{EC} = 2x$

Pythagorus $h^2 = e^2 + c^2$

$(2x)^2 = 3^2 + x^2$

$4x^2 - x^2 = 9$

$\frac{3x^2}{3} = \frac{9}{3}$

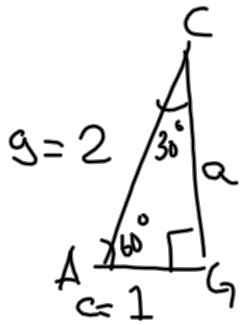
$x^2 = 3$

$x = \sqrt{3} = 1.732$ m

$\overline{EH} = 1.732$m

th 20 $\overline{AC} = 2(\overline{AG})$

$= 2(1) = 2$



Pythagorus $g^2 = c^2 + a^2$

$2^2 = 1^2 + a^2$

$4 - 1 = a^2 = 3$

$a = \sqrt{3} = 1.732 = \overline{CG}$

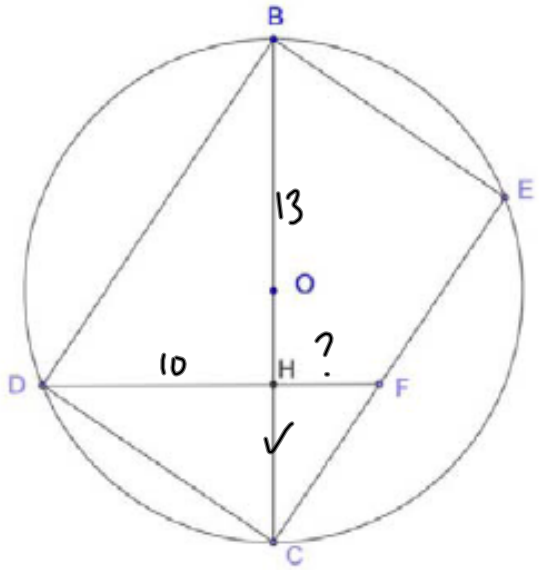
Loft $= \overline{GC} + \overline{EH}$

$= 1.732 + 1.732 = 3.464$ m

Question 14

In the following figure;

- Rectangle BDCE is inscribed in circle O
- DF is perpendicular to BC
- HB = 13cm and DH = 10cm



Find the length of HF (to the nearest hundredth of a centimeter). Show all your work.

Th 22 (left)

$\overline{DH}^2 = \overline{BH} \times \overline{CH}$

$\frac{10^2}{13} = \frac{13\,\overline{CH}}{13}$

$\overline{CH} = \frac{100}{13} = 7.69\text{ cm}$

Th 22 (Lower)

$\overline{CH}^2 = \overline{DH} \times \overline{HF}$

$\frac{7.69^2}{10} = \frac{10\,\overline{HF}}{10}$

$\overline{HF} = \frac{59.1}{10} = 5.91\text{cm}$