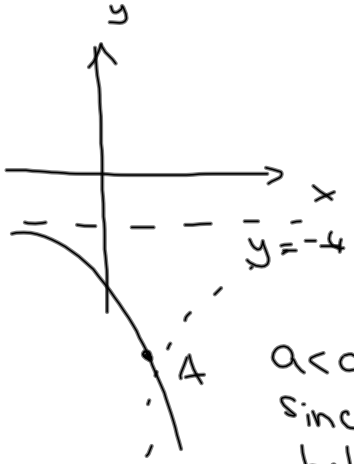
Question 1

(x, y)

Point A(2, -13) belongs to an exponential function of the form $f(x) = \pm c^x + k$. Knowing that the equation of the asymptote is $y = -4$, find the equation of the exponential function f.

$y = k$



a < 0 since below asymptote → a = -1

$$f(x) = \pm c^x + k$$

$$-13 = -c^2 + (-4)$$

$$\frac{-9}{-1} = \frac{-c^2}{-1}$$

$$\sqrt{c^2} = \sqrt{9}$$

$$c = 3$$

$$f(x) = -3^x - 4$$

Question 2

Solve the following equation algebraically.

$$7^{(-8-2x)} = 49^{(3x+4)}$$

$$7^{-8-2x} = (7^2)^{(3x+4)}$$

$$-8-2x = 2(3x+4)$$

$$-8-2x = 6x+8$$

$$\frac{-16}{8} = \frac{8x}{8}$$

$$x = -2$$

Question 3

Solve the following equation algebraically.

$$9^{(x-3)} = 125^{(x+1)}$$

$$\log 9^{x-3} = \log 125^{x+1}$$

$$(x-3)\log 9 = (x+1)\log 125$$

$$x\log 9 - 3\log 9 = x\log 125 + \log 125$$

$$x\log 9 - x\log 125 = \log 125 + 3\log 9$$

$$\frac{x\cancel{(\log 9 - \log 125)}}{\cancel{(\log 9 - \log 125)}} = \frac{\log 125 + 3\log 9}{\log 9 - \log 125}$$

$$x = -4.34$$

Question 4

Find the inverse of the following function:

$$f(x) = \log_7(x-3)$$

$$y = \log_7(x-3)$$

$\downarrow$ Inverse

$$x = \log_7(y-3)$$

$\downarrow$ Expo Form

$$7^x = y-3$$

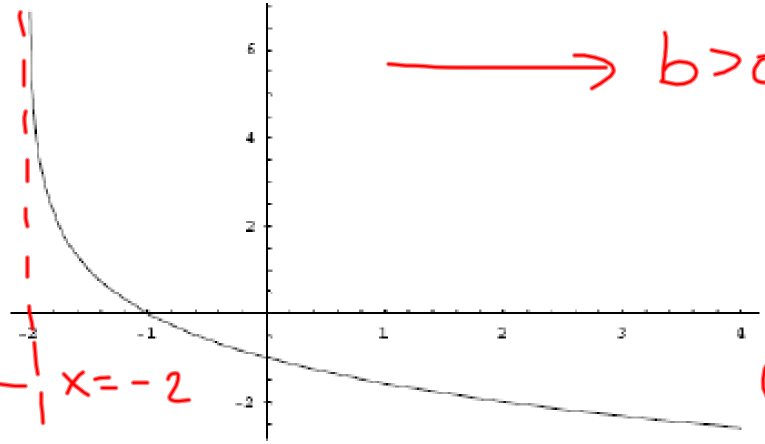
$$y = 7^x + 3$$

$$f^{-1}(x) = 7^x + 3$$

Question 5

$a = +1$

Given f, a logarithmic function defined by $f(x) = \log_c b(x-h)$ and whose graph is given below:



$b > 0$

$h = -2$ ← $x = -2$

$a = +1$

$a < 0$ → $0 < c < 1$

Which of the following is TRUE?

A) $b > 0$, $h < 0$ and $0 < c < 1$

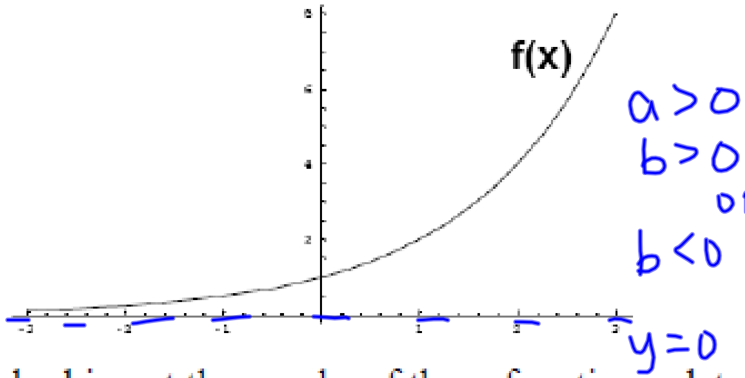
C) $b > 0$, $h < 0$ and $c > 1$

~~B) $b > 0$, $h > 0$ and $c > 1$~~

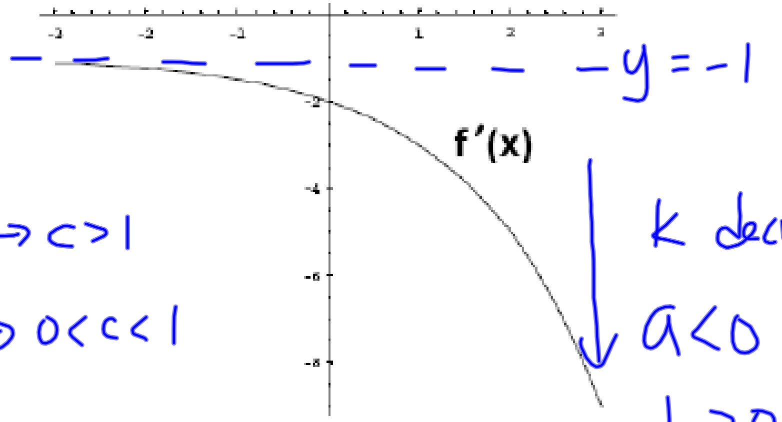
~~D) $b < 0$, $h < 0$ and $c < 1$~~

Question 6

Given the graphs of f, an exponential function of the form $f(x) = a(c^{bx}) + k$ and function f', obtained from f by modifying certain parameters:



$a>0$
$b>0 \rightarrow c>1$
OR
$b<0 \rightarrow 0<c<1$



k decreased
$a<0$
$b>0$

In looking at the graphs of these functions, determine among the following choices which combination allows us to transform f to f'.

~~A) The sign of a changed and k increased.~~
B) The sign of a changed and k decreased. ✓
C) The signs of a and b changed.
~~D) The sign of b changed and k increased.~~

Question 7

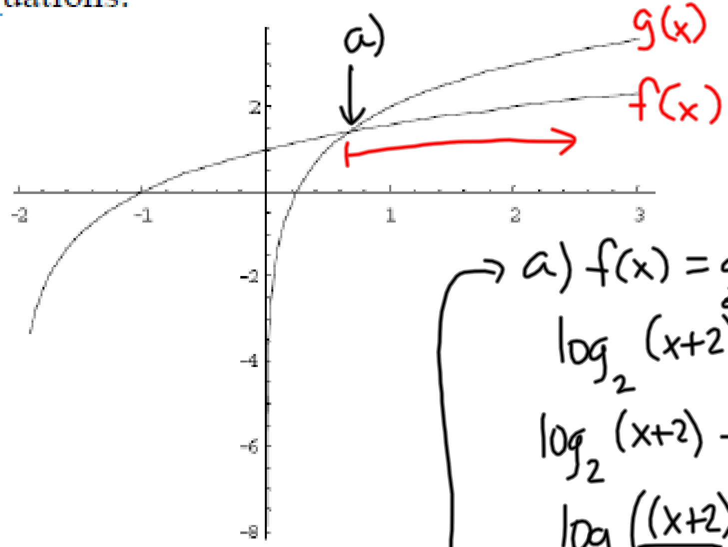
Given the two following graphs and equations:

$x = -2$

$f(x) = \log_2(x + 2)$

$g(x) = \log_2 x + 2$

$x = 0$



$\rightarrow$ a) $f(x) = g(x)$

$\log_2(x+2) = \log_2 x + 2$

$\log_2(x+2) - \log_2 x = 2$

$\log_2\left(\frac{(x+2)}{x}\right) = 2$

$\downarrow$ Expo Form

$2^2 = \frac{x+2}{x}$

$\frac{4}{1} = \frac{x+2}{x}$

$4x = x+2$

$\frac{3x}{3} = \frac{2}{3}$

$x = \frac{2}{3}$

a) Determine for which value of the domain f(x) = g(x).

$x = \frac{2}{3}$

b) Determine for which values of x $f(x) \leq g(x)$.

$\left[\frac{2}{3}, +\infty\right)$

c) Do these functions have the same asymptote?

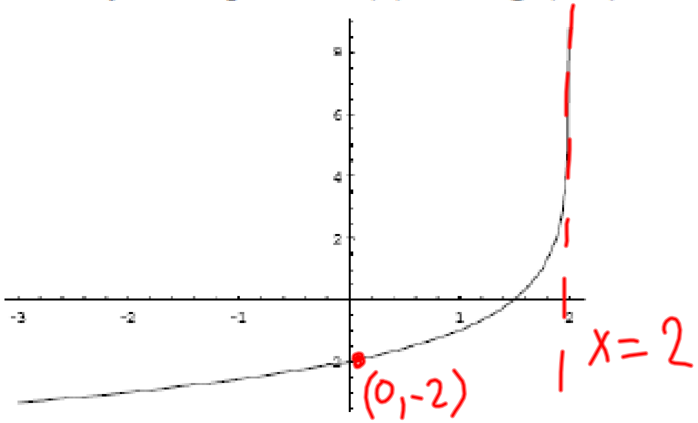
No!

d) The y-intercept of function f is y=1 and ~~[illegible]~~ g does not have a y-intercept. True or false?

TRUE!

Question 8

The graph of function f, defined by the equation $f(x) = -\log_2(2-x) - 1$, is given below:



Determine if the statements below are true or false. If they are false, correct the statement.

Statement	True or False	Correction (if necessary)
a) The point (0, -2) belongs to the graph of the function.	TRUE	
b) The equation of the asymptote is y = -4	False	x=2
c) The domain of the function is $-\infty$, 2 [	TRUE	
d) The range of the function is]-4, ∞	False	$\mathbb{R}$

Question 9

Over the next few years, we predict that the price of a house will increase 2% every 2 years.

If V(T) corresponds to the value of a house after t years, what is the equation that represents this situation if the initial value of the house was $125 000?

r = 2% every 2 years

r = 1% per year

$k = \frac{1}{2}$

$$V(t) = V_0\left(1+\frac{r}{k}\right)^{kt}$$

$$V(t) = 125000\left(1+\frac{0.01}{\frac{1}{2}}\right)^{\frac{1}{2}t}$$

$$V(t) = 125000(1.02)^{\frac{1}{2}t}$$

Question 10

Determine the value of the logarithmic expression below using the laws of logs.

$$5\ (\log_a a^3)^2 - 2\ \log_{1/a} 1 + \log_a a^{-3}$$

$$= 5(3)^2 - 2(0) + (-3)$$

$$= 45 - 0 - 3$$

$$= 42$$

Question 11

Simplify the following expression using the laws of logs.

$$\log_5(x^2 + x - 6) - \log_{1/5}5(x^2 - 9) - \log_5 (x + 3)^2(x^2 - 5x + 6)$$

$$= \log_5(x^2+x-6) + \log_5 5(x^2-9) - \log_5 (x+3)^2(x^2-5x+6)$$

$$= \log_5\left(\frac{\overset{(x+3)(x-2)}{(x^2+x-6)}(5)\overset{(x+3)(x-3)}{(x^2-9)}}{\underset{(x+3)(x+3)}{(x+3)^2}\underset{(x-3)(x-2)}{(x^2-5x+6)}}\right)$$

$$= \log_5\left(\frac{\cancel{(x+3)}\cancel{(x-2)}(5)\cancel{(x+3)}\cancel{(x-3)}}{\cancel{(x+3)}\cancel{(x+3)}\cancel{(x-3)}\cancel{(x-2)}}\right)$$

$$= \log_5 5 = 1$$

Question 12

Solve the following equation algebraically using the laws of logs.

$$\log_2 x + 9 \log_x 2 = 6$$

$$\frac{\log x}{\log 2} + \frac{9 \log 2}{\log x} = 6$$

LCD $=(\log 2)(\log x)$

$$\frac{(\log x)(\log x) + (9\log 2)(\log 2)}{(\log 2)(\log x)} = \frac{6}{1}$$

$$(\log x)^2 + 9(\log 2)^2 = 6 \log 2 \log x$$

$$(\log x)^2 - 6\log 2 \log x + 9(\log 2)^2 = 0$$

$$(\log x)^2 - 1.81 \log x + 0.82 = 0$$

Substitute $y = \log x$

$$y^2 - 1.81y + 0.82 = 0$$

$a = 1$ $\quad \Delta = b^2 - 4ac$

$b = -1.81$ $\quad = (-1.81)^2 - 4(1)(0.82)$

$c = 0.82$ $\quad = 0$

$$y = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-1.81) \pm 0}{2(1)}$$

$$y = \frac{1.81}{2} = 0.905$$

$$y = \log x = 0.905 \xrightarrow[\text{Form}]{\text{Expo}} 10^{0.905} = x$$

$$x = 8$$

Question 13

Among the following statements, correct those that are false.

A) $\log_3 x = \dfrac{1}{\log_x 3}$ True! $\dfrac{1}{\frac{\log 3}{\log x}} = \dfrac{\log x}{\log 3} = \log_3 x$

B) $\log_{1/a} b = \log_a\left(\dfrac{1}{b}\right)$ True! $-\log_a b = \log_a b^{-1} = \log_a\left(\frac{1}{b}\right)$

C) $\ln x \bullet \ln x = \ln x^2$ False! — $\ln x + \ln x = \ln x^2$

OR

$\ln x \cdot \ln x = (\ln x)^2$

DE) log x – log y = log (x – y) False!

$\log x - \log y = \log\left(\frac{x}{y}\right)$

Question 14

In an adult education centre, they noticed that the number of registrations decreases by a factor of 1/20 as compared to the year before. If there were originally 300 people registered;

a) Determine the equation that describes the number of registrants as a function of n, the number of years since the opening of the centre.

$$A(n) = A_0\left(1+\frac{r}{k}\right)^{kn}$$

$$\frac{1}{20} = 0.05 \rightarrow r = -0.05$$

$$k = 1$$

$$A_0 = 300$$

$$\left.\right\} \quad A(n) = 300\left(1+\frac{-0.05}{1}\right)^{1n}$$

$$A(n) = 300(0.95)^n$$

b) How many students will the centre lose between the 5th and 10th years it is open if this trend continues? Round to the nearest whole number.

$$A(5) = 300(0.95)^5 = 232$$

$$A(10) = 300(0.95)^{10} = 180$$

They lost 232 - 180

= 52 students

Question 15

Two brothers, Chris and Tony, each start a home-based business. Their profits approximate a logarithmic function:

	$f(x) = 2 \log x + 3$	for Chris
and	$g(x) = 5 \log x$	for Tony

Where ~~x represents the number of years since they started~~ and f(x) and g(x) represent the profit after x years.

a) After how many years will Tony start making a profit?

$$g(x) = \frac{0}{5} = \frac{\cancel{5}\log x}{\cancel{5}}$$

$$\log x = 0 \xrightarrow{\text{Expo Form}} 10^0 = x$$

$$x = 1 \text{ year}$$

b) After how many years will their profits be equal? How much will they be making at that point?

$$f(x) = g(x)$$

$$2\log x + 3 = 5\log x$$

$$3 = 5\log x - 2\log x$$

$$\frac{3}{3} = \frac{\cancel{3}\log x}{\cancel{3}}$$

$$\log x = 1 \xrightarrow{\text{Expo Form}} 10^1 = x = 10 \text{ years}$$

$$f(10) = 2\log 10 + 3$$

$$= 2(1) + 3$$

$$= 5$$