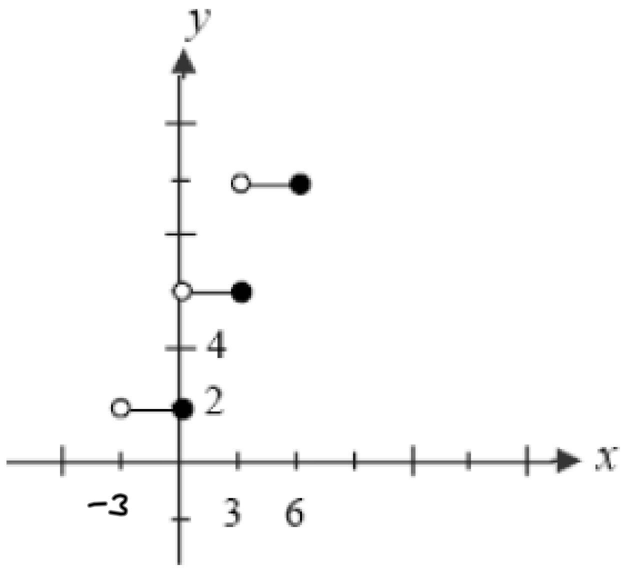
## Question 1

Given the graph of the following function:

$$f : [-3,6] \to [0,10]$$

$$x \mapsto -4\left[-\frac{1}{3}(x-3)\right]+6$$




Determine:

a) The type of function.

Greatest Integer

b) The value of parameter $a$. $a=-4$

c) The domain of the function.

$]-3,6]$

d) The range of the function.

$\{2,6,10\}$

e) The maximum of the function.

$y=10$

**Question 2**

a) Given the function $f(x) = 3|2(x-5)| + 7$, find $f(6.25)$.

$$f(6.25) = 3|2(6.25-5)| + 7$$
$$= 3|2.5| + 7$$
$$= 7.5 + 7 = 14.5$$

b) Given the function $h(x) = 6[2(x-3)] + 7$, find $h(5.25)$

$$h(5.25) = 6[2(5.25-3)] + 7$$
$$= 6[4.5] + 7$$
$$= 6(4) + 7 = 31$$

**Question 3**

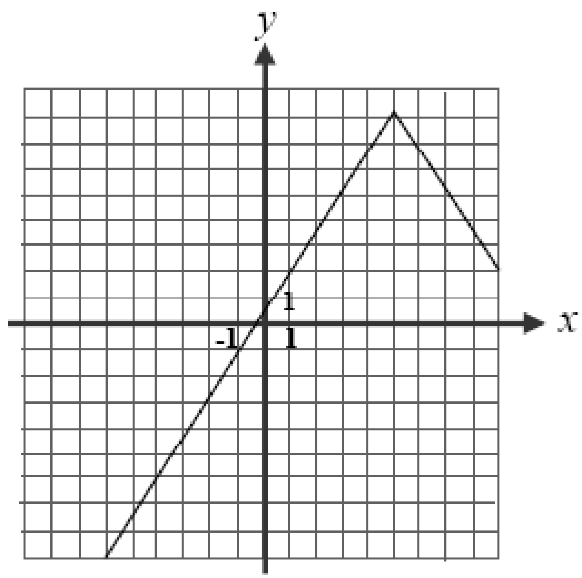
The following graph represents an absolute value function of the form:

$$f(x) = a|x - h| + k$$

Its vertex is the point (5,8) and it passes through point (3,5). → (x,y)

$\overbrace{(h,k)}$

What is the equation of this function?




$$5 = a|3-5| + 8$$

$$-3 = a|-2|$$

$$\frac{-3}{2} = \frac{2a}{2}$$

$$a = -\frac{3}{2}$$

$$f(x) = -\frac{3}{2}|x-5| + 8$$

## Question 4

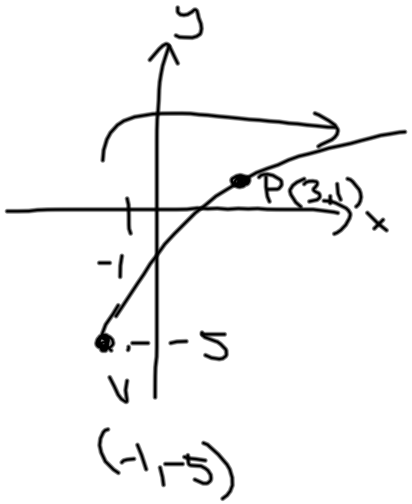
Find the equation of the square root function of the form:

$$f(x) = a\sqrt{(x-h)} + k$$

where the minimum is -5, its domain is the interval [-1, +∞ and passing through the point (3,1).

$V(-1,-5) \rightarrow (h,k)$

$P(3,1) \rightarrow (x,y)$




$1 = a\sqrt{3-(-1)} + (-5)$

$6 = a\sqrt{4}$

$\frac{6}{2} = \frac{\cancel{2}a}{\cancel{2}}$

$a = 3$

$f(x) = 3\sqrt{x+1} - 5$

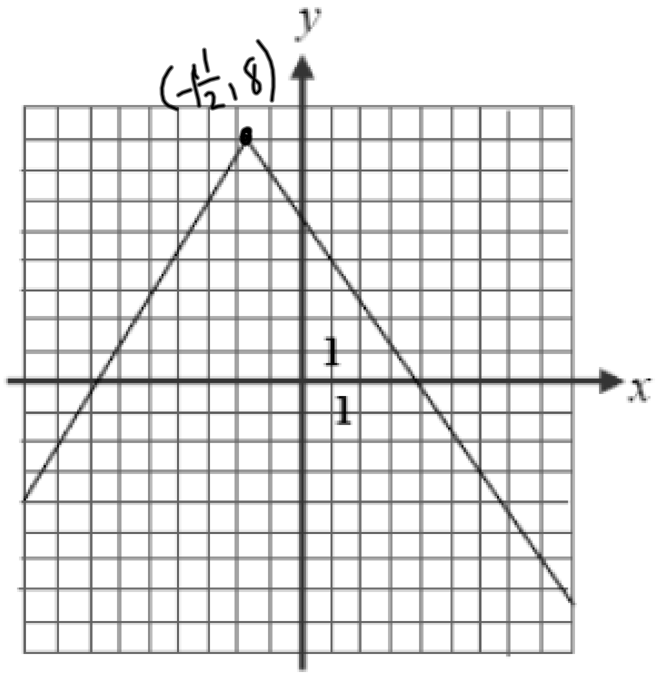
## Question 5

The following graph represents a function of the form:

$$g(x) = a|b(x-h)| + k$$

Which of the following points is true?

a) ~~$a < 0$ and $h > 0$~~
b) ~~$a > 0$ and $b > 0$~~
c) ~~$h > 0$ and $k > 0$~~
d) $a < 0$ and $h < 0$



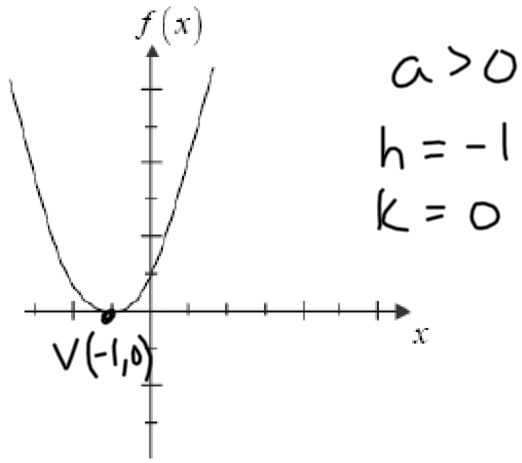

$a < 0$

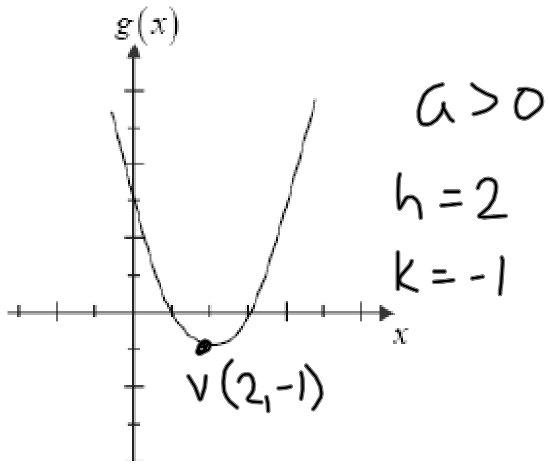
$h < 0$

$k > 0$

## Question 6

Real function $f$, of the form $f(x) = a(x-h)^2 + k$, is represented by the first graph below. The function $f$ undergoes a transformation, giving new function $g$, illustrated by the second graph.




Which of the following statements is true, concerning the changes to the parameters of $f$ in order to obtain $g$?

a) The parameter h increased and the parameter a decreased

~~b) The parameter h decreased and the parameter k increased~~

c) The parameter h increased and the parameter k decreased

~~d) The parameter a decreased and the parameter k increased~~

## Question 7

$h = -50 \quad k = 25$

Given the real function $f$ defined by $f(x) = -5\sqrt{2(x+50)} + 25$ :

$b > 0 \rightarrow R$

$a < 0 \rightarrow D$

- Determine if each of the following statements is true or false.
- Correct the false statements.


a) The function is decreasing since a < 0

True

b) The maximum of this function is -50

False, Max is 25

c) The function is negative over the interval [-50,+∞

False, negative ]-37.5,+∞

d) The zero of this function is (0, -25)

False, the zero is (-37.5, 0)

OR the y-intercept is (0, -25)

$f(x) = 0$

$0 = -5\sqrt{2(x+50)} + 25$

$\frac{-25}{-5} = \frac{-5\sqrt{2(x+50)}}{-5}$

$(5)^2 = \left(\sqrt{2(x+50)}\right)^2$

$\frac{25}{2} = \frac{2(x+50)}{2}$

$12.5 = x + 50$

$x = -37.5$

$(-37.5, 0)$

## Question 8

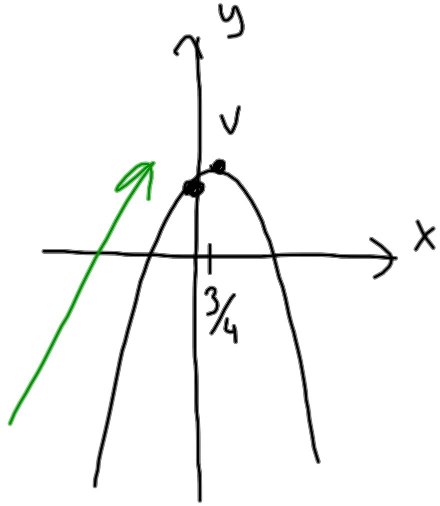
Given the function $f(x) = -2x^2 + 3x + 5$, defined over $\mathbb{R}\times\mathbb{R}$. Determine the values for which the function is ONLY increasing. Justify your answer by sketching two points.

$P_1(0,5)$

$a = -2 \quad \Delta = b^2 - 4ac$

$b = 3 \quad = 3^2 - 4(-2)(5)$

$c = 5 \quad = 9 + 40 = 49$




$V\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right)$

$V\left(\frac{-3}{2(-2)}, \frac{-49}{4(-2)}\right)$

$V\left(\frac{3}{4}, \frac{49}{8}\right)$

Increasing : $-\infty, \frac{3}{4}[$

**Question 9**

$f(x) = \frac{a}{x-h} + k$

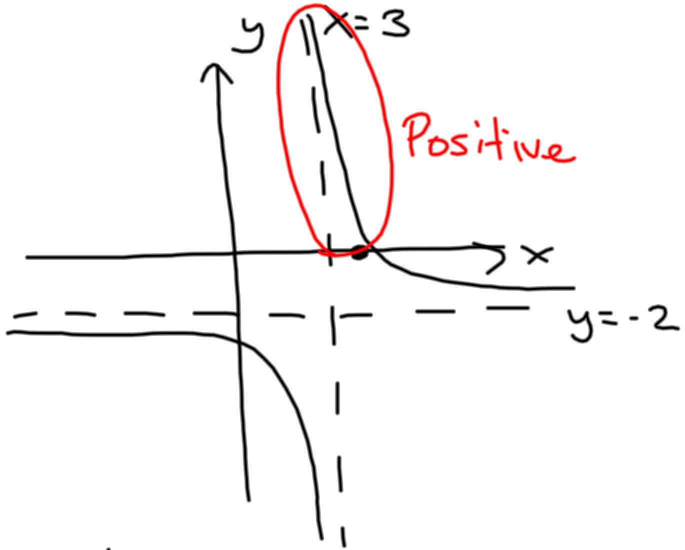
Over which interval is the following real function positive?

X-intercept ?

$$f(x) = \frac{3}{2x-6} - 2$$

$2(x-3)$

$= \frac{3/2}{x-3} - 2$




$f(x) = 0$

$0 = \frac{3}{2x-6} - 2$

$\frac{2}{1} = \frac{3}{2x-6}$

$2(2x-6) = 1 \times 3$

$4x - 12 = 3$

$\frac{4x}{4} = \frac{15}{4}$

$x = 3.75$

$(3.75, 0)$

Asymptotes: $x = h$ $\quad y = k$

$x = 3 \quad y = -2$

Positive: $]3, 3.75[$

**Question 10**

Gertrude would like to better control her weight. For several weeks she kept a log of her weight, then decided to consult a nutritionist to design for her a program to help her achieve her desired weight of 45 Kg. Her nutritionist calculated that her weight has varied according to the following function:

$$M(x) = -\frac{16}{3}\left|-\frac{1}{4}x+\frac{5}{2}\right|+\frac{127}{2} = -\frac{16}{3}\left|-\frac{1}{4}(x-10)\right|+\frac{127}{2}$$

Where $M(x)$ represents her weight in kilograms and $x$ represents the number of weeks she has been on her diet.

a) How much did Gertrude weigh when she started tracking her weight?

$x=0$ $\quad M(0)=50.2$ kg

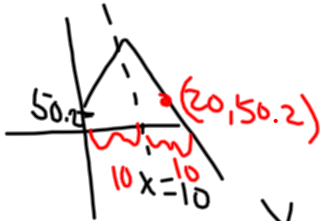
b) How many weeks will it take her to get back to her original weight?

$M(x)=50.2$ kg $=\frac{301}{6} \rightarrow 20$ weeks

c) What is her maximum weight?

Vertex! $\left(10, \frac{127}{2}\right) \rightarrow$ Max is 63.5 kg

d) The nutritionist has proposed her a 24 week program. Is that enough time to reach her desired weight?




$M(24) = -\frac{16}{3}\left|-\frac{1}{4}(24-10)\right|+\frac{127}{2}$

$= -\frac{16}{3}\left|-\frac{7}{2}\right|+\frac{127}{2}$

$= -\frac{16}{3}\times\frac{7}{2}+\frac{127}{2}$

$= -\frac{112}{6}+\frac{127}{2} = \frac{269}{6} = 44.8$ kg

Yes!

a) $M(0) = -\frac{16}{3}\left|-\frac{1}{4}(0)+\frac{5}{2}\right|+\frac{127}{2}$

$= -\frac{16}{3}\left|\frac{5}{2}\right|+\frac{127}{2} = -\frac{80}{6}+\frac{127}{2}$

$= \frac{301}{6} = 50.2$ kg

b) $\frac{301}{6} = -\frac{16}{3}\left|-\frac{1}{4}x+\frac{5}{2}\right|+\frac{127}{2}$

$\frac{\frac{-80}{6}}{\frac{-16}{3}} = \frac{\cancel{-\frac{16}{3}}\left|-\frac{1}{4}x+\frac{5}{2}\right|}{\cancel{-\frac{16}{3}}}$

$\frac{5}{2} = \left|-\frac{1}{4}x+\frac{5}{2}\right|$

$+$: $\quad -\frac{1}{4}x+\frac{5}{2}=\frac{5}{2}$

$-\frac{1}{4}x=0$

$x=0$

$-$: $\quad -\left(-\frac{1}{4}x+\frac{5}{2}\right)=\frac{5}{2}$

$\frac{1}{4}x-\frac{5}{2}=\frac{5}{2}$

$\frac{\cancel{\frac{1}{4}}x}{\cancel{\frac{1}{4}}} = \frac{\frac{10}{2}}{\frac{1}{4}}$

$x=20$

## Question 11

The Quebec Ramparts, a minor league hockey team, play 30 home games during the regular season. For the six first games of the year, the attendance increased from 1250 to 1625 fans. The attendance was represented by a functional relation of the following form:

$$f(x) = a\sqrt{(x-h)} + k$$

The vertex is the point (-3, 500). Assuming that the attendance will follow this same rule for the rest of the season, how many fans will attend the 22nd game?

$V(-3, 500) \rightarrow (h, k)$

$P_1(1, 1250) \rightarrow (x, y)$

$P_2(6, 1625)$

$1250 = a\sqrt{(1-(-3))} + 500$

$750 = a\sqrt{4}$

$\frac{750}{2} = \frac{\cancel{2}a}{\cancel{2}}$

$a = 375$

$f(x) = 375\sqrt{(x+3)} + 500$

$f(22) = 375\sqrt{22+3} + 500$

$= 375(5) + 500 = 2375$ fans

## Question 12

Find the inverse of the following function:

$$f : [-2,3] \to R$$
$$x \mapsto \frac{6x-8}{5}$$

Present your answer in functional notation.

$$y = \frac{6x-8}{5}$$

$\downarrow$ Inverse

$$x = \frac{6y-8}{5}$$

$$\times 5$$

$$5x = 6y - 8$$

$$\frac{5x+8}{6} = \frac{\cancel{6}y}{\cancel{6}}$$

$$y = \frac{5x+8}{6}$$

$$f^{-1} : \mathbb{R} \to [-2,3]$$
$$x \mapsto \frac{5x+8}{6}$$

## Question 13

Find the inverse of the following function:

$$h : R \to R^-$$
$$x \mapsto \frac{1}{4}(x-5)^2 - 3$$

Present your answer in functional notation.

$$y = \frac{1}{4}(x-5)^2 - 3$$

$\downarrow$ Inverse

$$x = \frac{1}{4}(y-5)^2 - 3$$

$$\frac{x+3}{\frac{1}{4}} = \frac{\cancel{\frac{1}{4}}(y-5)^2}{\cancel{\frac{1}{4}}}$$

$$\pm\sqrt{4(x+3)} = \sqrt{(y-5)^2}$$

$$\pm\sqrt{4(x+3)} = y-5$$

$$\pm\sqrt{4(x+3)} + 5 = y = h^{-1}(x)$$

$$h^{-1} : \mathbb{R}^- \to \mathbb{R}$$
$$x \mapsto \pm\sqrt{4(x+3)} + 5$$

**Question 14**

Solve the following equations algebraically. Clearly show all of your work.

a) $-2|x-4|+6=2$

$$\frac{\cancel{-2}|x-4|}{\cancel{-2}}=\frac{-4}{-2}$$

$$|x-4|=2$$

$+$

$$x-4=2$$
$$x=6$$

$-$

$$-(x-4)=2$$
$$-x+4=2$$
$$\frac{-x}{-1}=\frac{-2}{-1}$$
$$x=2$$

$$x\in\{2,6\}$$

b) $3+2\left|\frac{x}{3}-\frac{1}{2}\right|=5$

$$\frac{\cancel{2}\left|\frac{x}{3}-\frac{1}{2}\right|}{\cancel{2}}=\frac{2}{2}$$

$$\left|\frac{x}{3}-\frac{1}{2}\right|=1$$

$+$

$$\frac{x}{3}-\frac{1}{2}=1$$
$$\cancel{3}\left(\frac{x}{\cancel{3}}\right)=\left(\frac{3}{2}\right)3$$
$$x=\frac{9}{2}$$

$-$

$$-\left(\frac{x}{3}-\frac{1}{2}\right)=1$$
$$\frac{-x}{3}+\frac{1}{2}=1$$
$$\cancel{3}\left(\frac{-x}{\cancel{3}}\right)=\left(-\frac{1}{2}\right)(-3)$$
$$x=\frac{3}{2}$$

$$x\in\left\{\frac{3}{2},\frac{9}{2}\right\}$$

**Question 15**

Solve the following equations algebraically. Clearly show all of your work.

a) $2\sqrt{(3-2x)}+3=7$

$$\frac{\cancel{2}\sqrt{3-2x}}{\cancel{2}} = \frac{4}{2}$$

$$\left(\sqrt{3-2x}\right)^2 = 2^2$$

$$3-2x = 4$$

$$\frac{\cancel{-2x}}{\cancel{-2}} = \frac{1}{-2}$$

$$x = -\frac{1}{2}$$

Domain:

$$3-2x \geq 0$$

$$\frac{\cancel{-2x}}{\cancel{-2}} \geq \frac{-3}{-2}$$

$$x \leq 1.5$$

b) $-\frac{3}{2}\sqrt{(2x-1)}+2=-4$

$$\frac{\cancel{-\frac{3}{2}}\sqrt{2x-1}}{\cancel{-\frac{3}{2}}} = \frac{-6}{-\frac{3}{2}}$$

$$\left(\sqrt{2x-1}\right)^2 = 4^2$$

$$2x-1 = 16$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{17}{2}$$

$$x = \frac{17}{2} = 8.5$$

Domain:

$$2x-1 \geq 0$$

$$\frac{\cancel{2}x}{\cancel{2}} \geq \frac{1}{2}$$

$$x \geq \frac{1}{2}$$