

Question 1

Graph the following inequality:

$$x^2 - 6x + y^2 - 2y + 1 \leq 0$$

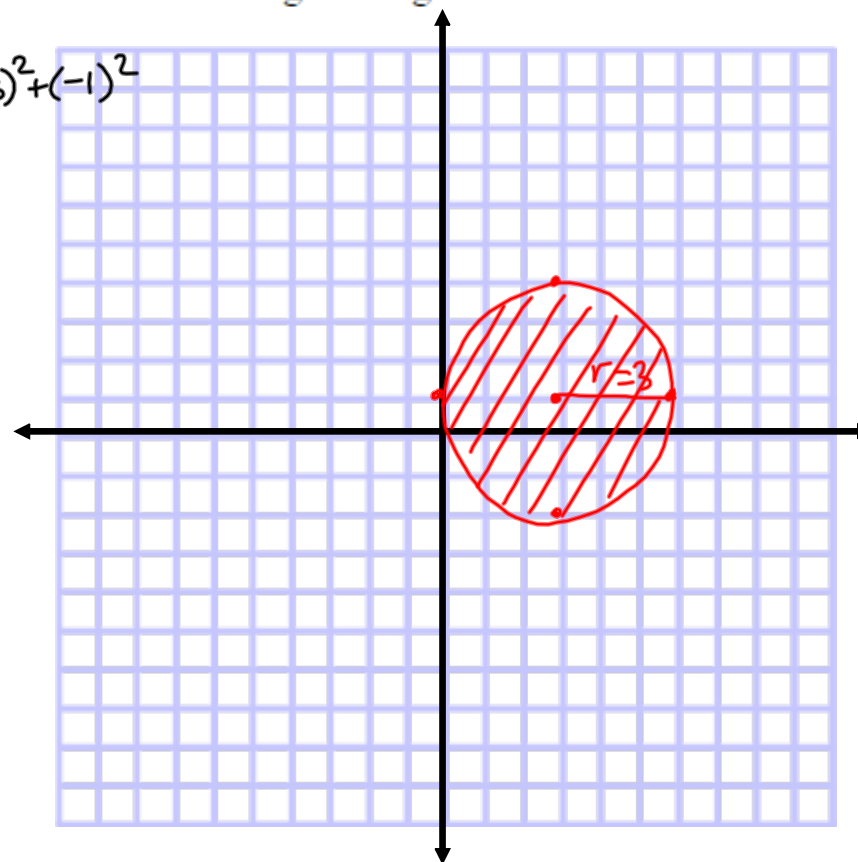
Clearly show the coordinates of the centre, and draw a radius indicating its length.

$$x^2 - \underbrace{6x + (-3)^2}_{\div 2} + y^2 - \underbrace{2y + (-1)^2}_{\div 2} \leq -1 + (-3)^2 + (-1)^2$$

$$(x-3)^2 + (y-1)^2 \leq 9$$

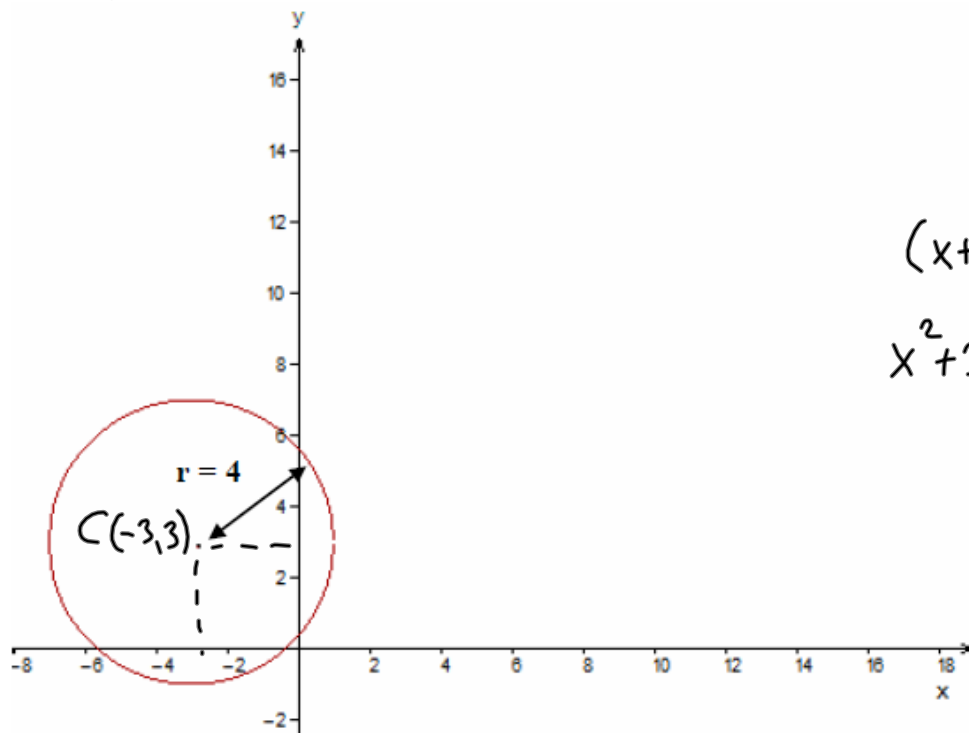
$$C(3,1)$$

$$r = 3$$



Question 2

Give the general form of the equation for the following circle.



$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x - (-3))^2 + (y - 3)^2 = 4^2$$

$$(x+3)^2 + (y-3)^2 = 16$$

Standard
Form

$$(x+3)(x+3) + (y-3)(y-3) - 16 = 0$$

$$x^2 + 3x + 3x + 9 + y^2 - 3y - 3y + 9 - 16 = 0$$

$$x^2 + y^2 + 6x - 6y + 2 = 0$$

Question 3

What is the equation of the line tangent to the circle $(x-1)^2 + (y-2)^2 = 5$ at point $(2,0)$?

Clearly show all your work.

1. Slope of radius: $C(1,2)$
 $P(2,0)$

$$m_R = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{1 - 2} = \frac{2}{-1} = -2$$

2. Slope of tangent:

$$m_T = \frac{-1}{m_R} = \frac{-1}{-2} = \frac{1}{2}$$

3. Equation of Tangent:

$$y = mx + b$$

$$P(2,0)$$

$$m_T = \frac{1}{2}$$

$$y = \frac{1}{2}x + b$$

$$0 = \frac{1}{2}(2) + b$$

$$b = -1$$

$$y = \frac{1}{2}x - 1$$

Question 4

Graph the following parabola:

$$\frac{(y-3)^2}{-8} \leq \frac{-8\left(x+\frac{1}{2}\right)}{-8}$$

$$\frac{(y-3)^2}{-8} = x + \frac{1}{2}$$

$$\frac{(y-3)^2}{-8} - \frac{1}{2} = x$$

x	y
-2.5	-1
-1	1
$-\frac{1}{2}$	3
-1	5
-2.5	7

Indicate the coordinates of the vertex, the focus as well as the equations of the axis of symmetry and the directrix.

$$V\left(-\frac{1}{2}, 3\right)$$

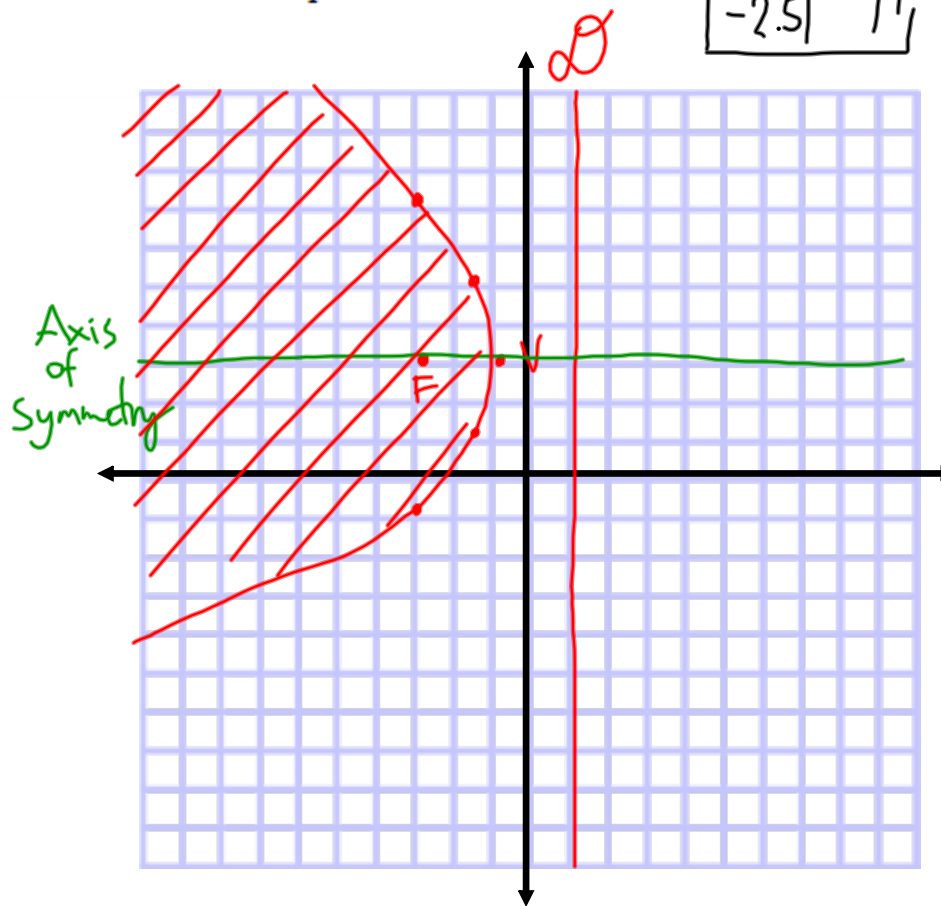
$$a = \frac{-8}{4} = -2$$

$$F(h+a, k) \rightarrow \left(-\frac{1}{2}-2, 3\right)$$

$$F(-2.5, 3)$$

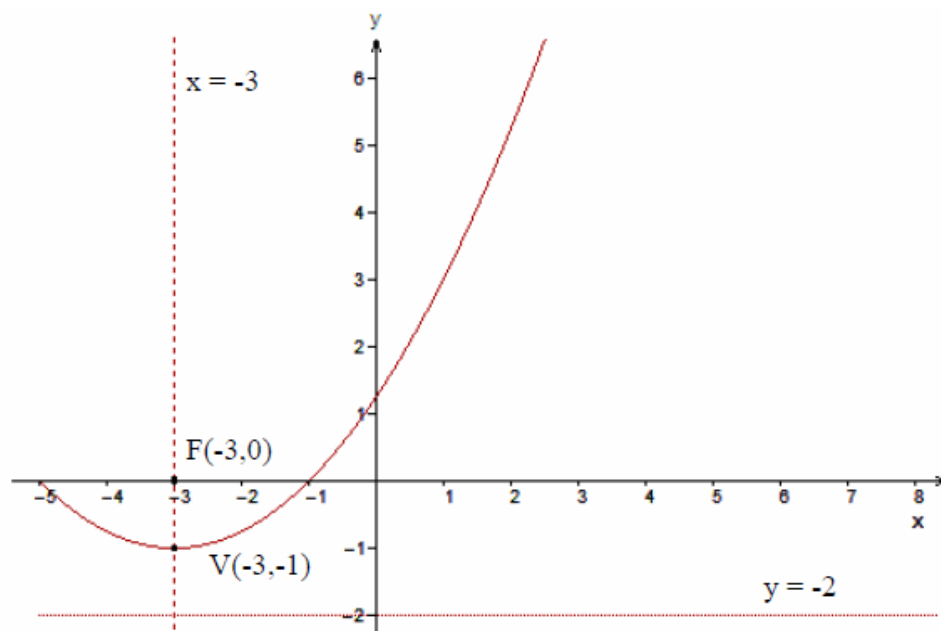
Directrix: $x = h - a$
 $x = -\frac{1}{2} - (-2)$
 $x = 1.5$

Axis of symmetry: $y = k$
 $y = 3$



Question 5

Find the equation in standard form of the following parabola:



$$(x-h)^2 = 4a(y-k)$$

$$V(-3, -1) \rightarrow (h, k)$$

$$\begin{array}{r} F(-3, 0) \\ - V(-3, -1) \\ \hline 0, 1 \leftarrow a = 1 \end{array}$$

$$(x - (-3))^2 = 4(1)(y - (-1))$$

$$(x+3)^2 = 4(y+1)$$

Question 6

Graph the following inequality:

$$\frac{y^2}{36} - \frac{x^2}{16} \leq 1$$

$$b=6 \quad a=4$$

test (0,0):

$$\frac{0^2}{36} - \frac{0^2}{16} \leq 1$$

$$0 \leq 1 \quad \text{TRUE!}$$

Give the coordinates of the vertices, the foci, and graph the asymptotes.

$$V(0, \pm b)$$

$$V(0, \pm 6)$$

$$\text{Focal length: } c^2 = a^2 + b^2$$

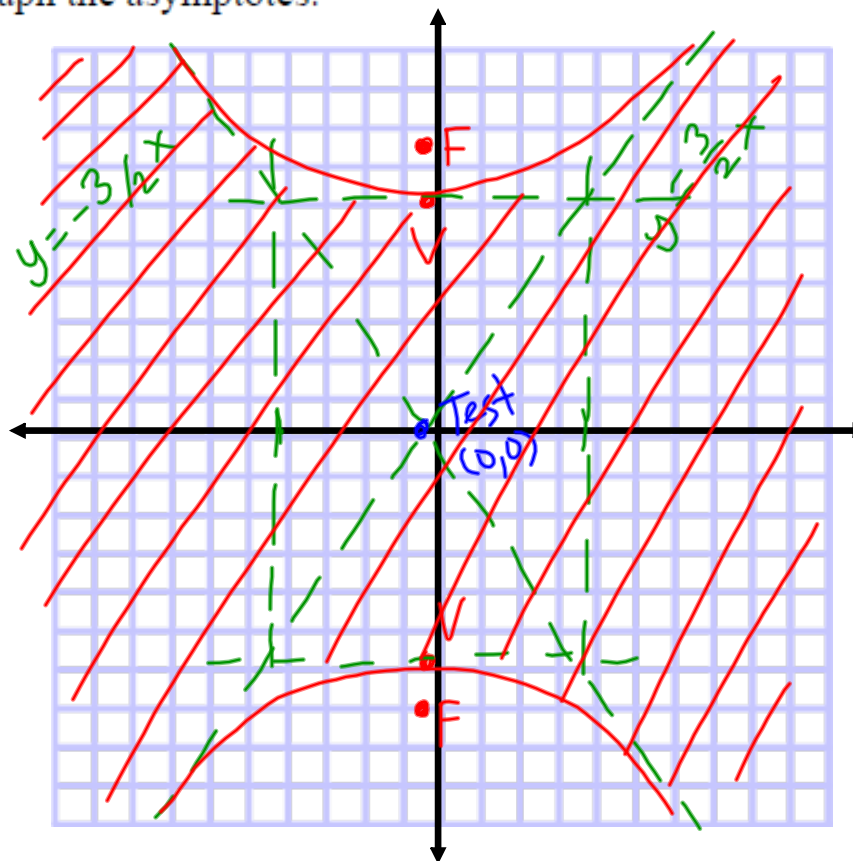
$$= 16 + 36 = 52$$

$$c = \sqrt{52} = 7.2$$

$$F(0, \pm 7.2)$$

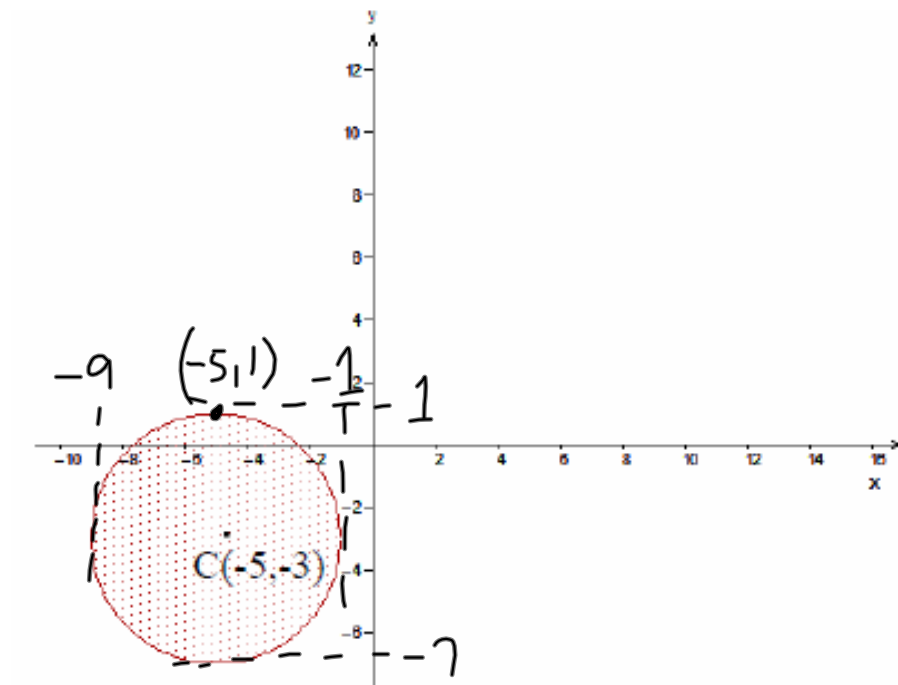
$$\text{Asymptotes: } y = \pm \frac{b}{a}x$$

$$y = \pm \frac{6}{4}x \rightarrow y = \pm \frac{3}{2}x$$



Question 7

Give the domain and range of the following relation in interval notation.

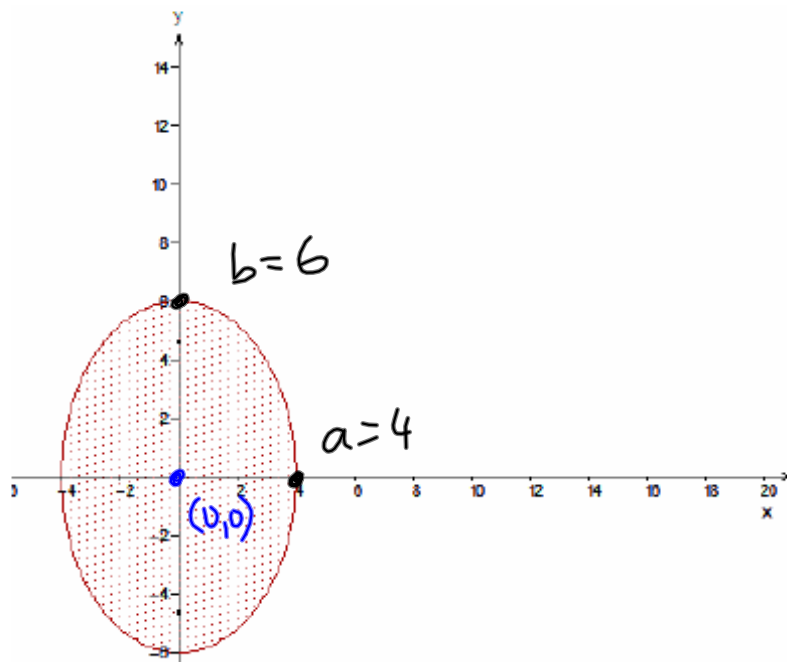


$$\text{Domain} = [-9, -1]$$
$$\text{Range} = [-7, 1]$$

Question 8

Determine the equation or inequality that represents each of the following relations. Give the answer in standard form.

a)



Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{36} = 1 \rightarrow \frac{x^2}{16} + \frac{y^2}{36} \leq 1$$

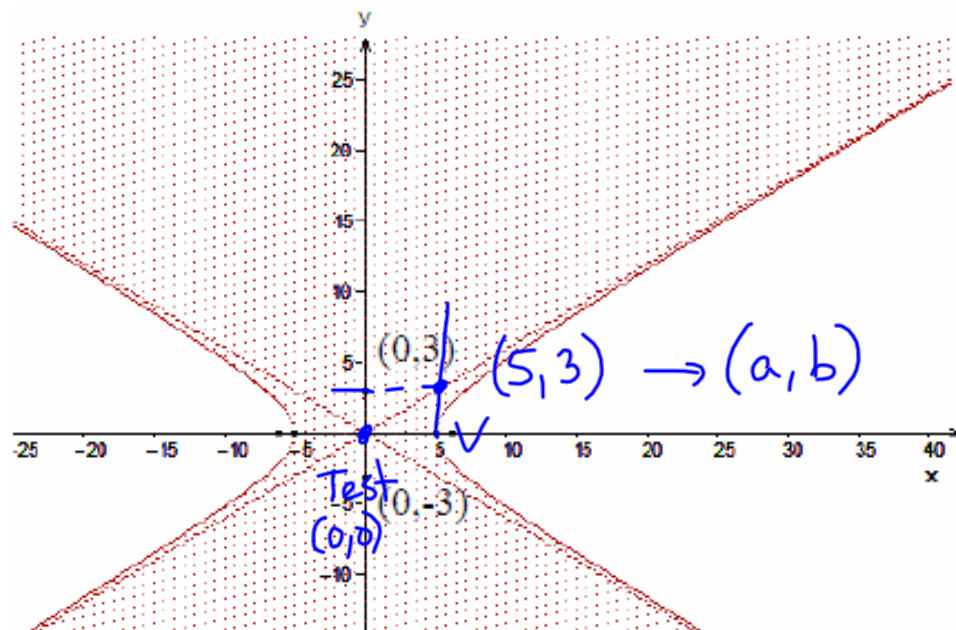
Test (0,0):

$$\frac{0^2}{16} + \frac{0^2}{36} ? 1$$

$$0 ? 1$$

$$0 \leq 1$$

b)



Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

$$\frac{x^2}{25} - \frac{y^2}{9} \leq 1$$

test (0,0)

$$\frac{0^2}{25} - \frac{0^2}{9} ? 1$$

$$0 ? 1$$

$$0 \leq 1$$

Question 9

Give the general form of the equation of a circle centred at $\left(-\frac{1}{2}, \frac{3}{4}\right)$ with a radius of 4 units.

$$C(h, k) \quad r = 4$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\left(x - \left(-\frac{1}{2}\right)\right)^2 + \left(y - \frac{3}{4}\right)^2 = 4^2$$

$$\left(x + \frac{1}{2}\right)\left(x + \frac{1}{2}\right) + \left(y - \frac{3}{4}\right)\left(y - \frac{3}{4}\right) - 16 = 0$$

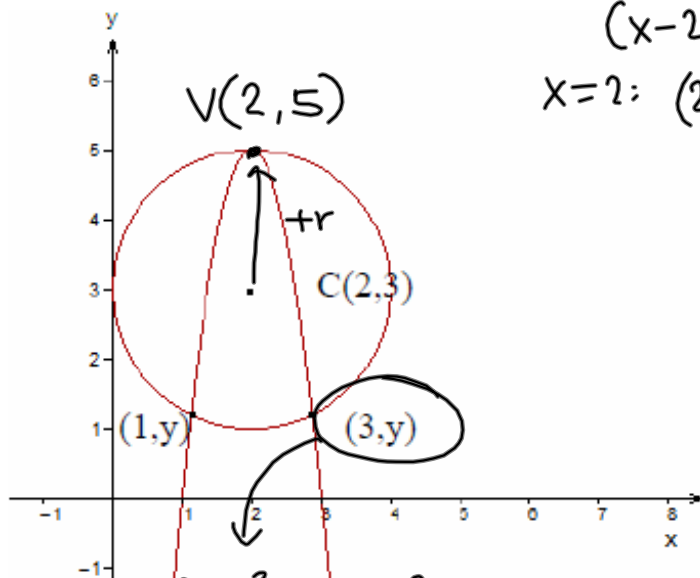
$$x^2 + \frac{1}{2}x + \frac{1}{2}x + \frac{1}{4} + y^2 - \frac{3}{4}y - \frac{3}{4}y + \frac{9}{16} - 16 = 0$$

$$x^2 + y^2 + x - \frac{3}{2}y - \frac{243}{16} = 0$$

Question 10

Abscissa = x-value
Ordinate = y-value

Determine the equation of the parabola in the graph below. The centre of the circle is (2,3) and its radius is 2 units. The x-value of the vertex of the parabola is 2, and the x-values of the points of intersection are 1 and 3.



$$(x-2)^2 + (y-3)^2 = 4$$

$$x=2: (2-2)^2 + (y-3)^2 = 4$$

$$\sqrt{(y-3)^2} = \pm\sqrt{4}$$

$$y-3 = \pm 2$$

$$y = 3 \pm 2$$

$$y_1 = 3+2 = 5$$

$$y_2 = 3-2 = 1$$

$$x=3: (3-2)^2 + (y-3)^2 = 4$$

$$1 + (y-3)^2 = 4$$

$$\sqrt{(y-3)^2} = 4-1 = \sqrt{3}$$

$$y-3 = \pm\sqrt{3}$$

$$y = 3 \pm \sqrt{3}$$

$$y_1 = 3 + \sqrt{3} = 4.73$$

$$y_2 = 3 - 1.73 = 1.27$$

$$(3, 1.27)$$

Parabola

$$(x-h)^2 = 4a(y-k)$$

$$V(2,5) \rightarrow (h,k)$$

$$P(3, 1.27) \rightarrow (x,y)$$

$$(3-2)^2 = 4a(1.27-5)$$

$$1 = 4a(-3.73)$$

$$1 = \frac{-14.92a}{-14.92}$$

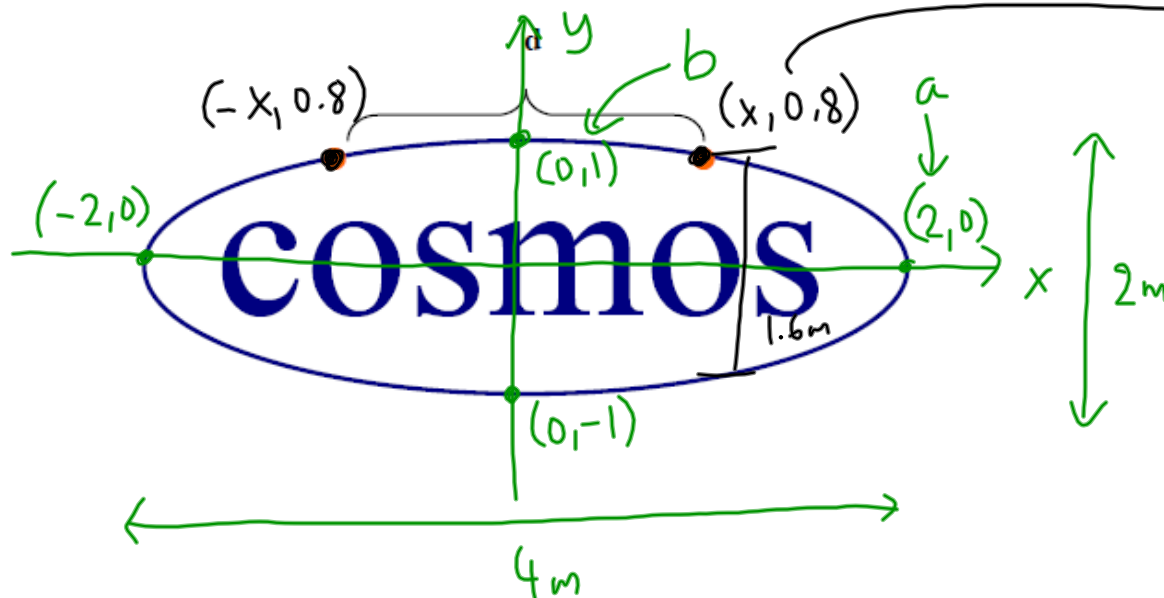
$$a = -0.067$$

$$(x-2)^2 = 4(-0.067)(y-5)$$

$$(x-2)^2 = -0.268(y-5)$$

Question 11

Cosmos' sign is in the shape of an ellipse. It measures 4m wide and 2m tall. The owner wants to place a mounting bracket above each O in the sign. Knowing that the sign is 1.6m high at that point, calculate the distance between the two brackets.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

$$\frac{x^2}{4} + (0.8)^2 = 1$$

$$\frac{x^2}{4} = \frac{1 - 0.64}{1}$$

$$x^2 = 4(0.36)$$

$$x^2 = 1.44$$

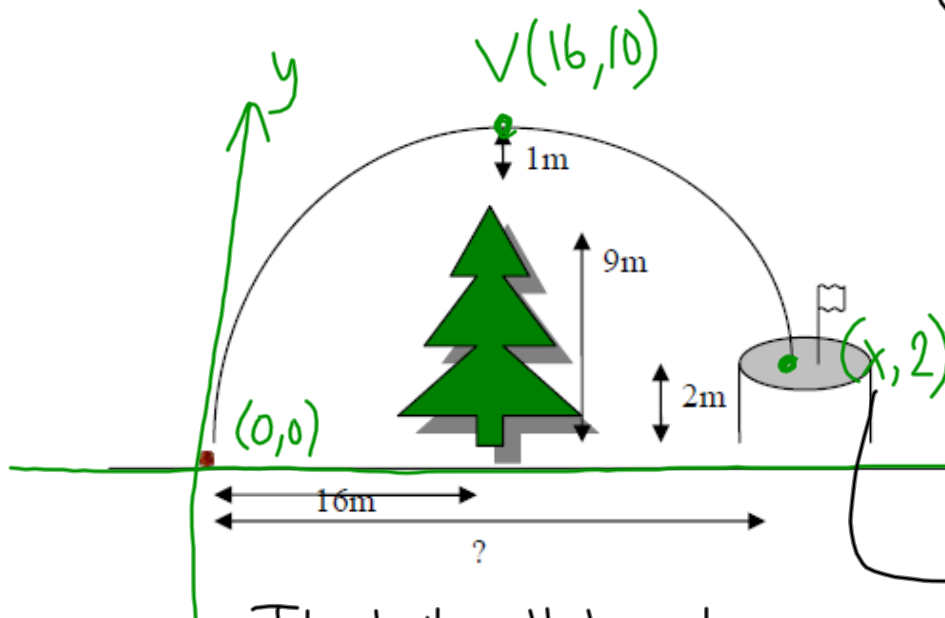
$$x = \pm\sqrt{1.44}$$

$$= \pm 1.2$$

The brackets are $2 \times 1.2 = 2.4$ m apart.

Question 12

Christian needs to hit his approach shot onto the green which is elevated 2m. To do this, he must hit his ball 1m over a tree 9m tall. How far will his ball travel, knowing that he is 16m from the tree and that his ball will travel in a parabolic trajectory?



Parabola

$$(x-h)^2 = 4a(y-k)$$

$$V(16,10) \rightarrow (h,k)$$

$$P(0,0) \rightarrow (x,y)$$

$$(0-16)^2 = 4a(0-10) \quad x_2 = 16 - 14.31$$

$$\frac{256}{-40} = \frac{-40a}{-40}$$

$$a = -6.4$$

$$(x-16)^2 = 4(-6.4)(y-10)$$

$$(x-16)^2 = -25.6(y-10)$$

$$(x-16)^2 = -25.6(2-10)$$

$$\sqrt{(x-16)^2} = \pm \sqrt{204.8}$$

$$x-16 = \pm 14.31 \rightarrow x = 16 \pm 14.31$$

$$x_1 = 16 + 14.31 = 30.31$$

$$x_2 = 16 - 14.31 = 1.69$$

The ball will travel 30.31m.