

1. The owner of an electronics store states that she cannot purchase more than 60 televisions of two different models for her store as she does not want to overstock items. At most, each 27" television occupies 0.7 m^3 and each 32" model uses 1.2 m^3 . She has a maximum storage space of 50 m^3 in the warehouse. Since the 32" model sells better than the 27" model, she keeps at least 30 in inventory. She makes a profit of 150\$ per 27" and 250\$ per 32".

Taking into account this situation, answer the following questions:

- a) Identify the constraints for this problem.
- At most 60 TVs in stock
 - At most 50 m^3 of storage
 - At least 30 32" TVs in inventory
- b) Identify the elements that would be used to optimize this function.
- Maximize Profit — 150\$ per 27" TV
— 250\$ per 32" TV
- c) Define the variables x and y

x : # of 27" TVs

y : # of 32" TVs

d) Supply the system of linear inequalities for the constraints

① $x \geq 0$

② $y \geq 0$

③ $x + y \leq 60$

④ $0.7x + 1.2y \leq 50$

⑤ $y \geq 30$

e) Supply the ~~objective~~ function to be optimized.

Profit: $P = 150x + 250y$

2. The owner of a sports store claims that he has at most 100 baseball mitts in stock, of which no more than 25 are catcher mitts. Moreover, he notices that the number of fielder mitts sold is at least as many as catcher mitts sold. For each catcher mitt he sells, he makes 30\$ and for each fielder mitt he sells, he makes 20\$. How many of each type of mitt must he sell to maximize his profit?

Let X : the number of catcher mitts
 Y : The number of fielder mitts

Given the system of inequalities:

- ① $x \geq 0$
- ② $y \geq 0$
- ③ $X + Y \leq 100$
- ④ $X \leq 25$
- ⑤ $Y \geq X$

a) Sketch the polygon of constraints indicated by this system of inequalities.

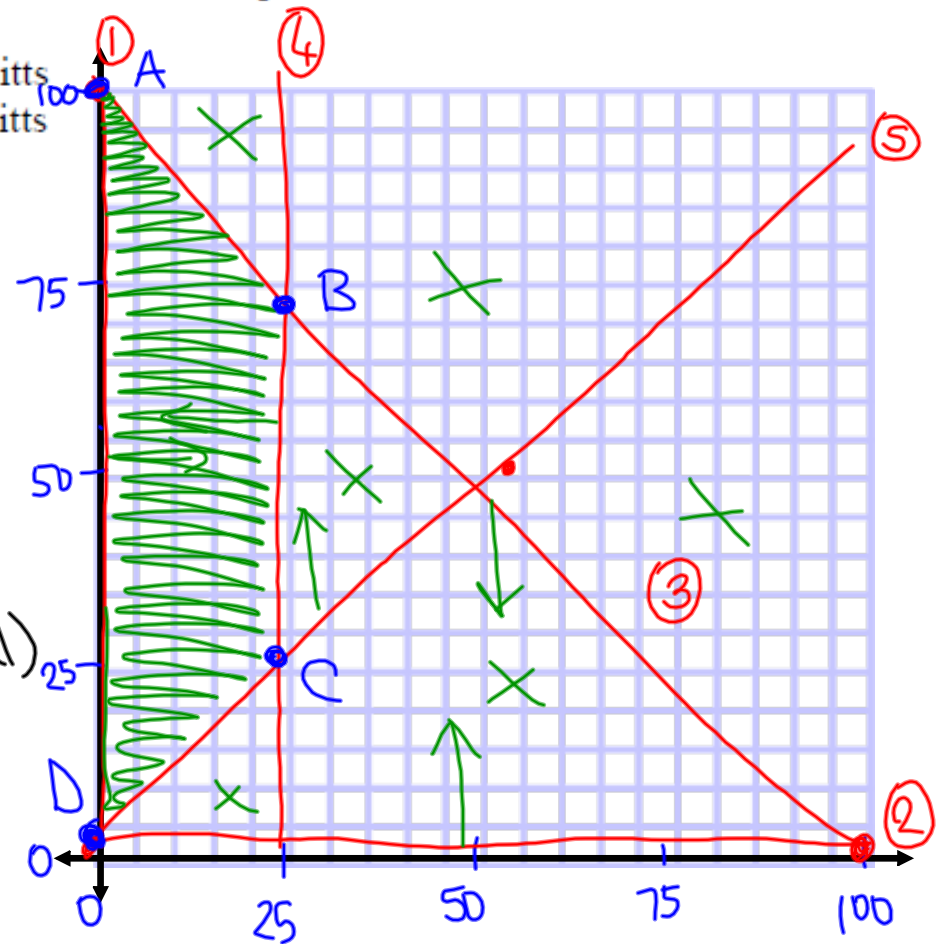
③ $x + y = 100$

x	y
0	100
100	0

④ $x = 25$ (vertical)

⑤ $y = x$

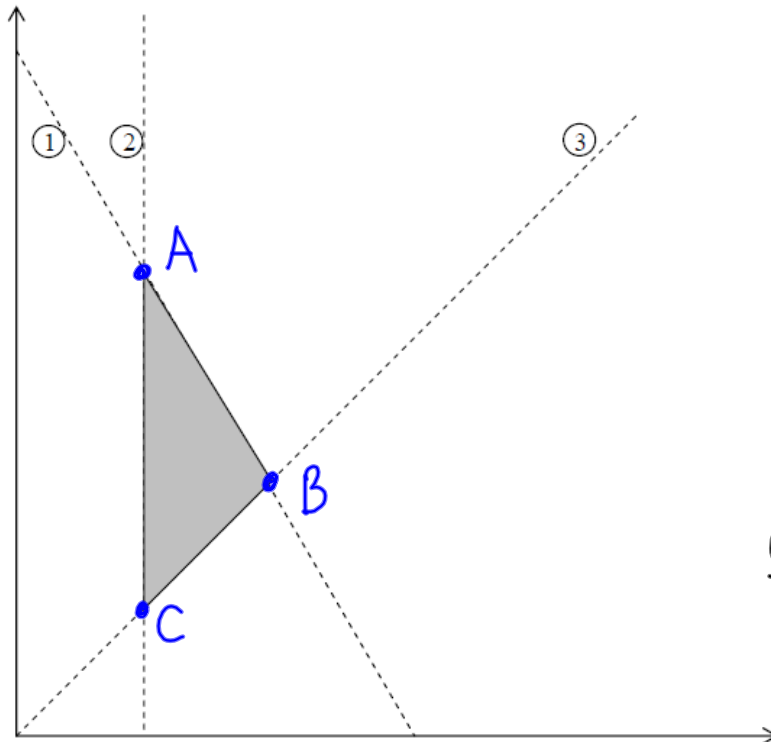
x	y
0	0
50	50



- b) Complete the following table and replace the value of X and Y in each inequality with the given values. Indicate in the final column whether the points belong to the polygon of constraints.

Point	$Y \geq X$	$X \leq 25$	$X + Y \leq 100$	Belongs (Y or N)
(50, 50)	T	F		N
(20, 60)	T	T	T	Y
(5, 99)	T	T	F	N

3. Algebraically determine the corner points of the polygon of constraints given below. The equations of the lines that determine the boundaries of the polygon are given below the figure.



- 1 $2x + y = 72$
 2 $x = 15$
 3 $y = x$

A: ① & ②
 ① $2x + y = 72$
 ② $x = 15$
 $2(15) + y = 72$
 $30 + y = 72$
 $y = 42$
 A (15, 42)

C: ② $x = 15$
 ③ $y = x$
 $y = 15$
 C (15, 15)

B: ① & ③
 ① $2x + y = 72$
 ③ $y = x$
 $2x + x = 72$
 $3x = 72$
 $x = 24$
 $y = 24$
 B (24, 24)

4. Solve the following problem and show all steps to your solution. Graph the results.

A bicycle design company produces at least 6 mountain bikes and at least 9 road bikes per week. The factory can produce up to a total of 35 bikes per week due to time constraints. Moreover, the factory produces at least 18 bikes per week due to demand. Each mountain bike costs 250\$ to produce, and each road bike costs 200\$ to produce. How many of each type of bikes must be produced to minimize production costs?

Variables

$x = \#$ of mountain bikes
 $y = \#$ of Road bikes

Function to be Optimized

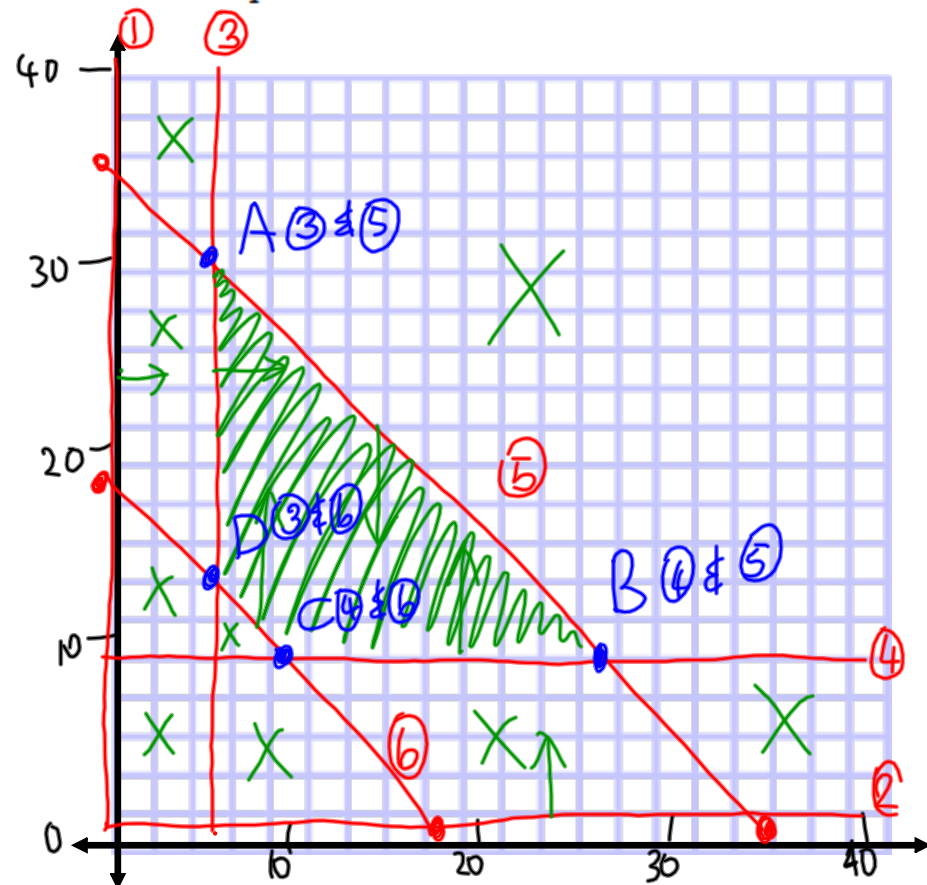
Cost: $C = 250x + 200y$

Constraints

- ① $x \geq 0$
- ② $y \geq 0$
- ③ $x \geq 6$
- ④ $y \geq 9$
- ⑤ $x + y \leq 35$
- ⑥ $x + y \geq 18$
- ③ $x = 6$ (vertical)
- ④ $y = 9$ (horizontal)
- ⑤ $x + y = 35$
- ⑥ $x + y = 18$

x	y
0	35
35	0

x	y
0	18
18	0



- ① $x \geq 0$
- ② $y \geq 0$
- ③ $x \geq 6$
- ④ $y \geq 9$
- ⑤ $x + y \leq 35$
- ⑥ $x + y \geq 18$

A: ③ & ⑤

$$\begin{aligned} \text{③ } x &= 6 \\ \text{⑤ } x + y &= 35 \\ \hline 6 + y &= 35 \\ y &= 29 \end{aligned}$$

A(6, 29)

B: ④ & ⑤

$$\begin{aligned} \text{④ } y &= 9 \\ \text{⑤ } x + y &= 35 \\ \hline x + 9 &= 35 \\ x &= 26 \end{aligned}$$

B(26, 9)

C: ④ & ⑥

$$\begin{aligned} \text{④ } y &= 9 \\ \text{⑥ } x + y &= 18 \\ \hline x + 9 &= 18 \\ x &= 9 \end{aligned}$$

C(9, 9)

D: ③ & ⑥

$$\begin{aligned} \text{③ } x &= 6 \\ \text{⑥ } x + y &= 18 \\ \hline 6 + y &= 18 \\ y &= 12 \end{aligned}$$

D(6, 12)

$$C = 250x + 200y$$

$$C_A = 250(6) + 200(29) = 7300 \$$$

$$C_B = 250(26) + 200(9) = 8300 \$$$

$$C_C = 250(9) + 200(9) = 4050 \$$$

$$C_D = 250(6) + 200(12) = 3900 \$$$

They should manufacture
6 mountain bikes and 12 road
bikes per week.

5. Solve the following problem and show all steps to your solution. Graph the results.

The organizers of a concert tour wish to manufacture at least 1000 concert tee-shirts in French for their tour stop in Montreal. There will be both long sleeve and short sleeve tee-shirts on sale. It is expected that they will sell at least twice as many short sleeve shirts as long sleeve shirts. The organizing committee has a maximum budget of 5000\$ to manufacture the shirts. A long sleeve shirt costs 7\$ to make and sells for 20\$, and a short sleeve shirt costs 3 dollars to make and sells for 10\$.

$$P_S = 10 - 3 = 7\$$$

$$P_L = 20 - 7 = 13\$$$

What is the maximum PROFIT that the organizers can expect to make if they sell all of their shirts?

Variables

$x = \#$ of short sleeve t-shirts

$y = \#$ of long sleeve t-shirts

Function to be optimized

↑ Profit: $P = 7x + 13y$

Constraints

- ① $x \geq 0$
- ② $y \geq 0$
- ③ $x + y \geq 1000$
- ④ $x \geq 2y$
- ⑤ $3x + 7y \leq 5000$

③ $x + y = 1000$

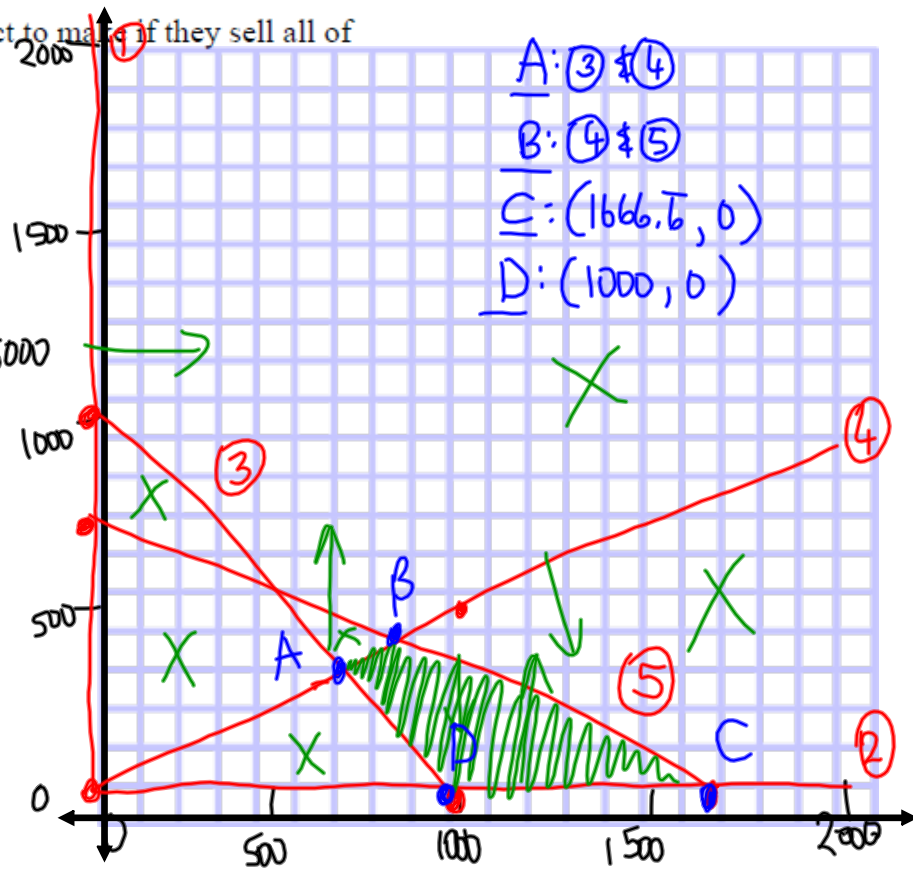
x	y
0	1000
1000	0

④ $x = 2y$

x	y
0	0
1000	500

⑤ $3x + 7y = 5000$

x	y
0	714
1667	0



- ① $x \geq 0$
- ② $y \geq 0$
- ③ $x + y \geq 1000$
- ④ $x \geq 2y$
- ⑤ $3x + 7y \leq 5000$

C: (1666, 0)
 D: (1000, 0)

A: ③ & ④
 ③ $x + y = 1000$
 ④ $x = 2y$

$$2y + y = 1000$$

$$\frac{3y}{3} = \frac{1000}{3}$$

$$y = 333.\bar{3}$$

↓ ④
 $x = 2(333.\bar{3})$
 $= 666.\bar{6}$

A(667, 333)

$$P = 7x + 13y$$

- $P_A = 7(667) + 13(333) = 8998 \$$
- $P_B = 7(769) + 13(385) = 10388 \$$
- $P_C = 7(1666) + 13(0) = 11662 \$ \leftarrow$
- $P_D = 7(1000) + 13(0) = 7000 \$$

B: ④ & ⑤

④ $x = 2y$
 ⑤ $3x + 7y = 5000$

$$3(2y) + 7y = 5000$$

$$6y + 7y = 5000$$

$$\frac{13y}{13} = \frac{5000}{13}$$

$$y = 384.6 \sim 385$$

↓ ④
 $x = 2(384.6) = 769.2$

B(769, 385) ~ 769

They should make 769 short sleeve and 385 long sleeve t-shirts.