1. Solve the following system of equations:

$$\begin{cases} y = \frac{1}{2}x^2 - x - \frac{3}{2} & ① \\ 5x - 4y - 10 = 0 & ② \end{cases}$$

substitute ① into ②

$5x - 4\left(\frac{1}{2}x^2 - x - \frac{3}{2}\right) - 10 = 0$

$5x - 2x^2 + 4x + 6 - 10 = 0$

$-2x^2 + 9x - 4 = 0$

$a = -2 \quad \Delta = b^2 - 4ac$

$b = 9 \quad = (9)^2 - 4(-2)(-4)$

$c = -4 \quad = 81 - 32 = 49$

2 solutions!

Q.F. $x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-9 \pm \sqrt{49}}{2(-2)}$

$= \frac{-9 \pm 7}{-4}$

$x' = \frac{-9+7}{-4} = \frac{-2}{-4} = \frac{1}{2} = 0.5$

$x'' = \frac{-9-7}{-4} = \frac{-16}{-4} = 4$

$\downarrow ①$

$x' = 0.5: \quad y' = \frac{1}{2}(0.5)^2 - 0.5 - 1.5$

$= 0.125 - 2$

$= -1.875$

$(0.5, -1.875)$

$x'' = 4: \quad y'' = \frac{1}{2}(4^2) - 4 - 1.5$

$= 8 - 5.5$

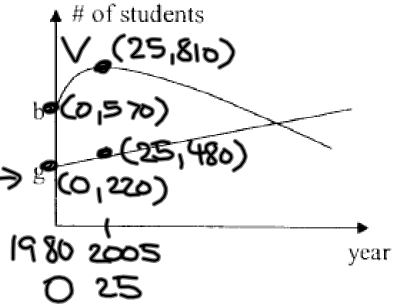
$= 2.5$

$(4, 2.5)$

2. In 1980, the student population at a CEGEP was 220 girls and 570 boys. Twenty-five years later, as the number of female students regularly increased, the total was 480 and the number of boys evolved quadratically and reached its maximum at 810.

If the tendency is maintained, what will the total number of students when the girls equal the boys.

In which year will this occur?



y-int → b=220

Girls

$y = mx + b$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{480-220}{25-0} = \dfrac{260}{25} = 10.4$

$\boxed{(g)\ y = 10.4x + 220}$

$y = a(x-h)^2 + k$

$V(25, 810) \rightarrow (h, k)$

$P(0, 570) \rightarrow (x, y)$

$570 = a(0-25)^2 + 810$

$-240 = a(-25)^2$

$\dfrac{-240}{625} = \dfrac{625a}{625} \rightarrow a = -0.384$

$\boxed{(b)\ y = -0.384(x-25)^2 + 810}$

(g) = (b)

$10.4x + 220 = 0.384(x-25)^2 + 810$

$10.4x + 220 = -0.384(x^2 - 25x - 25x + 625) + 810$

$10.4x - 590 = -0.384x^2 + 19.2x - 240$

$0.384x^2 - 8.8x - 350 = 0$

$a = 0.384$ $\quad \Delta = b^2 - 4ac$

$b = -8.8$ $\quad = (-8.8)^2 - 4(0.384)(-350)$

$c = -350$ $\quad = 615.04$

$\sqrt{\Delta} = 24.8$

Q.F. $x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-(-8.8) \pm 24.8}{2(0.384)}$

$x' = \dfrac{8.8 + 24.8}{0.768} = \dfrac{33.6}{0.768} = 43.75$

$x'' = \dfrac{8.8 - 24.8}{0.768} = \cancel{-20.8}$

(g)

$y = 10.4(43.75) + 220$

$= 675$ girls

& 675 boys

Total is 1350 students

in 2023.75

3. Two functions are described by:

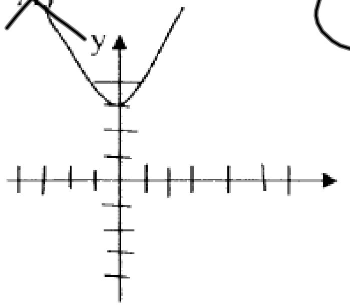
$f(x) = b_1$ where $b_1 < 0$ -4

$g(x) = ax^2 + b_2$ where $a > 0$ and $b_2 = -b_1$

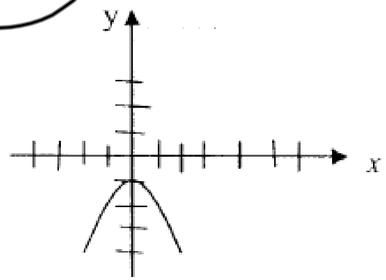
1 $b_2 = 4$

Which graph below could represent $g \cdot f$

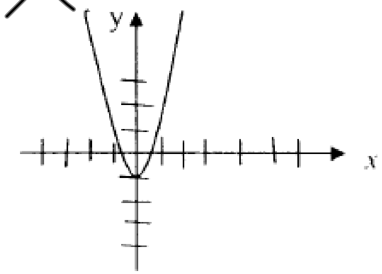
~~A)~~



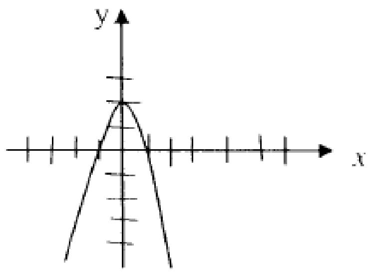
B)



~~C)~~



D)



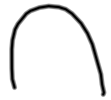
$f(x) = -4$

$g(x) = x^2 + 4$

$(g \cdot f)(x) = g(x) \cdot f(x)$

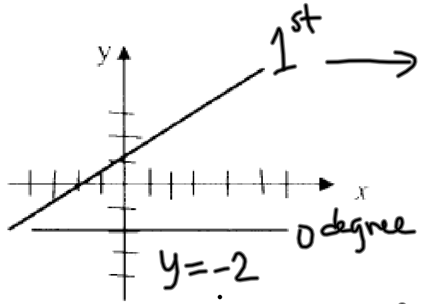
$= (x^2+4)(-4)$

$= -4x^2 - 16$

$a < 0 \rightarrow$ 

$C = -16 \rightarrow$ y-int $(0, -16)$

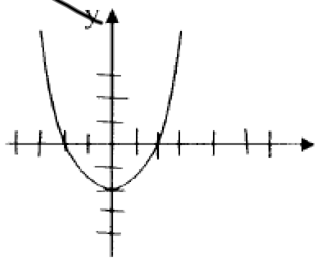
4. Given two functions, g and f, represented on the right



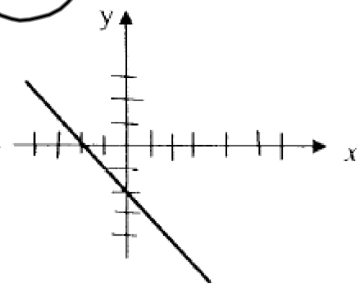
1st → +'ve slope
$y = 3x + 1$

Identify from the graphs below, the one that represents $f \cdot g$

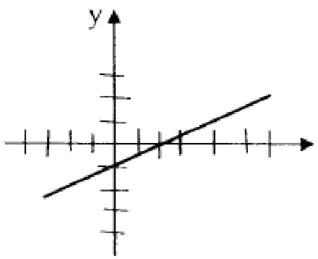
~~A)~~



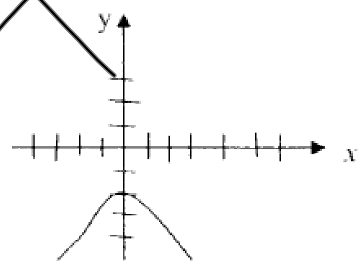
B)



C)



~~D)~~



$f \circ g = 1^{st} + 0^{deg} = 1^{st}$

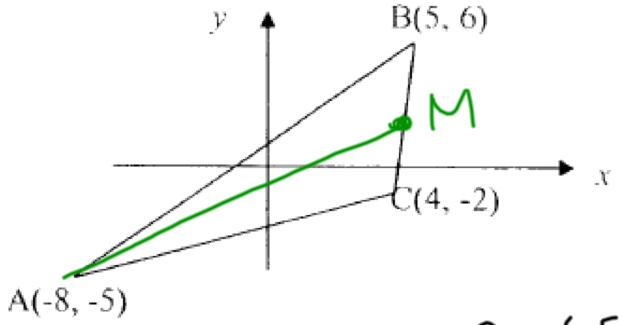
$f \circ g = -2(3x+1)$

$= -6x - 2$

-'ve slope -'ve y-int

5. Points A(-8, -5), B(5, 6) and C(4, -2) are the vertices of a triangle ABC

What is the equation of the median from A?



Mid-point

$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M\left(\frac{5+4}{2}, \frac{6+(-2)}{2}\right)$

$M(4.5, 2)$

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{2-(-5)}{4.5-(-8)} = \frac{7}{12.5}$

$= 0.56$

$y = mx + b$

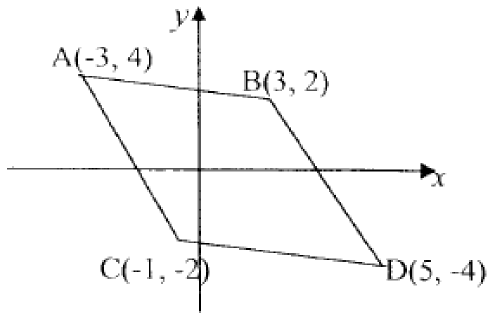
$-5 = 0.56(-8) + b$

$-5 = -4.48 + b$

$-0.52 = b$

$y = 0.56x - 0.52$

6. Find the area of the rhombus on the right. Round your answer to the nearest unit.



$$A = \frac{D \times d}{2}$$

$$D_{\overline{AD}} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5-(-3))^2 + (-4-4)^2}$$

$$= \sqrt{8^2 + (-8)^2} = \sqrt{64+64} = \sqrt{128} = 11.31$$

$$d_{\overline{BC}} = \sqrt{(3-(-1))^2 + (2-(-2))^2}$$

$$= \sqrt{4^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 5.66$$

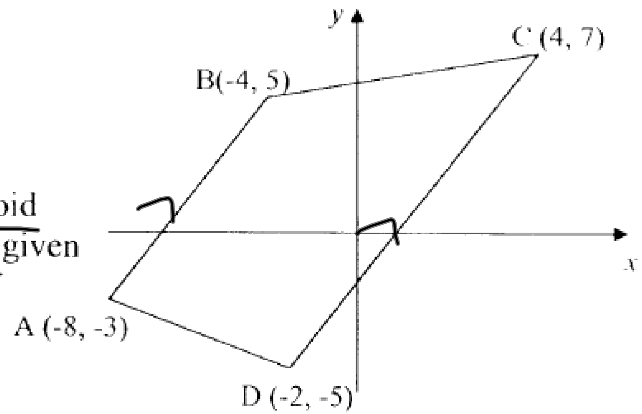
$$A = \frac{11.31 \times 5.66}{2}$$

$$= 32.01$$

$$= 32 \text{ units}^2$$

7. A quadrilateral has the following vertices: A(-8, -3), B (-4, 5), C (4, 7) and D (-2, -5)

Show that this quadrilateral is a trapezoid by using formulas and the information given relative to slopes and measurements of different segments.



Trapezoid = 2 parallel sides!

$$m_{\overline{AB}} = \frac{y_B - y_A}{x_B - x_A} = \frac{5-(-3)}{-4-(-8)} = \frac{8}{4} = 2$$

$$m_{\overline{CD}} = \frac{y_D - y_C}{x_D - x_C} = \frac{-5-7}{-2-4} = \frac{-12}{-6} = 2$$

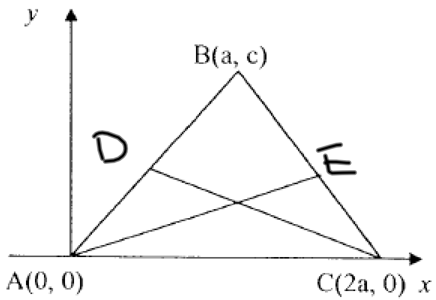
Since $m_{\overline{AB}} = m_{\overline{CD}}$, $\overline{AB} \parallel \overline{CD}$ so it is a trapezoid.

8. Complete the following table about the statement below using analytical geometry.

STATEMENT: The medians relative to the congruent sides of an isosceles triangle are congruent.

HYPOTHESIS
- the triangle ABC is isosceles
- $\overline{AB} \equiv \overline{BC}$
- $\overline{AE}$ and $\overline{CD}$ are medians

CONCLUSION
- $\overline{AE} \equiv \overline{CD}$



STATEMENTS	JUSTIFICATIONS
1. The coordinates of D and E are: $D\left(\frac{0+a}{2}, \frac{0+c}{2}\right) \Rightarrow D\left(\frac{a}{2}, \frac{c}{2}\right)$ $E\left(\frac{a+2a}{2}, \frac{c+0}{2}\right) = E\left(\frac{3a}{2}, \frac{c}{2}\right)$	1. Using the mid-point formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$d_{\overline{AE}} = \sqrt{(0-\frac{3a}{2})^2+(0-\frac{c}{2})^2}$ $= \sqrt{(\frac{-3a}{2})^2+(\frac{-c}{2})^2}$ $= \sqrt{\frac{9a^2}{4}+\frac{c^2}{4}}$	$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$d_{\overline{CD}} = \sqrt{(2a-\frac{a}{2})^2+(0-\frac{c}{2})^2}$$
$$= \sqrt{(\frac{3}{2}a)^2+(-\frac{c}{2})^2}$$
$$= \sqrt{\frac{9a^2}{4}+\frac{c^2}{4}}$$

$d_{\overline{AE}} = d_{\overline{CD}}$

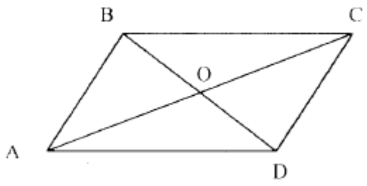
So $\overline{AE} \cong \overline{CD}$

9. Prove the following statement is correct.

The diagonals of a parallelogram intersect at their mid-points.

HYPOTHESIS
- ABCD is a parallelogram
- AC and BD are the diagonals

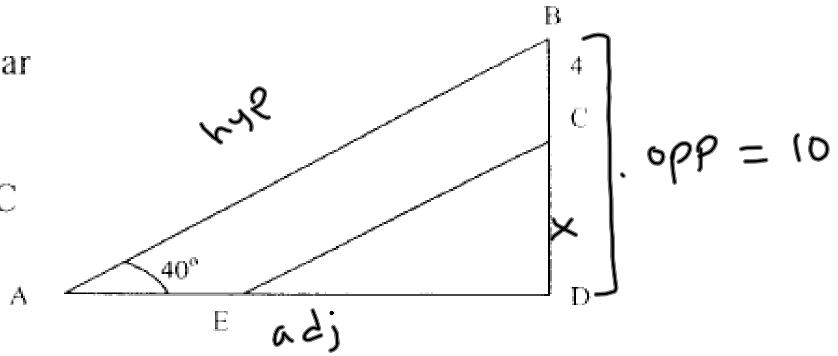
CONCLUSION
- AO ≡ OC and BO ≡ OD



STATEMENTS	JUSTIFICATIONS
$\overline{AB} \cong \overline{CD}$	opposite sides in Parallelogram
$\angle ABO \cong \angle CDO$	Alternate interior angles
$\angle BAO \cong \angle DCO$	Alternate interior angles
$\triangle ABO \cong \triangle CDO$	A-S-A
$\overline{OB} \cong \overline{OD}$	Corresponding sides of congruent triangles
$\overline{OA} \cong \overline{OC}$	

10. Triangles ABD and ECD are similar and the ratio of their areas is 0.36.

Angle A is 40° and the segment BC measures 4 cm. Determine the perimeter of triangle ABD.



$$\frac{A_1}{A_2} = 0.36 = k^2$$

$$k = \sqrt{0.36} = 0.6 \longrightarrow \frac{\text{Small}}{\text{Big}}$$

$$\frac{\overline{CD}}{\overline{BD}} = k$$

$$\frac{x}{x+4} = \frac{0.6}{1}$$

$$x = 0.6(x+4)$$

$$x = 0.6x + 2.4$$

$$\frac{0.4x}{\cancel{0.4}} = \frac{2.4}{0.4} \rightarrow x = 6$$

$$\sin 40^{\circ} = \frac{opp}{hyp}$$

$$\frac{\sin 40^{\circ}}{1} = \frac{10}{\overline{AB}}$$

$$\frac{10}{\sin 40^{\circ}} = \frac{\overline{AB} \times \cancel{\sin 40^{\circ}}}{\cancel{\sin 40^{\circ}}}$$

$$\overline{AB} = 15.5$$

$$\cos 40^{\circ} = \frac{adj}{hyp}$$

$$\frac{\cos 40^{\circ}}{1} = \frac{\overline{AD}}{15.5}$$

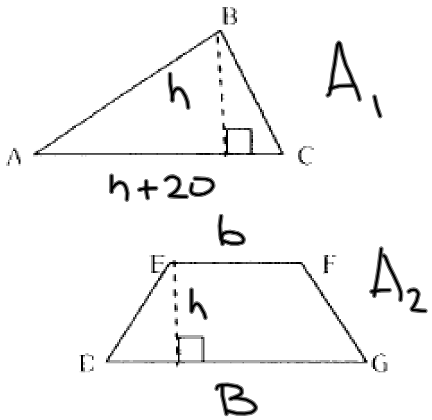
$$\overline{AD} = 15.5 \times \cos 40^{\circ}$$
$$= 11.87$$

$$\text{Perimeter} = 15.5 + 11.87 + 10$$
$$= 37.37 \text{ cm}$$

11. Triangle ABC is equivalent to the trapezoid DEFG

The base of the triangle is 20 cm longer than the height.

In the trapezoid, the long base is equal to the sum of the small base plus the height. The small base is two thirds of the height and the sum of the two bases plus the height is 160 cm.



$h = 64\text{ cm},\ b = 84\text{ cm}$

Find the measurements of the base and the height of the triangle.

$B+b+h=160$

$b=\frac{2}{3}h$

$B=b+h$

$=\frac{2}{3}h+h=\frac{5}{3}h$

$\frac{5}{3}h+\frac{2}{3}h+h=160$

$\frac{\frac{10}{3}h}{\frac{10}{3}}=\frac{160}{\frac{10}{3}} \Rightarrow h=48\text{ cm}$

$B=\frac{5}{3}h=\frac{5}{3}(48)=80$

$b=\frac{2}{3}h=\frac{2}{3}(48)=32$

Q.F. $x=\frac{-b\pm\sqrt{\Delta}}{2a}$

$=\frac{-20\pm 148}{2(1)}$

$x'=\frac{-20+148}{2}=64$

$x''=\frac{-20-148}{2}=\cancel{-84}$

$a=1$
$b=20$
$c=-5376$

$A_1=A_2$

$\frac{b\times h}{2}=\frac{(B+b)h}{2}$

$\frac{(h+20)h}{2}=\frac{(80+32)(48)}{2}$

$\frac{h^2+20h}{2} = \frac{2688}{1}$

$h^2+20h=2(2688)$

$h^2+20h-5376=0$

$\Delta=20^2-4(1)(-5376)$

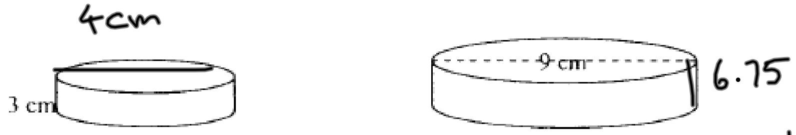
$=400+21504$

$=21904$

$\sqrt{\Delta}=148$

12. Two rounds of the same cheese are similar. The container of one has a diameter of 9 cm and this is 2.25 times bigger than the other container that has a height of 3 cm.

What is the difference in volume, to the nearest tenth, between these two rounds?



$\frac{d_1}{d_2} = 2.25 \quad \left(\frac{Big}{small}\right)$

$\frac{h_1}{h_2} = k = 2.25$

$\frac{9}{d_2} = \frac{2.25}{1}$

$\frac{h_1}{3} = \frac{2.25}{1}$

$h_1 = 6.75 \text{ cm}$

$\frac{9}{2.25} = \frac{\cancel{2.25} d_2}{\cancel{2.25}}$

$d_2 = 4 \text{ cm}$

$V_S = \pi r^2 h$
$= \pi (2)^2 (3)$
$= \pi (12) = 37.70 \text{ cm}^3$

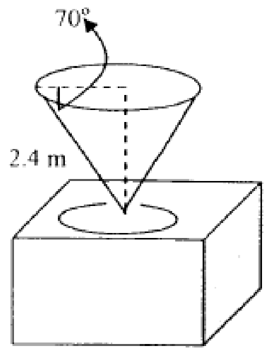
$V_B = \pi r^2 h$
$= \pi (4.5)^2 (6.75)$
$= 429.42 \text{ cm}^3$

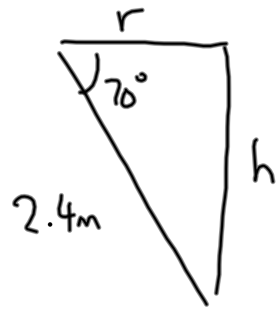
$V_B - V_S = 429.42 - 37.70$
$= 391.72 \text{ cm}^3$

13. A conic funnel used to fill cubic shipping crates has a capacity equivalent to each one of these:

The slant of the cone measures 2.4 m and forms an angle of 70° with the radius of the base.

What is the measurement, to the nearest tenth, of each side of the cube?





$$\frac{\cos 70^\circ}{1} = \frac{r}{2.4}$$

$$r = 2.4\cos 70^\circ$$
$$= 0.82\text{ m}$$

$$\frac{\sin 70^\circ}{1} = \frac{h}{2.4}$$

$$h = 2.4\sin 70^\circ$$
$$= 2.26$$

$$V_{cone} = V_{cube}$$

$$\frac{\pi r^2 h}{3} = s^3$$

$$\frac{\pi(0.82)^2(2.26)}{3} = s^3$$

$$\sqrt[3]{s^3} = \sqrt[3]{1.59}$$

$$s = 1.59^{\wedge}\frac{1}{3}$$

$$s = 1.17\text{ m} \Rightarrow 1.2\text{ m each side}$$