Question 1

Reduce the following algebraic fraction to lowest terms. Show all steps in the solution.

$$\frac{x^2+2x-15}{25-x^2} = \frac{(x+5)(x-3)}{(5+x)(5-x)}$$

$$= \frac{(x-3)}{(5-x)}$$

$x^2+2x-15$

$= (x+5)(x-3)$

$25-x^2$

$= (5+x)(5-x)$

OR

$$\frac{(x-3)}{-(x-5)}$$

Question 2

Divide the following algebraic fractions and reduce the result to lowest terms.
Show all the steps in the solution.

$$\frac{4x-2x^2}{x^2-5x+6} \div \frac{16x^3-4x^2}{2x^2-x+3}$$

$2x(2-x)$ $\quad 4x^2(4x-1)$

$(x-3)(x-2)$ $\quad 2\times 3=6$

$$= \frac{-1}{(x-3)(x-2)} \times \frac{(2x^2-x+3)}{2x} \quad \begin{matrix} 6\times 1 \\ 2\times 3 \end{matrix}$$

$$= \frac{-(2x^2-x+3)}{2x(x-3)(4x-1)}$$

Question 3

Perform the following operations and reduce the result to lowest terms. Show all the steps in the solution.

$$\frac{3}{4-4b+b^2}+\frac{4}{4-b^2}-\frac{2}{b-2}$$

$b^2-4b+4 \quad -b^2+4$

$=(b-2)(b-2) \quad =-(b^2-4)$

$=-(b+2)(b-2)$

$$=\frac{3}{(b-2)(b-2)}+\frac{4}{-(b+2)(b-2)}-\frac{2}{(b-2)}$$

$$=\frac{3}{(b-2)(b-2)}-\frac{4}{(b+2)(b-2)}-\frac{2}{(b-2)}$$

LCD $=(b-2)(b-2)(b+2)$

$$=\frac{3}{(b-2)(b-2)}\times\frac{(b+2)}{(b+2)}-\frac{4}{(b+2)(b-2)}\times\frac{(b-2)}{(b-2)}-\frac{2}{(b-2)}\times\frac{(b-2)(b+2)}{(b-2)(b+2)}$$

$$=\frac{3b+6}{(b-2)(b-2)(b+2)}-\frac{4b-8}{(b-2)(b-2)(b+2)}-\frac{2(b^2+2b-2b-4)}{(b-2)(b-2)(b+2)}$$

$$=\frac{3b+6-(4b-8)-(2b^2-8)}{(b-2)(b-2)(b+2)}$$

$$=\frac{3b+6-4b+8-2b^2+8}{(b-2)(b-2)(b+2)}$$

$$=\frac{(-2b^2-b+22)}{(b-2)(b-2)(b+2)}$$

Question 4

Determine the product of the following algebraic fractions and reduce the result to lowest terms. Show all steps in the solution.

$$\frac{36-x^4}{4x-24} \cdot \frac{3x^3-18x^2+2x-12}{x^4-4x^2-12} \cdot \frac{4x^2+8}{30+5x^2}$$

$$= \frac{(6+x^2)(6-x^2)}{4(x-6)} \cdot \frac{(x-6)(3x^2+2)}{(x^2-6)(x^2+2)} \cdot \frac{4(x^2+2)}{5(6+x^2)}$$

$$= \frac{-(3x^2+2)}{5}$$

Question 5

Perform the following operations and reduce the result to lowest terms. Show all the steps in the solution.

$$\frac{x^2-4}{x^2+xy} \bullet \frac{3x^3-3xy^2}{4x-8} \div \frac{x^2+2x-xy-2y}{16-4x}$$

$$= \frac{(x+2)(x-2)}{x(x+y)} \bullet \frac{3x(x-y)(x+y)}{4(x-2)} \bullet \frac{4(4-x)}{(x+2)(x-y)}$$

$$= \frac{3(4-x)}{1} = 3(4-x)$$

Question 6

Reduce the following algebraic expression to lowest terms, making sure to observe the order of operations. Show all steps in the solution.

$$\left(1+\frac{x+y}{x-y}\right)\div\frac{2x^2+6x}{x^2-2xy+y^2}$$

$2x(x+3)$

$(x-y)(x-y)$

$$\frac{1}{1}+\frac{(x+y)}{(x-y)}$$

LCD $= 1(x-y)$

$$=\frac{1}{1}\cdot\frac{(x-y)}{(x-y)}+\frac{(x+y)}{(x-y)}$$

$$=\frac{x-y}{x-y}+\frac{(x+y)}{(x-y)}$$

$$=\frac{x-y+x+y}{(x-y)}=\frac{2x}{(x-y)}$$

$$=\frac{2x}{(x-y)}\cdot\frac{(x-y)(x-y)}{2x(x+3)}$$

$$=\frac{(x-y)}{(x+3)}$$

Question 7

Reduce the following algebraic expressions to lowest terms, making sure to observe the order of operations. Show all steps in the solution.

$$\left(\frac{e^2-25}{(e-5)}+\frac{2}{1}-\frac{2e-4d}{e}\right)\div\frac{2e^2+8de}{e^2-3e-10}$$

$(e-5)(e+5)$ $2(e-2d)$ $2e(e+4d)$ $(e-5)(e+2)$

$$\frac{(e-5)(e+5)}{(e-5)}+\frac{2}{1}-\frac{2(e-2d)}{e}$$

$$=\frac{(e+5)}{1}+\frac{2}{1}-\frac{2(e-2d)}{e}$$

$LCD = e$

$$=\frac{(e+5)}{1}\times\frac{e}{e}+\frac{2}{1}\times\frac{e}{e}-\frac{2(e-2d)}{e}$$

$$=\frac{e^2+5e}{e}+\frac{2e}{e}-\frac{2e-4d}{e}$$

$$=\frac{e^2+5e+2e-(2e-4d)}{e}$$

$$=\frac{e^2+7e-2e+4d}{e}=\frac{e^2+5e+4d}{e}$$

$$=\frac{e^2+5e+4d}{e}\times\frac{(e-5)(e+2)}{2e(e+4d)}$$

$$=\frac{(e^2+5e+4d)(e-5)(e+2)}{2e^2(e+4d)}$$

Question 8

Reduce the following algebraic expressions to lowest terms, making sure to observe the order of operations. Show all steps in the solution.

$$\frac{r^2}{s^2-r^2}-\frac{s^2}{(r+s)}\bullet\frac{(s^2+r^2)}{2rs^2-2s^3}$$

$$\frac{s^2}{(r+s)}\cdot\frac{(s^2+r^2)}{2s^2(r-s)}$$

$$=\frac{(s^2+r^2)}{2(r+s)(r-s)}$$

$$=\frac{r^2}{(s-r)(s+r)}+\frac{(s^2+r^2)}{2(s+r)(s-r)}$$

$$=\frac{r^2}{(s-r)(s+r)}+\frac{(s^2+r^2)}{2(s+r)(s-r)}$$

LCD $=2(s-r)(s+r)$

$$=\frac{r^2}{(s-r)(s+r)}\times\frac{2}{2}+\frac{s^2+r^2}{2(s+r)(s-r)}$$

$$=\frac{2r^2}{2(s-r)(s+r)}+\frac{s^2+r^2}{2(s+r)(s-r)}$$

$$=\frac{2r^2+s^2+r^2}{2(s-r)(s+r)}$$

$$=\frac{3r^2+s^2}{2(s-r)(s+r)}$$