Question 1.

The product of two consecutive even integers is 528. Determine the second degree equation that represents this situation.

1^{st} number $= x$

2^{nd} number $= x+2$

$x(x+2) = 528$

$x^2 + 2x = 528$

$x^2 + 2x - 528 = 0$

Question 2.

A cable company serves 400 households and charges \$50 a month. Each \$1 drop in monthly charges leads to 10 new customers. Complete the table and determine the equation of the form $y = ax^2 + bx + c$ that represents the company's total monthly revenue.

Drop in price	Price per customer	Number of customers	Monthly revenue
0	50	400	$(50 \times 400) = 20000$
1	50-1 = 49	400+10(1) = 410	$(50-1)\times(400+10) =$ 49 × 410 = 20090
2	50-2=48	400+10(2) = 420	48×420 = 20160
3	50-3=47	400+10(3) = 430	47×430 = 20210
X	50 - x	400+10x	(50-x)(400+10x)=y

$y = (50-x)(400+10x)$

$y = 50(400) + 50(10x) + (-x)(400) + (-x)(10x)$

$= 20000 + 500x - 400x - 10x^2$

$y = -10x^2 + 100x + 20000$

Question 3

Solve using the quadratic formula.

$-3x^2 + 4x + 7 = 0$

$a = -3$
$b = 4$
$c = 7$

$\Delta = b^2 - 4ac$
$= (4)^2 - 4(-3)(7)$
$= 16 + 84$
$= 100 \rightarrow$ 2 solutions!

Q.F. $x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-4 \pm \sqrt{100}}{2(-3)} = \frac{-4 \pm 10}{-6}$

$x' = \frac{-4+10}{-6} = \frac{6}{-6} = -1$

$x'' = \frac{-4-10}{-6} = \frac{-14}{-6} = \frac{7}{3}$

Question 4

Solve using the quadratic formula.

$5x + 6x^2 - 20 = 0$

$6x^2 + 5x - 20 = 0$

$a = 6$ $\quad \Delta = b^2 - 4ac$

$b = 5$ $\quad = 5^2 - 4(6)(-20)$

$c = -20$ $\quad = 25 + 480 = 505 \rightarrow$ 2 solutions

Q.F. $x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-5 \pm \sqrt{505}}{2(6)} = \frac{-5 \pm 22.47}{12}$

$x' = \frac{-5 + 22.47}{12} = \frac{17.47}{12} = 1.45$

$x'' = \frac{-5 - 22.47}{12} = \frac{-27.47}{12} = -2.28$

Question 5

Solve by factoring.

$-x^2 + 8x - 16 = 0$

$\frac{-(x^2 - 8x + 16)}{-1} = \frac{0}{-1}$

$x^2 - 8x + 16 = 0$

$(x-4)(x-4) = 0$

$x - 4 = 0 \qquad x - 4 = 0$

$x = 4 \qquad x = 4$

Question 6

True or False?

a) If the discriminant of a quadratic is less than zero then the equation has one zero.

False! $\Delta < 0 \rightarrow$ No solution!

b) If the zeros of a quadratic function are 4 and 6 then the discriminant is 24.

False!

c) A quadratic equation with a discriminant equal to zero has zero zeros.

False! $\Delta = 0 \rightarrow$ 1 solution!

d) A positive discriminant means that the zeros will be positive.

False!

e) A quadratic equation with one zero has a discriminant equal to one.

False! $\Delta = 1 \rightarrow$ 2 solutions!

Question 7

Solve using the quadratic formula.

$\frac{1}{3}x^2 - 4x = 0$

$a = \frac{1}{3}$
$b = -4$
$c = 0$

$\Delta = b^2 - 4ac$
$= (-4)^2 - 4(\frac{1}{3})(0)^0$
$= 16 \longrightarrow$ 2 solutions!

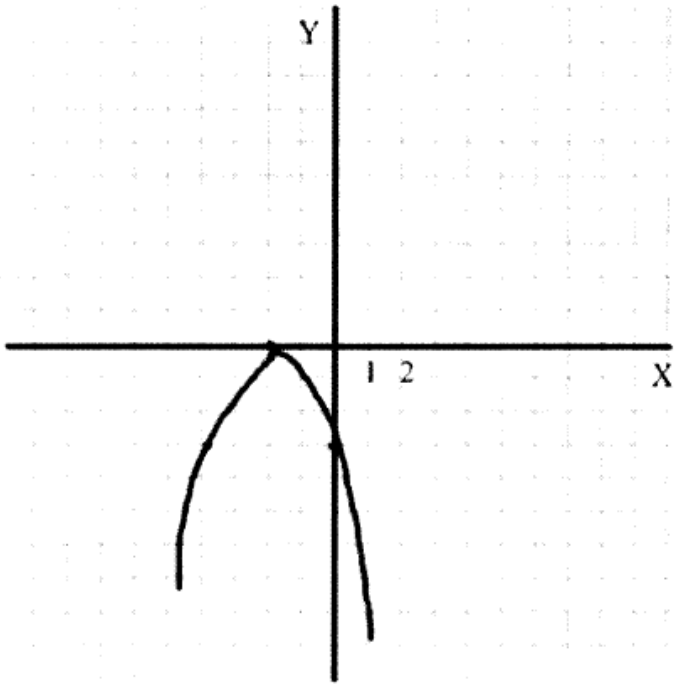
Q.F. $x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-4) \pm \sqrt{16}}{2(\frac{1}{3})} = \frac{4 \pm 4}{\frac{2}{3}}$

$x' = \frac{4+4}{\frac{2}{3}} = \frac{8}{\frac{2}{3}} = \frac{8}{2} \times 3 = 12$

$x'' = \frac{4-4}{\frac{2}{3}} = \frac{0}{\frac{2}{3}} = 0$

Question 8

Determine the characteristics listed below by referring to the graph.



Coordinates of the vertex (-2, 0)

Zero(s) (-2, 0)

Equation of the axis of symmetry $x = -2$

y-intercept (0, -3)

Maximum 0

Question 9

Graph the equation below:

$$y = -3x^2 - 2x + 4$$

$a = -3$
$b = -2$
$c = 4$

$\Delta = b^2 - 4ac$
$= (-2)^2 - 4(-3)(4)$
$= 4 + 48 = 52$

V

x	-2	-1	$-\frac{1}{3}$	0	1
y	-4	3	$\frac{13}{3}$	4	-1

$V\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right)$

$V\left(\frac{-(-2)}{2(-3)}, \frac{-52}{4(-3)}\right)$

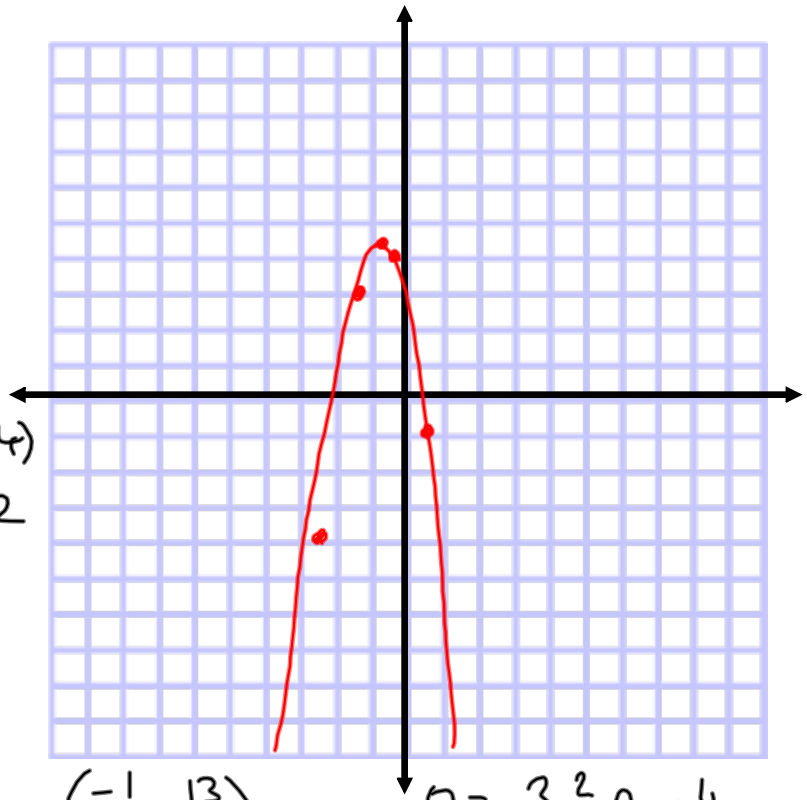
Coordinates of the vertex $V\left(-\frac{1}{3}, \frac{13}{3}\right)$ $\left(-\frac{1}{3}, \frac{13}{3}\right)$

Coordinates of the y-intercept $(0, 4)$

Coordinates of the point symmetric with the y-intercept $\left(-\frac{2}{3}, 4\right)$

Coordinates of the zero(s) $(-1.53, 0)$ & $(0.87, 0)$

Equation of the axis of symmetry $x = -\frac{1}{3}$



$0 = -3x^2 - 2x + 4$

QF $x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-2) \pm \sqrt{52}}{2(-3)}$

$x' = \frac{2 + 7.21}{-6} = \frac{9.21}{-6} = -1.53$

$x'' = \frac{2 - 7.21}{-6} = \frac{-5.21}{-6} = 0.87$

Question 10

Graph the equation below:

$$y = 2x^2 + 3$$

$a=2$ $b=0$ $c=3$

$\Delta = b^2 - 4ac$
$= (0)^2 - 4(2)(3)$
$= -24$

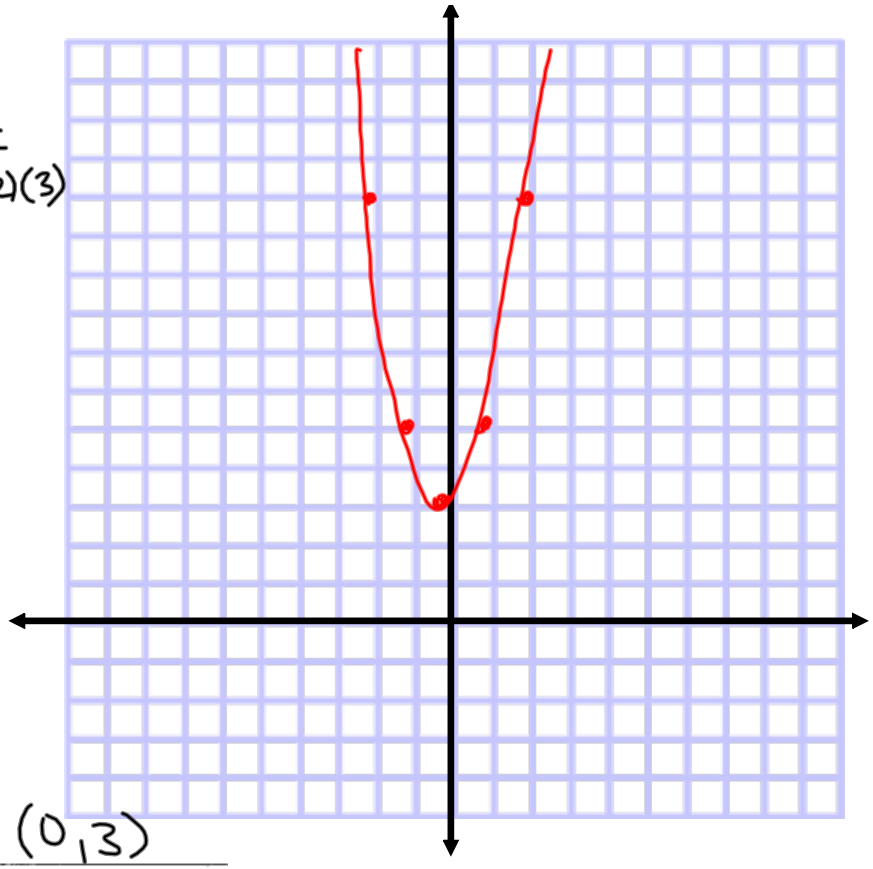
$V\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right)$

$V\left(\frac{-0}{2(2)}, \frac{-(-24)}{4(2)}\right)$

$V(0,3)$

V

x	-2	-1	0	1	2
y	11	5	3	5	11



Coordinates of the vertex (0,3)

Coordinates of the y-intercept (0,3)

Coordinates of the point symmetric with the y-intercept —

Coordinates of the zero(s) None!

Equation of the axis of symmetry $x = 0$

Question 11

Graph the equation below:

$$y = x^2 - x + \frac{1}{4}$$

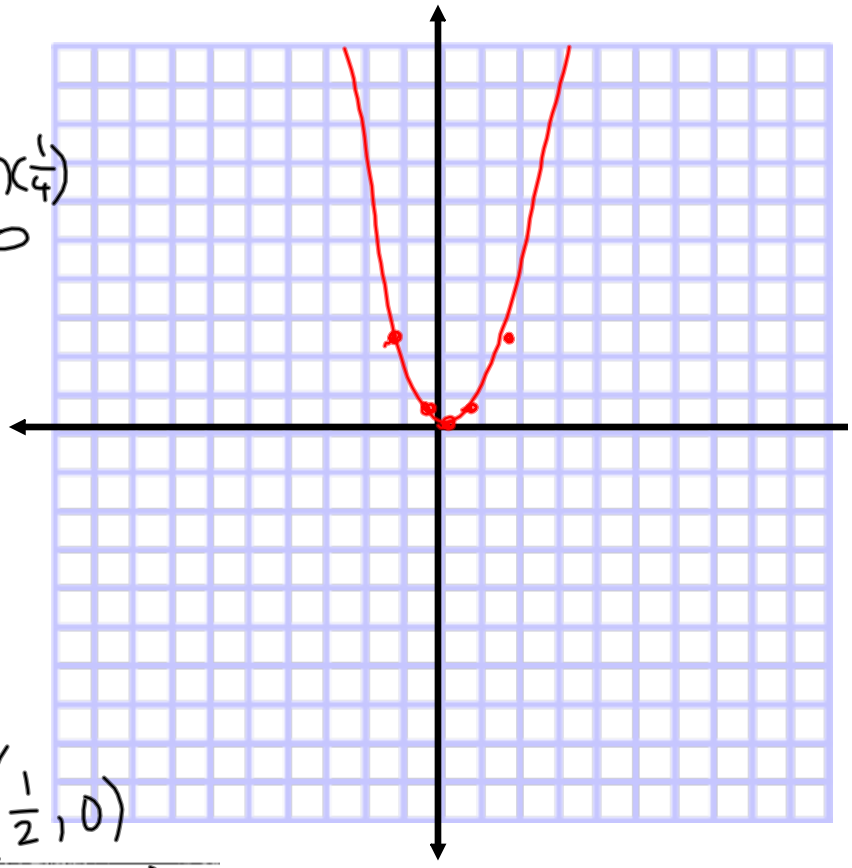
$a = 1$
$b = -1$
$c = \frac{1}{4}$

$\Delta = b^2 - 4ac$
$= (-1)^2 - 4(1)(\frac{1}{4})$
$= 1 - 1 = 0$

$V\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right)$
$V\left(\frac{-(-1)}{2(1)}, \frac{-0}{4(1)}\right)$
$V\left(\frac{1}{2}, 0\right)$

V

x	-1	0	½	1	2
y	2.25	0.25	0	0.25	2.25



Coordinates of the vertex $\left(\frac{1}{2}, 0\right)$

Coordinates of the y-intercept $(0, 0.25)$

Coordinates of the point symmetric with the y-intercept $(1, 0.25)$

Coordinates of the zero(s) $\left(\frac{1}{2}, 0\right)$

Equation of the axis of symmetry $x = \frac{1}{2}$

Question 12

Every day Benedicta drives her Ferrari to school from her home. It is 20km. One day the police are following her and so she reduces her speed by 40km/h to stay out of trouble with the police. If it takes her 90 seconds longer than usual to get to school then what is her usual average speed?

$x =$ usual average speed

Usual

$\frac{20}{x}$ = usual time

New

$\frac{20}{x-40}$ = usual time + 90 sec

$+0.025$ hrs

$90 \text{ sec} \times \frac{1}{60} = 1.5 \text{ min} \times \frac{1}{60} = 0.025$ hrs

$\frac{20}{x-40} = \frac{20}{x} + \frac{0.025}{1} \times \left(\frac{x}{x}\right)$

$\frac{20}{x-40} = \frac{20}{x} + \frac{0.025x}{x}$

$\frac{20}{x-40} \times \frac{20+0.025x}{x}$

$20(x) = (x-40)(20+0.025x)$

$20x = 20x + 0.025x^2 - 800 - x$

$0 = 0.025x^2 - x - 800$

$a = 0.025$ $\quad \Delta = b^2 - 4ac$

$b = -1$ $\quad = (-1)^2 - 4(0.025)(-800)$

$c = -800$ $\quad = 1 + 80 = 81$

Q.F. $x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-1) \pm \sqrt{81}}{2(0.025)}$

$x' = \frac{1+9}{0.05} = \frac{10}{0.05} = 200$

$x'' = \frac{1-9}{0.05} = \frac{-8}{0.05} = -160$

She usually drives 200 km/h

Question 13

A bathtub holds 260L. A dead rat has become trapped in the inflow pipe and the rate of flow has been reduced by 100l/h. it now takes seven and a half minutes longer to fill the tub than it did before the rat got wedged into the pipe. What is the original flow rate in L/h? x = original Flow rate

Usual

$\frac{260}{x}$ = usual time

New

$\frac{260}{x-100}$ = usual time + 7.5 min

+ 0.125 hrs

$7.5 \text{ min} \times \frac{1}{60} = 0.125 \text{ hrs}$

$\frac{260}{x-100} = \frac{260}{x} + \frac{0.125}{1} \times \left(\frac{x}{x}\right)$

$\frac{260}{x-100} = \frac{260}{x} + \frac{0.125x}{x}$

$\frac{260}{x-100} \times \frac{260+0.125x}{x}$

$260x = (x-100)(260+0.125x)$

$260x = 260x + 0.125x^2 - 26000 - 12.5x$

$0 = 0.125x^2 - 12.5x - 26000$

$a = 0.125$

$b = -12.5$

$c = -26000$

$\Delta = b^2 - 4ac$

$= (-12.5)^2 - 4(0.125)(-26000)$

$= 156.25 + 13000$

$= 13156.25$

Q.F. $x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-12.5) \pm \sqrt{13156.25}}{2(0.125)}$

$x' = \frac{12.5 + 114.7}{0.25} = \frac{127.2}{0.25} = 508.8$

$x'' = \frac{12.5 - 114.7}{0.25} = \frac{102.2}{0.25} = 408.8$

The original Flow rate is 508.8 L/h

Question 14

'Fatpig' is a competitive eater. In the competitions the contestants race to see who can wolf down 36 hotdogs in the shortest period of time. Fatpig has a tooth ache that has slowed down his/her consumption rate by one hotdog per minute. If Fatpig's new time to finish off the 36 hotdogs is 72 seconds longer than average then what is Fatpigs's average rate of consumption in hotdogs/minute?

x = Usual Average Rate

Usually

$\frac{36}{x}$ = usual time

New

$\frac{36}{x-1}$ = Usual time + 72 sec

+1.2 min

$72 \text{ sec} \times \frac{1}{60} = 1.2 \text{ min}$

$\frac{36}{x-1} = \frac{36}{x} + \frac{1.2}{1} \times \left(\frac{x}{x}\right)$

$\frac{36}{x-1} = \frac{36}{x} + \frac{1.2x}{x}$

$\frac{36}{x-1} \times \frac{36+1.2x}{x}$

$36x = (x-1)(36+1.2x)$

$\cancel{36x} = \cancel{36x} + 1.2x^2 - 36 - 1.2x$

$0 = 1.2x^2 - 1.2x - 36$

$a = 1.2$

$b = -1.2$

$c = -36$

$\Delta = b^2 - 4ac$

$= (-1.2)^2 - 4(1.2)(-36)$

$= 1.44 + 172.8$

$= 174.24$

Q.F. $x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-1.2) \pm \sqrt{174.24}}{2(1.2)}$

$x' = \frac{1.2 + 13.2}{2.4} = \frac{14.4}{2.4} = 6$

$x'' = \frac{1.2 - 13.2}{2.4} = \frac{-12}{2.4} = \cancel{-5}$

His usual average rate is 6 hotdogs per minute.