

**Question 1**

Factor the following polynomial:

$$10x^2 + 130xy^2$$
$$= 10x(x + 13y^2)$$

**Question 2**

Factor the following polynomial:

$$\begin{aligned} & d^2 - 2d - 24 \\ & \quad \quad \quad \underline{\quad} \\ & \quad \quad \quad 24 \times 1 \\ & \quad \quad \quad 12 \times 2 \\ & \quad \quad \quad 8 \times 3 \\ & \quad \quad \quad \textcircled{-6 \times 4} \\ = & \underline{d^2 - 6d} + \underline{4d - 24} \\ = & d(d-6) + 4(d-6) \\ = & (d-6)(d+4) \end{aligned}$$

**Question 3**

Factor the following polynomial:

$$\begin{aligned} & 10n^2 + 26nm + 12m^2 \\ &= 2(5n^2 + 13nm + 6m^2) \\ & \quad \begin{array}{c} \uparrow \quad \quad \quad \uparrow \\ \text{5x6=30} \\ \text{30x1} \\ \text{15x2} \\ \text{10x3} \\ \text{6x5} \end{array} \\ &= 2(\underline{5n^2 + 10nm} + \underline{3nm + 6m^2}) \\ &= 2(5n(n+2m) + 3m(n+2m)) \\ &= 2(n+2m)(5n+3m) \end{aligned}$$

**Question 4**

Factor the following polynomial:

$$\frac{25c^2}{64} - 144s^4$$
$$= \left( \frac{5c}{8} + 12s^2 \right) \left( \frac{5c}{8} - 12s^2 \right)$$

**Question 5**

Factor the following polynomial:

$$\begin{aligned} & \frac{3x^2 + xy - 12x - 4y}{=} \\ & = x(3x+y) - 4(3x+y) \\ & = (3x+y)(x-4) \end{aligned}$$

**Question 6**

Factor the following polynomial completely:

$$\begin{aligned} & 36x^2 + 48x + 16 \\ &= 4(9x^2 + 12x + 4) \\ & \quad \uparrow \quad \quad \quad \uparrow \\ & \quad \quad \quad \boxed{9 \times 4 = 36} \quad \uparrow \\ & \quad \quad \quad \begin{array}{c} 36 \times 1 \\ 18 \times 2 \\ 12 \times 3 \\ 9 \times 4 \\ \underline{6 \times 6} \end{array} \\ &= 4(\underline{9x^2 + 6x} + \underline{6x + 4}) \\ &= 4(3x(3x+2) + 2(3x+2)) \\ &= 4(3x+2)(3x+2) \end{aligned}$$

**Question 7**

Factor the following polynomial completely

$$\begin{aligned} & 80a^5b - 5ab^5 \\ &= 5ab(16a^4 - b^4) \\ &= 5ab(4a^2 + b^2)(4a^2 - b^2) \\ &= 5ab(4a^2 + b^2)(2a + b)(2a - b) \end{aligned}$$

**Question 8**

Reduce the following algebraic fraction to its lowest terms. Show all steps in the solution.

$$\begin{aligned} 9a^2 - 16b^2 &= (3a+4b)(3a-4b) \\ 15a^2 - 20ab &= 5a(3a-4b) \end{aligned} \quad \frac{9a^2 - 16b^2}{15a^2 - 20ab} = \frac{(3a+4b)\cancel{(3a-4b)}}{5a\cancel{(3a-4b)}} = \frac{(3a+4b)}{5a}$$



**Question 9**

Express the quotient of the following algebraic fractions in lowest terms. Show all steps to your solution.

$$\begin{aligned}
 & \frac{9-y^2}{y^2+3y-18} \div \frac{b^2y+3b^2}{1} = \frac{\cancel{(3+y)}\cancel{(3-y)}}{(y+b)\cancel{(y-3)}} \times \frac{1}{\cancel{b^2}\cancel{(y+3)}} \\
 & \frac{9-y^2}{y^2+3y-18} = \frac{9 \times 1 - y^2}{y^2 + 3y - 18} \\
 & \quad \quad \quad \begin{array}{l} 18 \times 1 \\ 9 \times 2 \\ \underline{6 \times 3} \end{array} \\
 & = \frac{y^2 + 6y - 3y - 18}{y^2 + 3y - 18} \\
 & = \frac{y(y+6) - 3(y+6)}{y^2 + 3y - 18} \\
 & = \frac{(y+6)(y-3)}{y^2 + 3y - 18} \\
 & \quad \quad \quad \begin{array}{l} b^2y + 3b^2 \\ \underline{\phantom{b^2y} + 3b^2} \\ b^2(y+3) \end{array} \\
 & = \frac{(y+6)(y-3)}{b^2(y+3)}
 \end{aligned}$$

**Question 10**

Express the difference of the following algebraic fractions in lowest terms. Show all steps to your solution.

$$\frac{7x^4}{2x^2} - \frac{(4+x)(4-x)}{(x-4)}$$

$$= \frac{7x^2}{2} - \frac{(4+x)(4-x)}{(x-4)}$$

$$= \frac{7x^2}{2} + \frac{(4+x)}{1}$$

LCD = 2

$$= \frac{7x^2}{2} + \frac{(4+x)}{1} \times \frac{2}{2}$$

$$= \frac{7x^2}{2} + \frac{8+2x}{2}$$

$$= \frac{7x^2 + 2x + 8}{2}$$

$$7x^2 + 2x + 8$$

↑ 7x8 = 56 ↑

56x1  
28x2  
14x4  
7x8

Not factorable

**Question 11**

Express the product of the following algebraic fractions in lowest terms. Show all steps to your solution.

$$\begin{aligned}
 & x^2 + 7x + 12 \\
 & \quad \quad \quad \begin{array}{l} 12 \times 1 \\ 6 \times 2 \\ \hline 4 \times 3 \end{array} \\
 & = \underline{x^2 + 4x} + \underline{3x + 12} \\
 & = x(x+4) + 3(x+4) \\
 & = (x+4)(x+3)
 \end{aligned}$$

$$\begin{aligned}
 & 9x^2 + 18x \\
 & = 9x(x+2)
 \end{aligned}$$

$$\begin{aligned}
 & x^2 - x - 12 \\
 & \quad \quad \quad \begin{array}{l} 12 \times 1 \\ 6 \times 2 \\ \hline -4 \times 3 \end{array} \\
 & = \underline{x^2 - 4x} + \underline{3x - 12} \\
 & = x(x-4) + 3(x-4) \\
 & = (x-4)(x+3)
 \end{aligned}$$

$$\begin{aligned}
 \frac{x^2 + 7x + 12}{3x} \times \frac{9x^2 + 18x}{x^2 - x - 12} &= \frac{(x+4)\cancel{(x+3)}}{\cancel{3x}} \times \frac{\overset{3}{\cancel{9}}x(x+2)}{\cancel{(x-4)}\cancel{(x+3)}} \\
 &= \frac{3(x+4)(x+2)}{(x-4)}
 \end{aligned}$$

**Question 12**

Express the sum of the following algebraic fractions in lowest terms. Show all steps to your solution.

$$\begin{aligned}
 & \frac{3}{(4-x)} + \frac{(x+2)}{2x(x-4)} \\
 &= \frac{3}{(4-x)} + \frac{(x+2)}{2x(x-4)} \\
 &= \frac{3}{(4-x)} - \frac{(x+2)}{2x(4-x)} \\
 & \text{LCD} = (4-x)(2x) \\
 &= \frac{3}{(4-x)} \cdot \frac{(2x)}{(2x)} - \frac{(x+2)}{2x(4-x)} \\
 &= \frac{6x}{2x(4-x)} - \frac{(x+2)}{2x(4-x)} \\
 &= \frac{6x - (x+2)}{2x(4-x)} = \frac{6x - x - 2}{2x(4-x)} \\
 &= \frac{(5x-2)}{2x(4-x)}
 \end{aligned}$$

**Question 13**

The following two algebraic expressions are equivalent. Demonstrate their equivalence by transforming the expression on the left side. Show all steps to your solution.

$$\begin{aligned} & \frac{(x+5)}{(x+6)} - \frac{9}{(x+6)(x-3)} \\ \text{LCD} &= (x+6)(x-3) \\ &= \frac{(x+5)(x-3)}{(x+6)(x-3)} - \frac{9}{(x+6)(x-3)} \\ &= \frac{x^2 - 3x + 5x - 15 - 9}{(x+6)(x-3)} \\ &= \frac{x^2 + 2x - 24}{(x+6)(x-3)} = \frac{(x+6)(x-4)}{(x+6)(x-3)} = \frac{(x-4)}{(x-3)} \end{aligned}$$

$$\begin{aligned} & \frac{(x+5)}{(x+6)} - \frac{9}{x^2 + 3x - 18} = \frac{(x-4)}{(x-3)} \\ & \frac{x^2 + 6x - 3x - 18}{(x+6)(x-3)} \end{aligned}$$

$$\begin{aligned} & x^2 + 2x - 24 \\ & \quad 24 \times 1 \\ & \quad 12 \times 2 \\ & \quad 8 \times 3 \\ & \quad \underline{6 \times 4} \\ & = x^2 + 6x - 4x - 24 \\ & = x(x+6) - 4(x+6) \\ & = (x+6)(x-4) \end{aligned}$$

**Question 14**

The following algebraic expressions are equivalent. This time, demonstrate their equivalence by transforming both expressions. Show all steps to your solution.  $s^2$

$$\frac{(s+4)}{(s-4)} - \frac{2s}{(s+4)(s-4)} = \frac{s^3}{s(s^2-16)} + \frac{2(3s+8)}{s^2-16} = \frac{\cancel{s}}{\cancel{s}(s+4)(s-4)} + \frac{2(3s+8)}{(s+4)(s-4)}$$

$$= \frac{s^2}{(s+4)(s-4)} + \frac{2(3s+8)}{(s+4)(s-4)}$$

$$= \frac{s^2 + 6s + 16}{(s+4)(s-4)}$$
  

LCD =  $(s-4)(s+4)$

$$= \frac{(s+4)}{(s-4)} \times \frac{(s+4)}{(s+4)} - \frac{2s}{(s+4)(s-4)}$$

$$= \frac{s^2 + 4s + 4s + 16}{(s-4)(s+4)} - \frac{2s}{(s+4)(s-4)}$$

$$= \frac{s^2 + 8s + 16 - 2s}{(s-4)(s+4)} = \frac{s^2 + 6s + 16}{(s-4)(s+4)}$$

Same!

  

$$s^2 + 6s + 16$$

$$\begin{array}{r} 16 \times 1 \\ 8 \times 2 \\ 4 \times 4 \end{array}$$