Question 1

Perform the following operation by applying the laws of exponents. Make sure your answer contains only positive exponents. Show all steps to the solution.

$$\left(16a^2b^{-3}\right)^2 \div \left(4b^2a^3\right)^{-\frac{2}{3}}$$

$$= 16^2(a^2)^2(b^{-3})^2 \div \left(4^{-\frac{2}{3}}(b^2)^{-\frac{2}{3}}(a^3)^{-\frac{2}{3}}\right)$$

$$= (2^4)^2 a^4 b^{-6} \div (2^2)^{-\frac{2}{3}} b^{-\frac{4}{3}} a^{-2}$$

$$= 2^8 a^4 b^{-6} \div 2^{-\frac{4}{3}} b^{-\frac{4}{3}} a^{-2}$$

$$= 2^{8-\left(-\frac{4}{3}\right)} a^{4-(-2)} b^{-6-\left(-\frac{4}{3}\right)}$$

$$= 2^{\frac{28}{3}} a^6 b^{-\frac{14}{3}} = \frac{2^{\frac{28}{3}} a^6}{b^{\frac{14}{3}}}$$

Question 2

Perform the following operation by applying the laws of exponents. Make sure your answer contains only positive exponents. Show all steps to the solution.

$$\left(m^{-3}n^{4}o^{2}\right)^{\frac{3}{4}} \times \left(m^{2}n^{-2}o^{-4}\right)^{-1}$$

$$= \left(m^{-3}\right)^{\frac{3}{4}}\left(n^{4}\right)^{\frac{3}{4}}\left(o^{2}\right)^{\frac{3}{4}} \times \left(m^{2}\right)^{-1}\left(n^{-2}\right)^{-1}\left(o^{-4}\right)^{-1}$$

$$= m^{-\frac{9}{4}}\, n^{3}\, o^{\frac{3}{2}} \times m^{-2}\, n^{2}\, o^{4}$$

$$= m^{-\frac{9}{4}+(-2)}\, n^{3+2}\, o^{\frac{3}{2}+4}$$

$$= \frac{m^{-\frac{17}{4}}\, n^{5}\, o^{\frac{11}{2}}}{1} = \frac{n^{5}\, o^{\frac{11}{2}}}{m^{\frac{17}{4}}}$$

Question 3

Perform the following operation by applying the laws of exponents. Make sure your answer contains only positive exponents. Show all steps to the solution.

$$\left(\frac{x^3}{4}\right)\times\left(\frac{16}{x}\right)^{-2}$$

$$= \frac{x^3}{2^2}\times\left(\frac{2^4}{x}\right)^{-2}$$

$$= \frac{x^3}{2^2}\times\left(\frac{x}{2^4}\right)^2 = \frac{x^3}{2^2}\times\frac{x^2}{(2^4)^2} = \frac{x^3}{2^2}\times\frac{x^2}{2^8} = \frac{(x^3)(x^2)}{(2^2)(2^8)}$$

$$= \frac{x^{3+2}}{2^{2+8}} = \frac{x^5}{2^{10}}$$

Question 4

Perform the following operation by using scientific notation and the laws of exponents. Express your answer using scientific notation. Show all steps to the solution.

$$\left(\frac{6.2\times10^{6}}{0.008}\right)$$ ✓

0.008

$= 8 \times 10^{-3}$

$= \frac{6.2\times10^{6}}{8\times10^{-3}}$

$= \frac{6.2}{8} \times 10^{6-(-3)}$

$= 0.775 \times 10^{9}$

$= 7.75\times10^{-1}\times10^{9}$

$= 7.75\times10^{-1+9}$

$= 7.75\times10$

$= 7.75\times10^{8}$

Question 5

Simplify the following expression. Make sure your answer contains only positive exponents. Show all the steps in the solution.

$$\left(\frac{x^3y^5z^{-2}}{27x^{-2}y^7z}\right)^{\frac{3}{2}}$$

$$=\left(\frac{x^{3-(-2)}\,y^{5-7}\,z^{-2-1}}{27}\right)^{-\frac{3}{2}}$$

$$=\left(\frac{x^5\,y^{-2}\,z^{-3}}{27}\right)^{-\frac{3}{2}}=\frac{\left(x^5\right)^{-\frac{3}{2}}\left(y^{-2}\right)^{-\frac{3}{2}}\left(z^{-3}\right)^{-\frac{3}{2}}}{27^{-\frac{3}{2}}}=\left(\frac{x^{-\frac{15}{2}}\,y^{3}\,z^{\frac{9}{2}}}{27^{-\frac{3}{2}}}\right)$$

$$=\frac{27^{\frac{3}{2}}\,y^{3}\,z^{\frac{9}{2}}}{x^{\frac{15}{2}}}$$

Question 6

Determine if the following two expressions are equivalent by applying the laws of exponents. Show all steps to your solution.

$$\left(\frac{16}{125}\right)^{-3} \times \left(\frac{25}{8}\right)^{2} \times \left(\frac{5}{2}\right)^{4} \quad \text{and} \quad \left(\frac{625}{64}\right)^{-1} \times \left(\frac{256}{625}\right)^{-\frac{1}{4}} \times \left(\frac{2}{5}\right)^{10}$$

$$= \left(\frac{125}{16}\right)^{3} \times \left(\frac{25}{8}\right)^{2} \times \left(\frac{5}{2}\right)^{4}$$

$$= \left(\frac{5^3}{2^4}\right)^{3} \times \left(\frac{5^2}{2^3}\right)^{2} \times \left(\frac{5}{2}\right)^{4}$$

$$= \frac{(5^3)^3}{(2^4)^3} \times \frac{(5^2)^2}{(2^3)^2} \times \frac{5^4}{2^4}$$

$$= \frac{5^9}{2^{12}} \times \frac{5^4}{2^6} \times \frac{5^4}{2^4} = \frac{5^{9+4+4}}{2^{12+6+4}}$$

$$= \frac{5^{17}}{2^{22}}$$

$$= \left(\frac{64}{625}\right) \times \left(\frac{625}{256}\right)^{\frac{1}{4}} \times \left(\frac{2}{5}\right)^{10}$$

$$= \frac{2^6}{5^4} \times \left(\frac{5^4}{2^8}\right)^{\frac{1}{4}} \times \left(\frac{2}{5}\right)^{10}$$

$$= \frac{2^6}{5^4} \times \frac{(5^4)^{\frac{1}{4}}}{(2^8)^{\frac{1}{4}}} \times \frac{2^{10}}{5^{10}}$$

$$= \frac{2^6}{5^4} \times \frac{5^1}{2^2} \times \frac{2^{10}}{5^{10}} = \frac{2^{6+10-2}}{5^{4-1+10}}$$

$$= \frac{2^{14}}{5^{13}}$$

NOT EQUIVALENT!

Question 7

$x = -4$

If x is an even negative integer, determine if the following statements are true or false by replacing the variable with the number of your choice.

a) $2^x \geq 1$

$2^{-4} \geq 1$

$\frac{1}{2^4} \geq 1$

$\frac{1}{16} \geq 1$ False!

b) $\left(-\frac{1}{2}\right)^x \leq 4$

$\left(-\frac{1}{2}\right)^{-4} \leq 4$

$\left(-\frac{2}{1}\right)^4 \leq 4$

$16 \leq 4$ False!

c) $\left(\frac{1}{2}\right)^{-x} \leq \frac{1}{2}$

$\left(\frac{1}{2}\right)^{-(-4)} \leq \frac{1}{2}$

$\left(\frac{1}{2}\right)^4 \leq \frac{1}{2}$

$\frac{1}{16} \leq \frac{1}{2}$ TRUE

d) $(-2)^x \leq 1$

$(-2)^{-4} \leq 1$

$\frac{1}{(-2)^4} \leq 1$

$\frac{1}{16} \leq 1$ TRUE

Question 8

Among the following algebraic expressions, circle those that are equivalent. In the space provided under each expression, show how you arrived at your conclusion.

$-a^2b^6$	$(-ab^2)^3$ $-(ab^2)^3$ $-(a^3)(b^2)^3$ $-a^3b^6$	$-\dfrac{b^3}{(a^{-2}b^{-9})}$ $=-\dfrac{b^{3-(-9)}}{a^{-2}}$ $=-a^2b^{12}$
$-(a^4b^{12})^{\frac{1}{4}}$ $-(a^4)^{\frac{1}{4}}(b^{12})^{\frac{1}{4}}$ $-a^1b^3$	$(ab^3)^2$ $a^2(b^3)^2$ a^2b^6	$-ab^3 \times ab^3$ $-(a^{1+1})(b^{3+3})$ $-a^2b^6$

Question 9

Perform the operations indicated in the expression below and simplify your answer. Show all steps in the solution.

$$\left(3\sqrt{8}+4\right)\bullet\left(-5\sqrt{32}-2\right)$$

$= 3\sqrt{8}(-5\sqrt{32}) + 3\sqrt{8}(-2) + 4(-5\sqrt{32}) + 4(-2)$

$= -15\sqrt{256} - 6\sqrt{8} - 20\sqrt{32} - 8$

$= -15(16) - 6\sqrt{4*2} - 20\sqrt{16*2} - 8$

$= -240 - 6(2)\sqrt{2} - 20(4)\sqrt{2} - 8$

$= -248 - 12\sqrt{2} - 80\sqrt{2}$

$= -248 - 92\sqrt{2}$

Question 10

Perform the operations indicated in the expression below and simplify your answer. Show all steps in the solution.

$$\sqrt{72} - \sqrt{576} + \sqrt{512}$$

$= 24$

$72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, (36), 72$

$\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$

$512: 1, 2 \quad \ldots \quad (256), 512$

$\sqrt{512} = \sqrt{256 \times 2} = 16\sqrt{2}$

$= 6\sqrt{2} - 24 + 16\sqrt{2}$

$= 22\sqrt{2} - 24$

Question 11

Perform the operations indicated in the following expression. Simplify your answer and rationalize the denominator, if necessary. Show all steps in the solution.

$$\frac{4\sqrt{2}}{5\sqrt{2}+4} \times \frac{(5\sqrt{2}-4)}{(5\sqrt{2}-4)}$$

$$= \frac{4\sqrt{2}(5\sqrt{2}) + 4\sqrt{2}(-4)}{5\sqrt{2}(5\sqrt{2}) + 5\sqrt{2}(-4) + 4(5\sqrt{2}) + 4(-4)}$$

$$= \frac{20\sqrt{4} - 16\sqrt{2}}{25\sqrt{4} - \cancel{20\sqrt{2}} + \cancel{20\sqrt{2}} - 16}$$

$$= \frac{20(2) - 16\sqrt{2}}{25(2) - 16} = \frac{40 - 16\sqrt{2}}{50 - 16} = \frac{40 - 16\sqrt{2}}{34} \begin{matrix} \div 2 \\ \div 2 \end{matrix} = \frac{20 - 8\sqrt{2}}{17}$$

Question 12

Perform the operations indicated in the following expression. Simplify your answer and rationalize the denominator, if necessary. Show all steps in the solution.

$$\left(\frac{-2\sqrt{96}}{1\sqrt{27}}\right)$$

$$= -\frac{2}{1} \times \sqrt{\frac{96}{27}} \quad \begin{matrix} \div 3 \\ \div 3 \end{matrix}$$

$$= -\frac{2}{1} \times \sqrt{\frac{32}{9}}$$

$$= \frac{-2\sqrt{32}}{\sqrt{9}} = \frac{-2\sqrt{16 \cdot 2}}{3}$$

$$= \frac{-2(4)\sqrt{2}}{3} = -\frac{8\sqrt{2}}{3}$$

Question 13

Determine if the two following expressions are equivalent. Show all the steps in the solution.

$(5-3\sqrt{3})\bullet(3\sqrt{3}+5)$ and $4\sqrt{9}-2\sqrt{49}$

$= 5(3\sqrt{3}) + 5(5) + (-3\sqrt{3})(3\sqrt{3}) + (-3\sqrt{3})(5)$

$= 15\sqrt{3} + 25 - 9\sqrt{9} = 15\sqrt{3}$

$= 25 - 9(3)$

$= 25 - 27$

$= -2$

$= 4(3) - 2(7)$

$= 12 - 14$

$= -2$

YES

Question 14

Determine if the two following expressions are equivalent. Show all the steps in the solution.

$x^{\frac{5}{2}}\sqrt[2]{x^5}$ and $\left(\frac{1}{x^2}\right)^{-2}\sqrt[4]{x^4}$

$= x^{\frac{5}{2}} \cdot x^{\frac{5}{2}}$

$= x^{\frac{5}{2}+\frac{5}{2}}$

$= x^{\frac{10}{2}} = x^5$

$= \left(\frac{x^2}{1}\right)^2 \cdot x^{\frac{4}{4}}$

$= \frac{(x^2)^2}{1^2} \cdot x^1$

$= x^4 \cdot x^1$

$= x^{4+1} = x^5$

YES