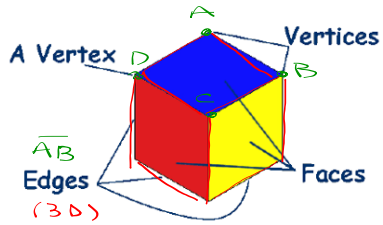
Lesson 9: Areas and Volume of Prisms (mini-tasks)

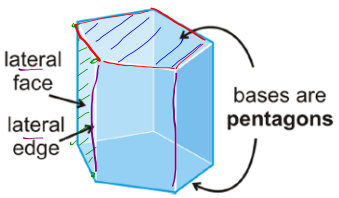
June 5th, 2023

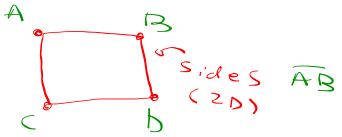
(tricky → review notes @ end of day, esp. for tomorrow)

1.1 The Total Surface Area of a Prism

Review Terminology: Before beginning, let's review some terminology







Recall:

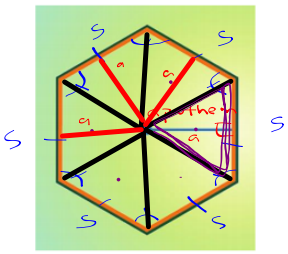
Total Area = Lateral Area + 2 × Area of Base

simplified formula later

recall

lateral area = area of lat face 1 + area of lat face 2 + ... + area of lat face 5

Review of The Area of any regular polygon: We will need to be able to find the area of any regular polygon.



For any regular polygon, the area can be computed as follows:

$$A = \frac{p \times a}{2}$$

- p is the perimeter of the polygon
- a is the apothem in a 2D shape

Proof and Example:

hexagon → 6 isosceles triangles

$\therefore A_{hex} = 6 \times A_{triangles}$

$= 6 \times \frac{b \times h}{2}$

$= 6 \times \frac{s \times a}{2}$

$A = \frac{p \times a}{2}$

a = apothem
s = side of hex
p = perimeter

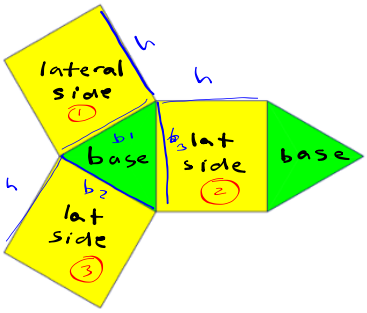
$s + s + s + s + s + s$
$= 6s$

$s \times s \times s \times s \times s \times s$
$= s^6$

$s \times s = s^2$
$s + s = 2s$

1.2 Discovering the Formula for the Lateral Area (A_l)

Let's consider the case of a triangular prism. We will see that calculating the lateral area for any prism is done in exactly the same way.



Lateral Area = sum of area of lateral sides

$= (1) + (2) + (3)$

$= b_1 \times h + b_2 \times h + b_3 \times h$

$= h(b_1 + b_2 + b_3)$

= (perimeter of base of 3D solid) $\times$ h (of solid)

$= P_B \times h$

gcf = h

- Lateral Area of any prism:

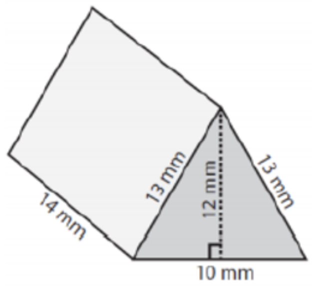
$$A_\ell = P_B \times h$$

- Total Surface Area of any prism:

$$A_T = A_\ell + 2A_B$$

$$A_T = P_B \times h + 2A_B$$

1.4 Example: Robin wants to make a toy tent in the shape of a triangular prism. He bought 3.5 bags of canvas. Each bag (per bag) covers 170 mm^2 and cost him \$2.50. Will the 3.5 bags be enough to make his toy tent?



In notebook, write "refer to handout 1, pg 2, # 1.4"

mini - task

WANT:

i. area of toy tent

ii. area of the amount of canvas in bags (mm^2)

TOOL:

$A_T = A_\ell + 2A_B$

TIP: look @ unit of numbers.

3.5 bags, 170 $\frac{mm^2}{bag}$, 2.50 $\frac{\#}{bag}$

$3.5 \text{ bags} \times 170 \frac{mm^2}{\text{bags}}$ ✓

$3.5 \text{ bags} \times 170 \frac{mm^2}{\text{bags}} \times 2.5 \frac{\#}{\text{bag}}$ ✗

$3.5 \text{ bags} \times 2.50 \frac{\#}{\text{bags}}$ (#) ✗

∴ amount canvas that we have:

$3.5 \text{ bags} \times 170 \frac{mm^2}{\text{bag}}$

$= 595 \ mm^2$

enough?

Find A_T of prism

$A_T = A_\ell + 2A_B$

A_ℓ (3D) $= P_B \times h$ (height of prism)

height of triangle (of base)

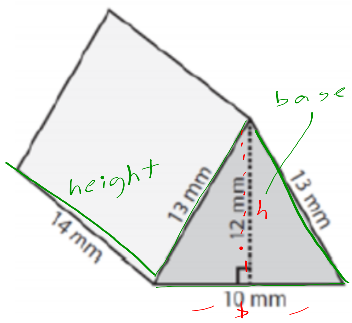
$A_B = \frac{b \times h}{2}$

$A_\ell = (13 + 13 + 10) \times 14$

$A_B = \frac{10 \times 12}{2}$

$A_\ell = 504 \ mm^2$

$A_B = 60 \ mm^2$



$A_T = 504 + 2 \cdot 60$

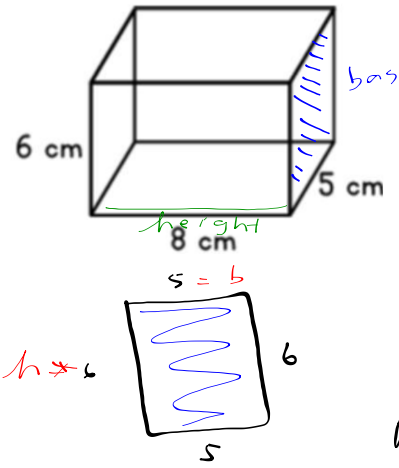
$A_T = 624 \ mm^2$

and we have $595 \ mm^2$

∴ No, we won't have enough canvas since $595 < 624$

You do 1.7 practice

1.7 Practice: Youssef makes boxes of matches in the shape of a rectangular prism shown below. He purchases 1.5 packs of cardboard to make one box. Each pack will cover 160 cm^2. Each box of matches will be made with the cardboard and will contain 25 matches. Will Youssef have enough cardboard to make a box?



- Youssef has

$1.5 \text{ pack} \times 160 \frac{cm^2}{\text{pack}}$

$240\ cm^2$ of cardboard

- Prism

$A_T = A_\ell + 2A_B$

$A_\ell = P_B \times h$

$A_\ell = (5+5+6+6) \times h$

$A_\ell = 22 \times 8$

$A_\ell = 176\ cm^2$

$A_B = b \times h$

$A_B = 5 \times 6$

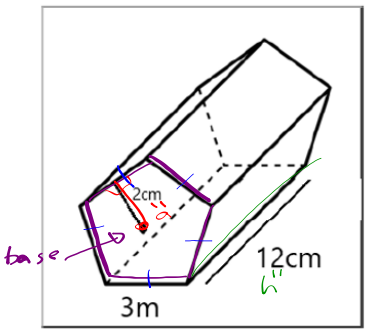
$A_B = 30\ cm^2$

$A_T = 176 + 2(30)$

$A_T = 236\ cm^2$

∴ Yes, Youssef does have enough cardboard since $240\ cm^2 > 236\ cm^2$

1.5 Example: Paulina has a company that makes erasers in the form of pentagonal prims. She thinks 2.2 sheets of plastic should be enough to wrap them. She pays \$0.10 per sheet and each one covers $100cm^2$. Does she have enough plastic?



Paulina has : want cm^2

2.2 sheets · 0.10 \$/sheet · 100 cm^2/sheet

2.2×0.10 sheets $\times$ \$/sheet (don't want \$)

2.2 sheets $\times$ 100 cm^2/sheet

220 cm^2 of plastic

Find area of prism

$A_T = A_\ell + 2 \cdot A_B$

$A_\ell = P_B \times h$ (height of prism = 12)

$A_\ell = (5 \times 3) \times 12$

$A_\ell = 180$

$A_B = \frac{P \times a}{2}$ (pentagon)

$= \frac{(5 \times 3) \times 2}{2}$

$A_B = 15$

P = perimeter
a = apothem

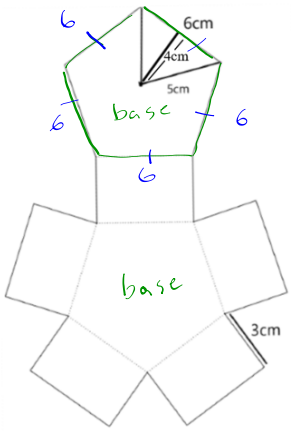
$A_T = 180 + 2 \times 15$

$A_T = 210\ cm^2$

∴ Paulina does have enough plastic is $220\ cm^2 > 210\ cm^2$

You do Practice 1.6

1.6 Practice: Paulina has a new design for the eraser and buys 5 sheets of paper wrapper that cost \$0.50 more. Below is the net of the new pentagonal prisms. If each sheet of paper cover $50cm^2$, will the 5 sheets be enough to wrap the eraser?



- Paulina has $250\ cm^2$
- Prism needs $210\ cm^2$

∴ yes 5 sheets will be enough!

Volume of Prisms

Write all formula for prisms

Formula sheet.

Write all formulas for pyramids

$A_\ell = P_B \times h$

$A_T = A_\ell + 2A_B$

$A_T = P_B \times h + 2A_B$

$V = A_B \times h$

Where,

A_ℓ = Lateral Area of prism (3D)

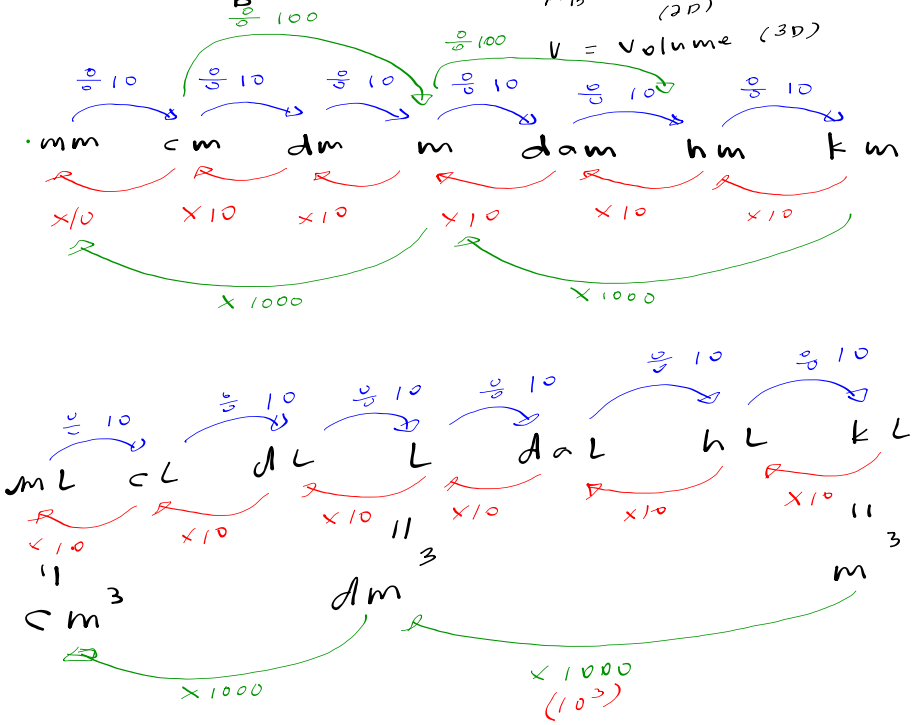
P_B = Perimeter of base (2D)

h = height of prism (3D)

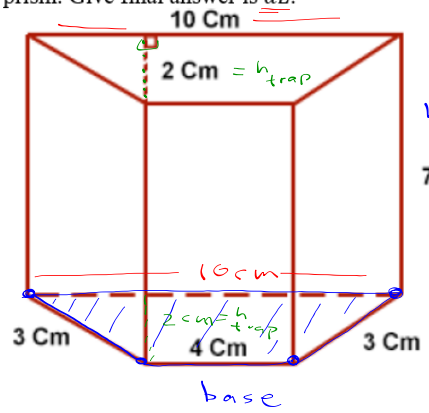
A_T = Total Area of prism (3D)

A_B = Area of base (2D)

V = Volume (3D)



1.10 Example: Find the volume of the following prism. Give final answer is *dL*.



h prism = 7 Cm

$\Rightarrow V = A_B \times h$ ← height of ... prism (7 cm)
(3D)

• Find A_B

$A_B = \frac{(b + B) \times h}{2}$ ← height of ... trapezoid (2 cm)
(2D)

$A_B = \frac{(4 + 10) \times 2}{2}$

$A_B = 14\ cm^2$

• Find volume

$V = A_B \times h$

$V = 14 \times 7$

$V = 98\ cm^3$

now convert to dL (many "paths") (1st to mL)

$V = 98\ mL$

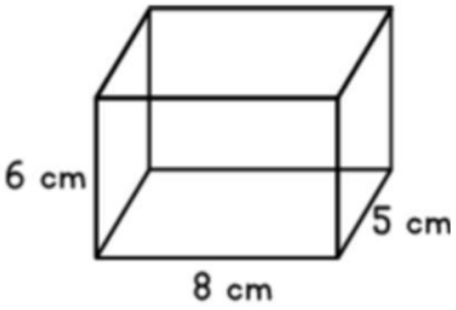
$98 \div 10 \div 10$

$0.98\ dL$

You do

P. 1.11

1.11 Practice: Find the volume of the following prism. Give the final answer in *cL*.



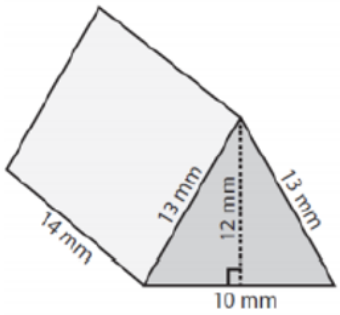
You do!
and
1.12 and 1.13

$V = 240\,cm^3$
$= 240\,mL$
$= 24\,cL$

You do 1.12 and 1.13

1.12 Practice

Find the volume of the following prism. Give the final answer in mL.



$V = A_B \times h$

$A_B = \frac{b \times h}{2}$

$= \frac{10 \times 12}{2}$

$= 60$

height of prism = 14

$V = 60 \times 14$

$V = 840\ mm^3$

mL

cubed since volume

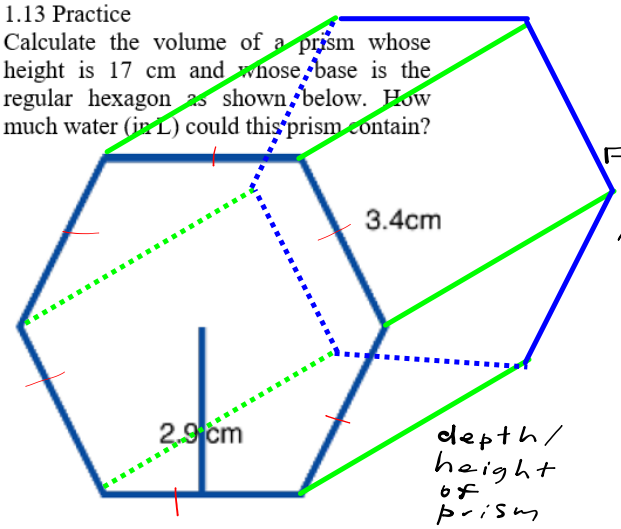
$840 \frac{2}{5} 10^3$

$0.84\ cm^3$

$0.84\ mL$

1.13 Practice

Calculate the volume of a prism whose height is 17 cm and whose base is the regular hexagon as shown below. How much water (in L) could this prism contain?



depth / height of prism = 17cm

$V = A_B \times h$

Find A_B

$A_B = \frac{P \cdot a}{2}$

$A_B = \frac{(6 \times 3.4) \times 2.9}{2}$

$A_B = 29.58\ cm^2$

$V = A_B \times h$

$V = 29.58 \times 17$

$V = 502.86\ cm^3$

$= 502.86\ mL$

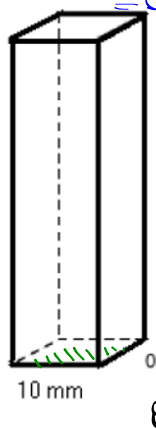
$= 502.86 \div 10 \div 10 \div 10$

$= 0.50286\ L$

Mini – tasks using Areas and Volumes

Pg 4

2.1.1 Example: Determine the volume (in mL) of the following rectangular prism if its total surface area is 952 mm^2.



convert to mm ×10

8 mm

WANT: the volume

TOOL: $V = A_B \times h$

$A_B = \ell \times w$

INFO: $\ell = 10$ $h = ?$
$w = 8$

WANT: height (unit)

TOOL: 1 eq (which one? volume or Total Area? TIP: pick eq you have most info for)

$A_T = A_\ell + 2A_B$

$A_T = P_B \times h + 2A_B$

Info: have all :)

start here

(4) $A_T = A_\ell + 2A_B$

- $A_\ell = P_B \times h$
- $A_\ell = (10 + 10 + 8 + 8) \times h$

$A_\ell = 36h$ (1)

- $A_B = \ell \cdot w$ (or $b \times h$)
- $A_B = 10 \times 8$

$A_B = 80$ (2)

$A_T = 952$ (3)

sub (1) (2) (3) into (4)

$952 = 36h + 2(80)$

$952 = 36h + 160$ (−160 −160)

$792 = 36h$

$\frac{792}{36} = \frac{36h}{36}$

$h = 22$ mm

- sub
- simplify
- solve
- w/ 0.0.

simplify BEDMAS solve

Find volume

$V = A_B \times h$

$A_B = \ell \cdot w$
$= 10 \times 8$
$A_B = 80\ mm^2$

$h = 22$ mm

$V = 80 \times 22$

$V = 1760\ mm^3$

$\frac{0}{0}\ 10^3$

$1.76\ cm^3$

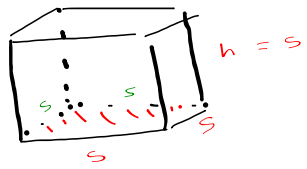
$\therefore V = 1.76$ mL

You do 2.1.2

2.1.2 Practice: Determine the volume (in L) of a cube if its total surface area is 216 m^2.

2cm x 2cm x 2cm

÷ 6 ↑ 2



For cube

$A_T = 6s^2$

· sub
· simplify
· solve for s

$\frac{216}{6} = \frac{6 \cdot s^2}{6}$

$\sqrt{36} = \sqrt{s^2}$

$s = 6$

Find volume

$V = s^3$

$V = 6^3$

$V = 216\ m^3$ → convert to L

$\times 1000$

$= 216\,000\ dm^3$

$= 216\,000\ L$

Want: V

$V = A_B \times h$

$V = s \times s \times s$

tool: $V = s^3$

info: $s = ?$

want side (s) link

tool | eq (vol? area?)

$A_T = A_\ell + 2A_B$

$A_T = P_B \times h + 2(s \cdot s)$

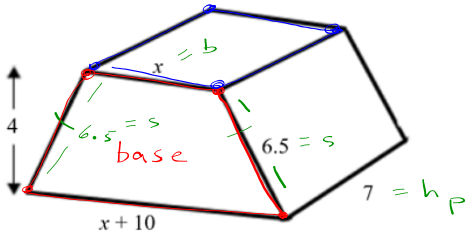
$A_T = 4s \times s + 2s^2$

$A_T = 4s^2 + 2s^2$

$A_T = 6s^2$

INFO: $A_T = 216\ m^2$

2.2.1 Example: Determine the lateral area of the following isosceles trapezoidal prism if the below units are in cm and its volume is 0.224 *litres*.



WANT : lateral area

TOOL :

$A_\ell = P_B \times h$

$A_\ell = (b + B + s + s) \times h_P$

INFO

$b = ? = x$

$B = ? = x + 10$

$s = 6.5$

$h = 7$

WANT : $b = x$

TOOL : $V = A_B \times h$

have all info

convert to cm^3

$V_{olume} = 0.224\ L$

$= 0.224\ dm^3$

$\times 1000$

(3) $V = 224\ cm^3$

find b w volume

(4) $V = A_B \times h$ ← height of prism (7)

$A_B = \frac{(b + B) \times h}{2}$ ← height of trapezoid (4)

$A_B = \frac{(x + (x+10)) \times 4}{2}$ · sub · simplify

$A_B = (2x + 10) \times 2$

(1) $A_B = 4x + 20$ (2) $h = 7\ cm$

sub (1) (2) (3) into (4)

$224 = (4x + 20) \times 7$

· Simplify
· solve for x
· find lateral area

· HMWK 2.2.2 + 2.2.3 (from handout)
· from textbook
pg 91 #2.21 / 2.23
pg 109 #3.8
Bonus pg 92 #2.25
pg 96 #2.31

2.2.2 Practice: Determine the lateral area of the following isosceles trapezoidal prism if the below units are in cm and its volume is $0.0315\ daL$.