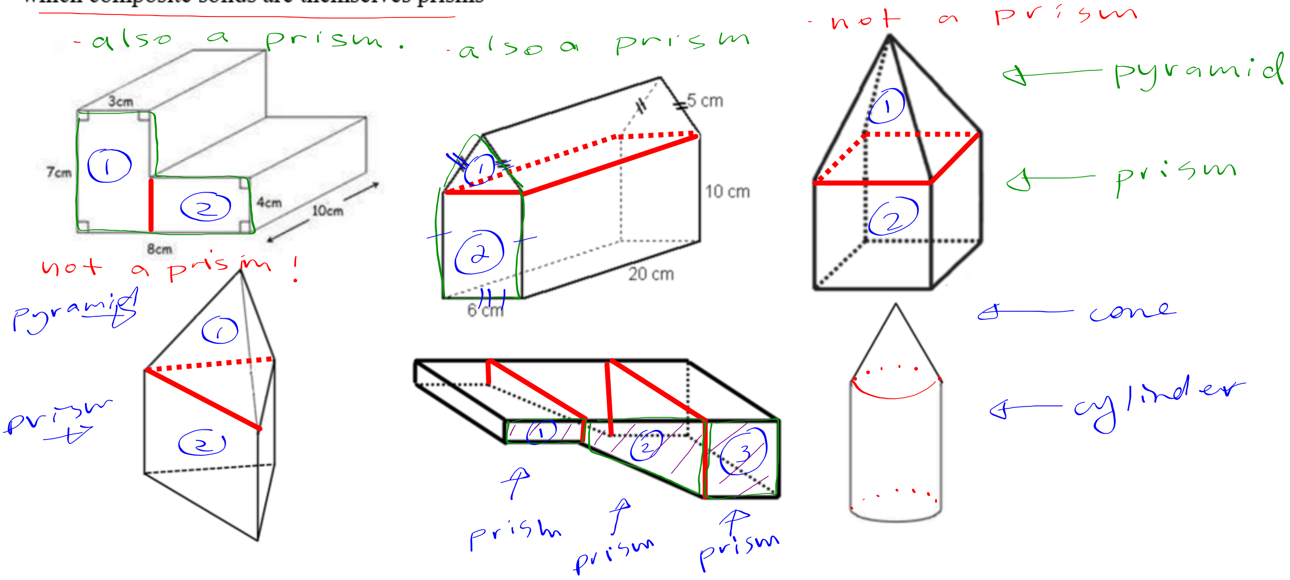
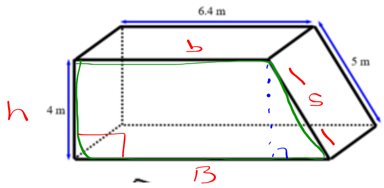
Lesson 12: Volume and Areas of Composite Solids (plus tasks) June 9th, 2023

Definition: Composite solids are decomposable three dimensional (3D) objects that are composed of simplier 3D objects.

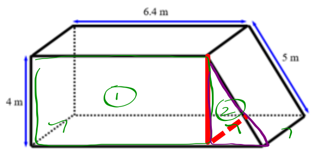
Warm-up: Decompose/break up the following solids into simpler prisms or pyramids. State which composite solids are themselves prisms



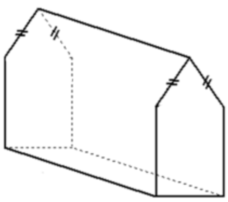
1


6.4 m
5 m
4 m
h
b
s
B

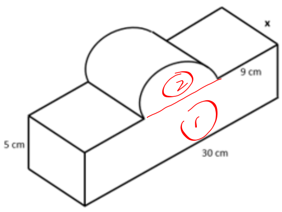
1


6.4 m
5 m
4 m
1
2

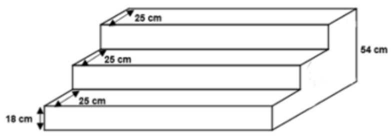
2



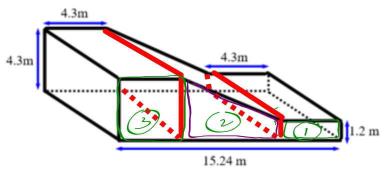
3


x
9 cm
5 cm
30 cm
2
1

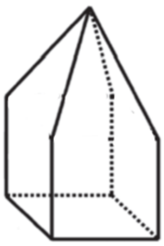
4


25 cm
25 cm
25 cm
54 cm
18 cm

5


4.3m
4.3m
4.3m
1.2 m
15.24 m
3
2
1

6



Finding Volumes of Composite Prisms and composite Solids

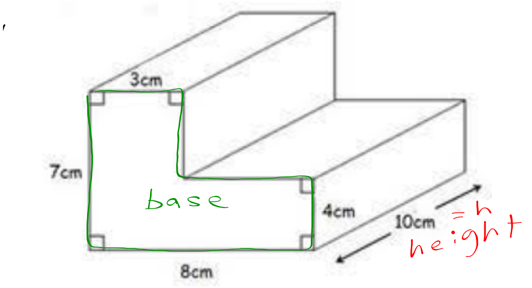
simplier procedure

1.1.1 Example

prisms

Consider the following composite solid. Determine its volume.

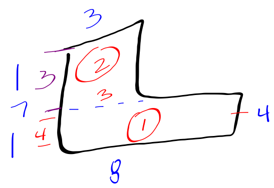
since it's also a prism, consider the two procedures to find volume:



1st way: treat as prism

height of prism

$V = A_B \times h$ label



Find: A_B Base is a L:

$A_B = A_1 + A_2$

$A_B = l \cdot w + s^2$

$A_B = 8 \cdot 4 + 3^2$

$A_B = 41\ cm^2$ $h = 10$

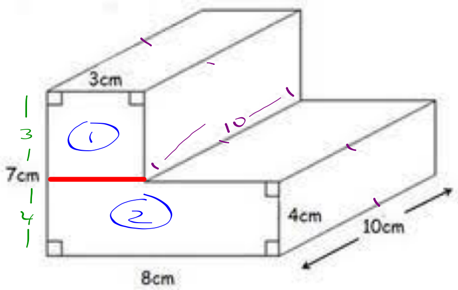
$V = A_B \times h$

$V = 41 \times 10$

$V = 410\ cm^3$

2nd way → works for all our composite figure

$V = V_1 + V_2$

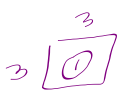


$V_1 = A_{B_1} \times h$ $V_2 = A_{B_2} \times h$

$V_1 = s^2 \times h$ $V_2 = l \cdot w \times h$

$V_1 = 3^2 \times 10$ $V_2 = 8 \cdot 4 \cdot 10$

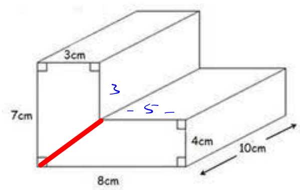
$V_1 = 90\ cm^3$ $V_2 = 320\ cm^2$



$V = V_1 + V_2$

$= 90 + 320$

$= 410\ cm^3$



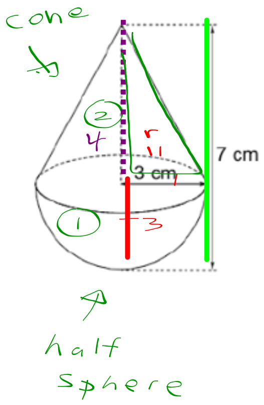
$V = \left(\frac{(3+7)3}{2} + \frac{(5+3)4}{2}\right) \times 10$

$V =$

not a prism

1.1.2 Example

Consider the following container. If its base is a hemisphere, determine its capacity in L.



not a prism, ∴ use way 2

cone

$$V = V_① + V_②$$

$$V_① = \frac{V_{sphere}}{2}$$

$$V_① = \frac{\frac{4\pi r^3}{3}}{2}$$

$$V_① = \frac{\left(\frac{4\pi(3)^3}{3}\right)}{2}$$

$$V_① = 18\pi$$

$$V_① \approx 56.55\ cm^3$$

pyramid

$$V_② = \frac{A_B \times h}{3}$$

$$V_② = \frac{\pi r^2 \times h}{3}$$

$$V_② = \frac{\pi(3)^2 \times 4}{3}$$

$$V_② = 12\pi$$

$$V_② = 37.699$$

$$V = 18\pi + 12\pi$$

$$V = 30\pi\ cm^3$$

$$V = 56.55 + 37.699$$

$$V = 94.25\ cm^3$$

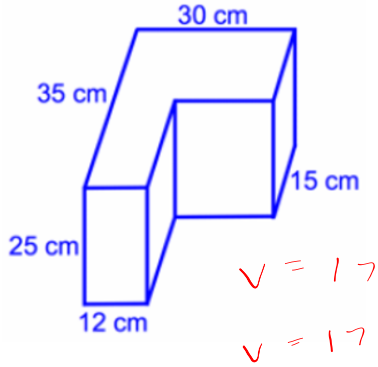
$$V = 94.25\ mL$$

÷ 10 ÷ 10 ÷ 10

$$V = 0.09425\ L$$

You do 1.1.3 and 1.1.4

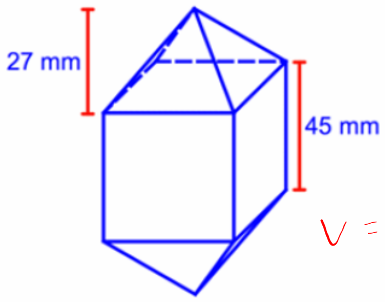
1.1.3 Practice: Consider the following composite solid. Determine its capacity in *L*.



$V = 17\,250 \text{ cm}^3$

$V = 17.25 \text{ L}$

1.1.4 Practice: Consider the following composite solid composed of a cube and two identical pyramids. Determine its capacity in *mL*.



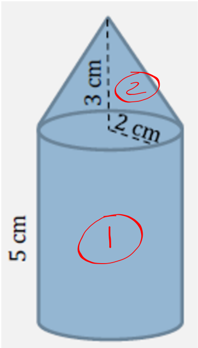
$V = 127575 \text{ mm}^3$

$V = 127.575 \text{ mL}$

Finding Areas of Composite Solids

* not prisms *

1.2.1 Example: Consider the following composite solid. Determine its total surface area.



$A_T = A_{T\,\text{cylinder}} + A_{T\,\text{cone}}$?

no (cause the cone's base is not counted)

$A_T = A_{T\,\text{cylinder}} + A_{\ell\,\text{cone}}$

no (cause one of cylinder's bases is covered)

① $A_T = A_{T\,\text{cylinder}} - A_B + A_{\ell\,\text{cone}}$

or

② $A_T = A_{\ell\,\text{cylinder}} + A_{\ell\,\text{cone}} + A_B$

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① $A_T = A_{\ell\,\text{cyl}} + 2A_B - A_B + A_{\ell\,\text{cone}}$

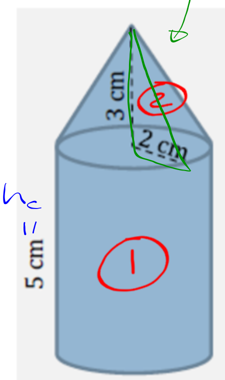
$A_T = P_B \times h_{\text{cyl}} + A_B + \dfrac{P_B \times a}{2}_{\text{cone}}$

$A_T = 2\pi r \times h_{\text{cyl}} + \pi r^2 + \dfrac{\cancel{2}\pi r \times a}{\cancel{2}}$ prism

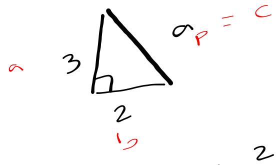
$A_T = 2\pi(2)\times 5 + \pi 2^2 + \pi(2)\,3.6$

$A_T \cong 98.02\text{ cm}^2$

pyramid



find apothem of cone:



$c^2 = a^2 + b^2$

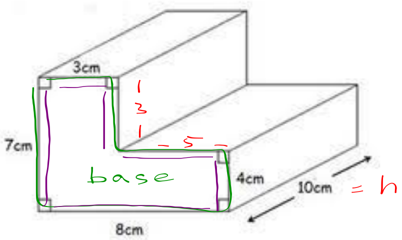
$\sqrt{c^2} = \sqrt{(2^2 + 3^2)}$

$c \doteq 3.6\text{ cm}$

Areas of a Composite Prism

a prism

1.2.2 Example: Consider the following composite solid. Determine its total surface area.



. don't decompose it!

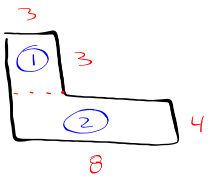
. keep it simple!

$A_T = A_\ell + 2A_B$

height of prism

3D $A_T = P_B \times h + 2A_B$ 4

come back



Find

$A_B = A_1 + A_2$

$A_B = s^2 + \ell \cdot w$

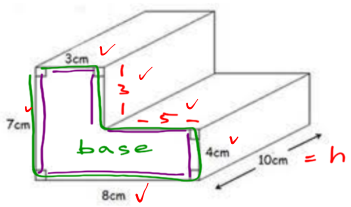
$A_B = 3^2 + 8 \cdot 4$

1 $\boxed{A_B = 41\text{ cm}^2}$

$\boxed{h = 10}$ 2

3 $P = 8 + 7 + 3 + 3 + 5 + 4$

$\boxed{P = 30}$



4 $A_T = P_B \times h + 2A_B$

sub 1 2 3 into 4

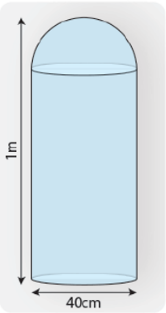
$A_T = 30 \times 10 + 2(41)$

$A_T = 382\text{ cm}^2$

you do 1.2.3 and 1.2.4

and then tasks

1.2.3 Practice: The following composite solid is composed of a cylinder and half a sphere. Determine its total surface area in cm^2.



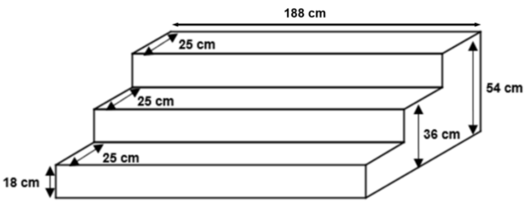
Answer:

$A_T = 4400\pi \ cm^2$

or

$A_T \cong 13\ 823.008\ cm^2$

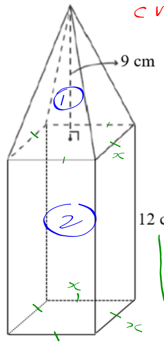
1.2.4 Practice: Consider the following composite solid. Determine its total surface area.



$A_T = 53\ 904\ cm^2$

lateral area of the pyramid.

2.1.1 Example: An architect is making a shed whose roof he will shingle with 2.5 bags of shingles. The shed is represented in the diagram below which pictures a pyramid whose base is the same as that of the prism. Their bases have congruent sides. The total capacity of the shed is ~~60 000 dm^3~~ $= V_T$ 60 000 mm^3
If each bag covers 12 ~~m^2~~ cm^2 and contains 28 shingles, will the ~~3.5~~ 2.5 bags be enough to shingle the roof?



WANT:
- amount (area) (m^2) of shingle we have
- lateral area just of pyramid (roof)

2.5×12 — bags, $\frac{m^2}{bag}$, $\frac{28\ shin}{bag}$

TOOL

$30\ m^2$ of shingles is what we have!

TOOL

$A_\ell = \frac{P_B \times a}{2}$

IWFO $P_B = 4x$

$x = ?$

$a = ?$

want: x ✓

TOOL: V_T

want: apothem

$c^2 = a^2 + b^2$

$\sqrt{c^2} = \sqrt{1^2 + 9^2}$

$c = \sqrt{82}$

$\boxed{a_p \simeq 9.06\ cm}$

c " ap, 9 " b, $1 = a$

$V_T = V(1) + V(2)$

$V_T = \frac{A_B \times h_1}{3} + A_B \times h_2$

$V_T = \frac{s^2 \times h_1}{3} + s^2 \times h_2$

$60 = \frac{x^2 \times \cancel{9}^{3}}{\cancel{3}} + x^2 \times 12$

* simplify!

$60 = 3x^2 + 12x^2$

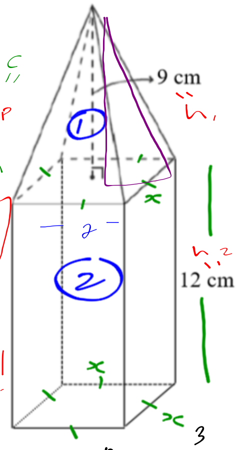
* add like terms

$\frac{60}{15} = \frac{15 \cdot x^2}{15}$

* solve for x w o.o.

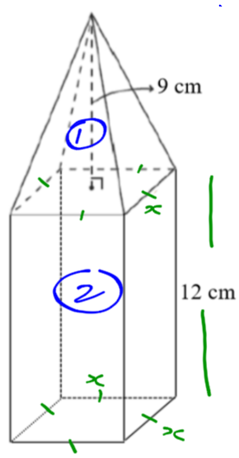
$\sqrt{4} = \sqrt{x^2}$

$x = 2\ cm$



$V = 60\ 000\ mm^3$
$\div 1000$
$60\ cm^3$

B E D M A S



Find A_ℓ of pyramid

$A_\ell = \frac{P_B \times a}{2}$

$A_\ell = \frac{(4 \cdot 2) \times 9.0\cancel{5}6}{2}$

$A_\ell = 36.22\ cm^2$

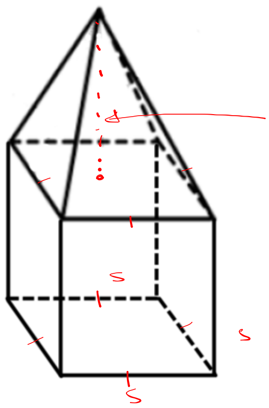
∴ The roof has an area of $36.22\ cm^2$

And we have $30\ cm^2$ of shingles.

∴ No the architect won't have enough!

You do 2.1.2

2.1.2 Practice: The architect is now making a smaller shed whose exterior walls and roof he plans to paint with 4 cans of paint. The shed is represented in the diagram below which pictures a pyramid on top of a cube. The height of the pyramid is congruent to each side length of the cube. The total capacity of the smaller shed is 13.5 dm^3. If each can of paint cost \$22.50 and covers 1.5 dm^2, will the 4 cans be enough?



$s = 2.16$

$A_{\ell\,shed} = A_{\ell\,cube} + A_{\ell\,pyr}$

$A_{\ell\,shed} = 29.07\ dm^3$ to be covered

But we have $(4\text{ cans} \times 1.5\ dm^2/\text{can})$ 6 dm^2 of paint

∴ no we won't have enough to cover.

HMWK

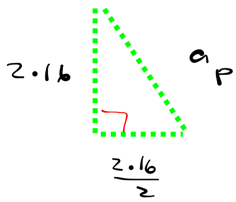
pg 91 #2.22 (great one)

pg 92 #2.24

pg 139 (top)

pg 143-4 #3.26-#3.27

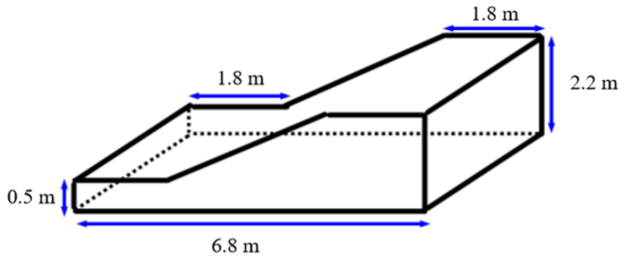
pg 115 (not a))



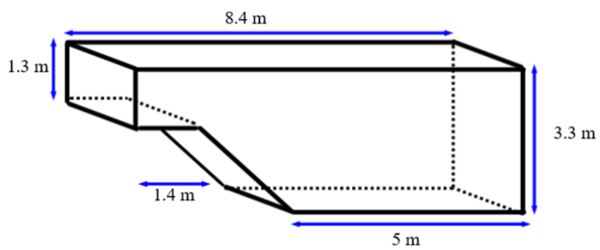
$c = \sqrt{2.16^2 + \left(\frac{2.16}{2}\right)^2}$

$c = 2.41$ ∴ apothem is 2.41 dm

2.1.3 Example: The below work of art will be placed in the garden of a museum in a section that measures $7\ m \times 2\ m$. The total amount of space that the work of art takes up is 27.54 kL. Is the garden section big enough to accommodate the structure?



2.1.4 Practice: A water tank holds a maximum capacity of 45 840 litres of water and must be placed in a section of a shed that measures $9\ m \times 4\ m$. Is the section of the shed big enough to accommodate the tank?



Answers:

2.1.3:

- missing measurement in structure: 3m
- therefore, the garden section ($7\ m \times 2\ m$) will NOT be big enough to accommodate the structure

2.1.4:

- missing measurement in water tank: 2m
- therefore, the section in shed $9\ m \times 4\ m$ will be big enough to accommodate the structure