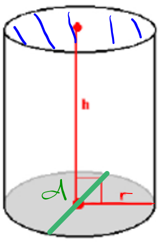
Lesson 11: Volume and Areas of Cylinders, Cones, and Spheres (plus mini-tasks)

June 7th, 2023

3.1 Volume of a Cylinder



d = diameter

$d = 2r$

$r = \frac{1}{2} \cdot d$

$$V = A_B \times h$$

$$A_{B\,\text{circle}} = \pi r^2$$

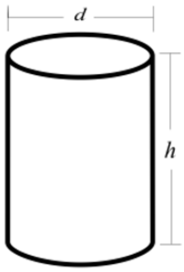
$$V = \pi r^2 \times h$$

$$V = \pi r^2 h$$

r = radius

h = height of prism

3.1.1 Example: Calculate the volume (in dL) of the following cylinder if $d = 4cm$ and $h = 10cm$:



$V = A_B \times h$

$V = \pi r^2 \times h$

$V = \pi (2)^2 \times 10$

$V = 40\pi \ cm^3$

$V \cong 125.66 \ cm^3$

$V = 125.66 \ mL$

$125.66 \div 10 \div 10$

$V = 1.2566 \ dL$

$h = 10cm$

$r = \frac{1}{2} d$

$r = \frac{1}{2} \cdot 4$

$r = 2$

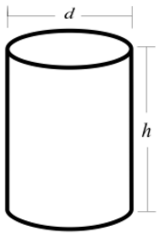
You do:

3.1.2

and

3.1.3

3.1.2 Practice: 3.1.1 Example: Calculate the volume (in kL) of the following cylinder if $d = 7dam\ and\ h = 13\ dam$:



$V = A_B \times h$

$V = \pi r^2 \times h$

$V = \pi\ 3.5^2 \times 13$

$V = 500.2986\ dam^3$

$\times 10^3$

$= 500\ 298.63\ m^3$

$= 500\ 298.63\ kL$

3.1.3 Practice: Calculate the capacity (in mL) of a cylinder if $r = 20cm\ and\ h = 75cm$:

$V = A_B \times h$

$V = \pi r^2 \times h$

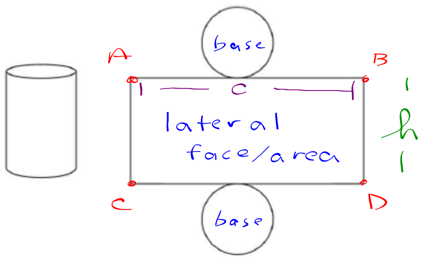
$V = \pi\ 20^2 \times 75$

$V = 94\ 247.7796\ cm^3$

$V = 94\ 247.7796\ mL$

Simplified of Total Area for Cylinder

3.2.1 Discovering the Surface Area of a Cylinder



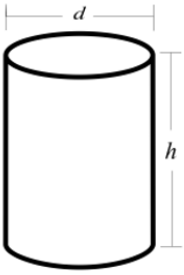
$$A_T = A_\ell + 2A_B$$

$$A_T = P_B \times h + 2A_B$$

$$A_T = 2\pi r \cdot h + 2\pi r^2$$

prism
cn 2
2 bases!

3.2.1 Example: Calculate the surface area (in cm^2) of the following cylinder if $d = 3\ cm$ and $h = 70\ mm$:



$= 70\,mm$
$= 7\,cm$

You do:

3.2.2
and
3.2.3

$$A_T = A_\ell + 2A_B$$

$$A_T = P_B \times h + 2A_B$$

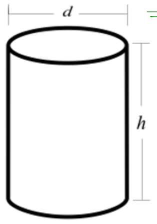
$$A_T = 2\pi r \times h + 2\pi r^2$$

$$A_T = 2\pi\left(\frac{3}{2}\right) \times 7 + 2\pi(1.5)^2$$

$$A_T = \frac{51\pi}{2}\ cm^2$$

$$A_T \simeq 80.11\ cm^2$$

3.2.2 Practice: Calculate the surface area (in m^2) of the following cylinder if $d = 1\ dam\ and\ h = 17\ m$:



$d = 1\ dam$

$= 10\ m$

$\therefore r = 5\ m$

$h = 17\ m$

$A_T = 691.1504\ m^2$

3.2.3 Practice: Calculate the surface area (in mm^2) of a cylinder if $r = 25\ mm\ and\ h = 7\ cm$:

$h = 7\ cm$

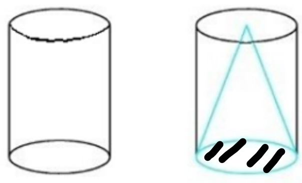
$h = 70\ mm$

$A = 14\ 922.5651\ mm^2$

Simplified formula for cone (pyramid)

3 The Volume of Cones

3.1 Thinking about the Volume of a Cone Relative to a Cylinder

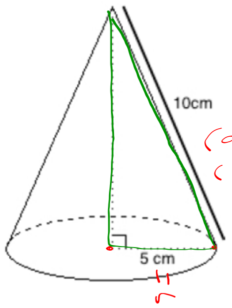


$V = A_B \times h$ $V = \frac{A_B \times h}{3}$

$$V = \frac{\pi r^2 \cdot h}{3}$$

3.2.1 Example

Calculate the volume of the following cone:

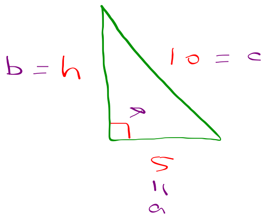


= a (apothem) (slanted height)

$h = ?$

Find it with:

$c^2 = a^2 + b^2$



$10^2 = 5^2 + h^2$

$100 - 25 = \cancel{25} - \cancel{25} + h^2$

$\sqrt{75} = \sqrt{h^{\cancel{2}}}$

$h = \sqrt{75}$

$h = 5\sqrt{3}$

$h \cong 8.66 \text{ cm}$

$\sqrt{75}$

$= \sqrt[2]{(25 \times 3)}$ $\frac{1}{2}$

$= (25 \times 3)^{1/2}$

$= 25^{1/2} \times 3^{1/2}$

$= \sqrt{25} \times \sqrt{3}$

$= 5\sqrt{3}$

$(ab)^n = a^n b^n$

$V = \dfrac{A_B \times h}{3}$

label info

$V = \dfrac{\pi r^2 \cdot h}{3}$

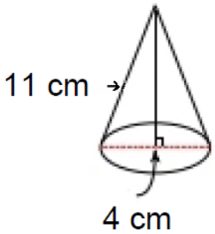
$V = \dfrac{\pi (5)^2 \times 8.66}{3}$

$V = 226.72 \text{ cm}^3$

you do pg 3 3.2.2 3.2.3

3.2.2 Example

Calculate the volume of the following cone:

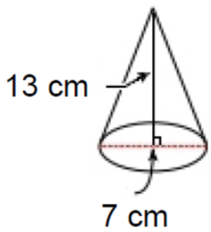


$h = 10.82$ cm

$V = 45.32\ cm^3$

3.2.3 Practice

Calculate the volume of the following cone

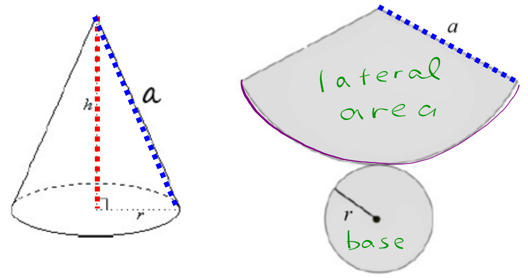


$V = 166.77\ cm^3$

Simplify the Area Formula for cone.

4 The Surface Area of Cones

4.1 Thinking about the Total Area and Lateral Area of Cones



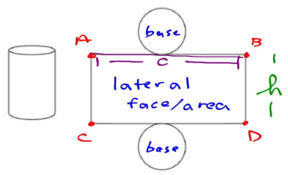
$$A_T = A_\ell + A_B$$

$$A_T = \frac{P_B \times a}{2} + A_B$$

$$A_T = \frac{\cancel{2}\pi r \cdot a}{\cancel{2}} + \pi r^2$$

$$A_T = \pi r \cdot a + \pi r^2$$

a = apothem



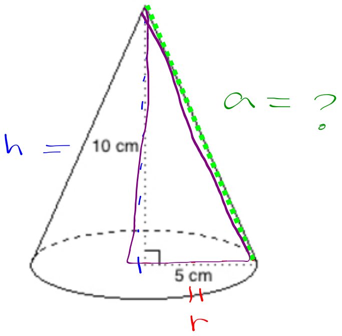
$$A_T = A_\ell + 2A_B$$

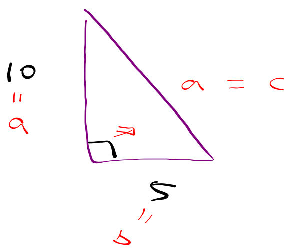
$$A_T = P_B \times h + 2A_B$$

$$A_T = 2\pi r \cdot h + 2\pi r^2$$

4.2.1 Example

Calculate the surface area of the following cone





$$A_T = A_L + A_B$$

$$A_T = \frac{P_B \times a}{2} + A_B$$

$$A_T = \frac{2\pi r \times a}{2} + \pi r^2$$

$$c^2 = a^2 + b^2$$

$$a_p^2 = 10^2 + 5^2$$

$$a_p = \sqrt{(100 + 25)}$$

$$a_p = 11.18 \text{ cm}$$

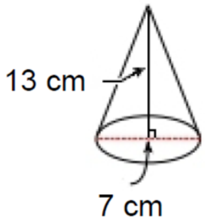
$$A_T = \left(\frac{2\pi(5) \times 11.18}{2}\right) + \pi 5^2$$

$$A_T = 254.15 \text{ cm}^2$$

You do
4.2.2
4.2.3

4.2.2 Practice

Calculate the surface area of the following cone



$r = 3.5$ cm

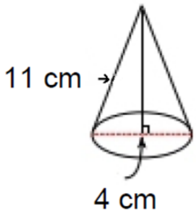
$a \cong 13.46$ cm

slant height

$A_T = 186.48 \text{ cm}^2$

4.2.3 Practice

Calculate the surface area of the following cone:



$r = 2$ cm

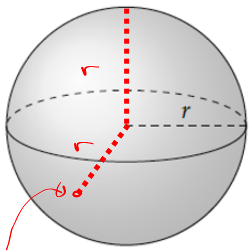
$a = 11$ cm

$A = 81.68 \text{ cm}^2$

check answer and take early break! back @ 10:45

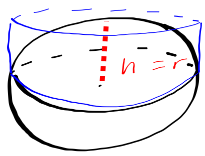
Discovering the Volume of a sphere

1.1 The Volume of a Sphere



point on the 3D surface

$$V = \frac{4\pi r^3}{3}$$



$V_B = A_B \times h$

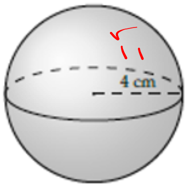
$V_B = \pi r^2 \times r$

$V_{Blue} = \pi r^3$

The sphere is $\frac{4}{3}$ greater than blue cylinder.

1.1.1 Example

Calculate the volume of the following sphere.



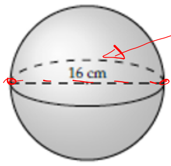
$$V = \frac{4\pi r^3}{3}$$

$$V = \frac{(4\pi(4)^3)}{3}$$

$$V = 268.08\,cm^3$$

1.1.3 Practice

Calculate the volume of the following sphere.



$d = 16$

$\therefore r = \frac{16}{2}$

$r = 8\,cm$

$$V = \frac{4\pi r^3}{3}$$

$$V = \frac{4\pi(8)^3}{3}$$

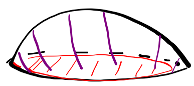
$$V = 2144.66\,cm^3$$

Discovery the Area Formula

1.2 The Surface Area of a Sphere

$$A_T = 4\pi r^2$$

half sphere



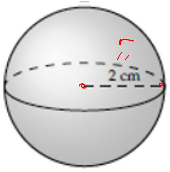
$A_B = \pi r^2$

$= 2\pi r^2$

the surface is double area of base!

1.2.1 Example

Calculate the surface area of the following sphere



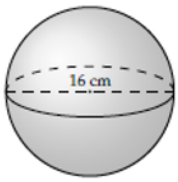
$A_T = 4\pi r^2$

$A_T = 4\pi(2)^2$

$= 50.27\,cm^2$

1.2.2 Practice

Calculate the surface area of the following sphere.

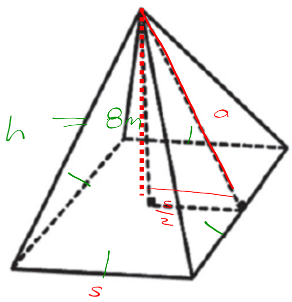


$A_T = 804.25\,cm^2$

Mini - Tasks

(lesson 10's handout last pg)

3.1.1 Example: Robin is now making the actual tent that is in the shape of a square pyramid whose volume is 180 000 daL. He buys 2.5 bags of canvas. It cost him \$50.60 per bag and each one contains 600 m^2 of canvas. If the height of the pyramid is 8 m, will the 2.5 bags be enough to make his tent?



(what math concept/thing do we want)

③ WANT: (volume) (m^2)

(area) amount (area) of canvas we have ; total area of pyramid

TOOL:

2.5 bags × 600 $\frac{m^2}{bag}$

1500 m^2

$A_T = A_\ell + A_B$

$A_T = \frac{P_B \times a}{2} + s^2$

$A_T = \frac{4s \times a}{2} + s^2$

Info: $s = ?$

$a = ?$

Find s:

$V = \frac{s^2 \times h}{3}$

$1800 \times 3 = \left(\frac{s^2 \times 8}{\cancel{3}}\right) \cancel{3}$

$\frac{5400}{8} = \frac{s^2 \times 8}{8}$

B.E.D.M.A.S

$\sqrt{675} = \sqrt{s^2}$

$s = 15\sqrt{3}$

$s \doteq 25.98$ ✓

① WANT $s = ?$

TOOL: 1 eq.

$V = \frac{A_B \times h}{3}$

$V = \frac{s^2 \times h}{3}$

INFO $h = 8\ m$

$V = 180\,000\ daL$

÷10 ÷10 ÷10

$= 1800\ kL$

$V = 1800\ m^3$

② WANT $a = ?$ (apothem) 1 unk

TOOL: 1 unk

$c^2 = a^2 + b^2$

convert ↓ height = b = 8 ; apothem c ; $\frac{s}{2}$ = a

INFO $h = 8$, $s = ?$

② Find apothem

a = 8 m ; $a_p = c$; 12.99 = b

$\frac{s}{2} = \frac{25.98}{2}$

$c^2 = a^2 + b^2$

$\sqrt{c^2} = \sqrt{(8^2 + 12.99^2)}$

$a_{pothem} = 15.256\ m$

③

$A_T = \frac{4s \times a}{2} + s^2$

$A_T = \frac{4(25.98) \times 15.256}{2} + 25.98^2$

$A_T = 1467.66\ m^2$

~~No~~ Yes the ~~won't~~ will have enough since $1467.66\ m^2 < 1500\ m^2$

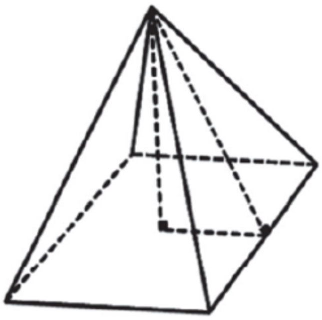
what we need (dimensions of tent) ; what we have

Last hour:

- review this example
- try ex 3.1.2
- try ex 3.1.3

HMWK:
page 184 (not #4.17)
page 186
page 220 #5.11
BONUS: p170 and p 221 #5.16

3.1.2 Practice: Robin also makes the actual thriftier tent still in the shape of a square ~~prism~~ pyramid with reduces measurements. The tent has a capacity of 16 000 hL. He buys 3.5 bags of plastic tent material and 2 highlighters. If each bag contains $9m^2$ of plastic and one side of the base is 18 m, will he have enough to make his tent?



$h = 14.814\ m$

apothem $= 17.33\ m$

$A_T = 948.04\ m^2$

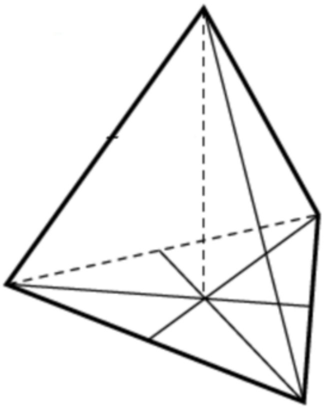
(what you need to cover)

Robin $3.5 \text{ bags} \times 9 \frac{m^2}{\text{bag}}$

$= 31.5\ m^2$

no he won't get

3.1.3 Example: The architect is now making the actual triangular pyramid whose roof he will shingle with 3.5 bags of shingles. The total surface area of the pyramid is $96.588m^2$. Each bag covers $25m^2$ and contains 28 shingles. If the apothem of the pyramid is $9\ m$ and the height of the base is $3\sqrt{3}\ m$ $(or\ 5.196m)$, will the 3.5 bags be enough to shingle the roof?



side base = 6 m

$A_\ell = 81\ m^2$
(what you need to cover)

He bought $87.5\ m^2$ of shingles

Yes he has enough.