Lesson 7: Representations of 3D solids using Projections (5 different types)

May 31st, 2023

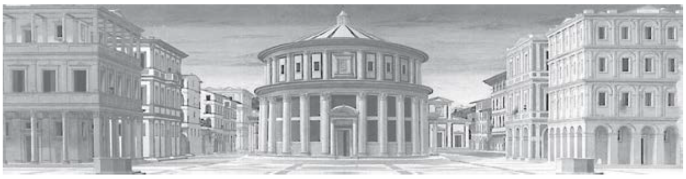
definition: projection – a drawing of a 3D object (solid) on a 2D plane (i.e. a piece of paper)

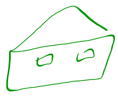
- 2 types of central Projections

#1. Perspective w̄ one Vanishing Point.

ex. pg 62

Here is a reproduction of a painting from the 15th century that has long been credited to Italian painter Piero della Francesca and entitled *Ideal City*. Thanks to his rigourous use of perspective, he was able to achieve a remarkable illusion of depth.





· look flat

← illusion of depth.

How?

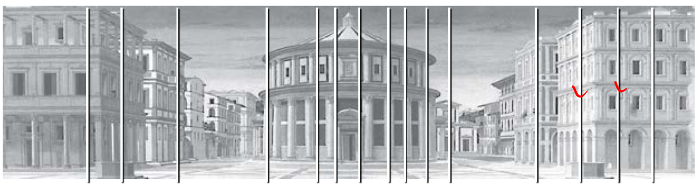
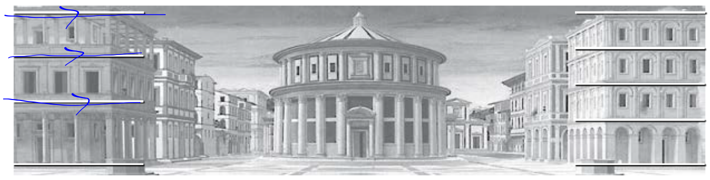


Figure 2.3

← all vertical lines are parallel



← some horizontal lines are parallel (in the drawing)

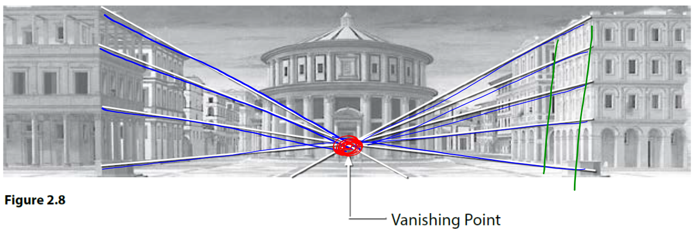
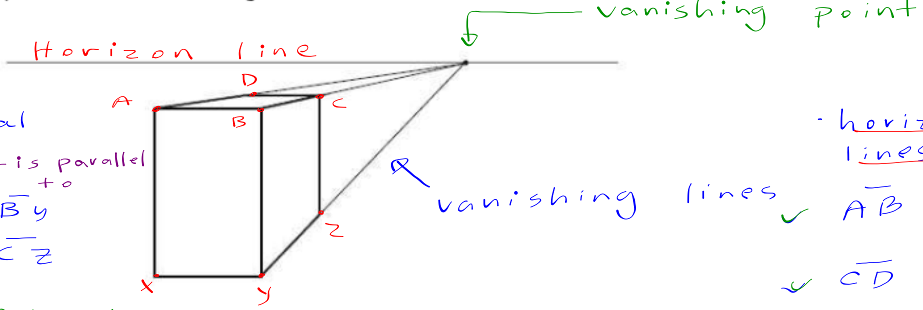


Figure 2.8

← horizontal lines on non-front face of solid are not parallel in drawing (even tho parallel in reality)

Terminology:

1.1 Perspective With One Vanishing Point



· vertical lines is parallel to

$\overline{AX} \parallel \overline{BY}$

$\overline{BY} \parallel \overline{CZ}$

line segment

· horizontal lines

✓ $\overline{AB} \parallel \overline{XY}$

✓ $\overline{CD} \parallel \overline{AB}$

BUT!

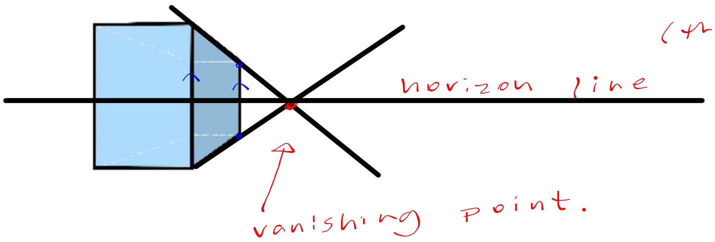
$\overline{BC}$ not $\parallel \overline{YZ}$

(tho in reality $\overline{BC} \parallel \overline{YZ}$)

1.1.1 Example

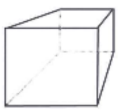
Determine the vanishing point and horizon line for the following prism:

· extend the non-parallel horizontal lines.



1.1.2 Practice

Determine the vanishing point and horizon line for the following prism:

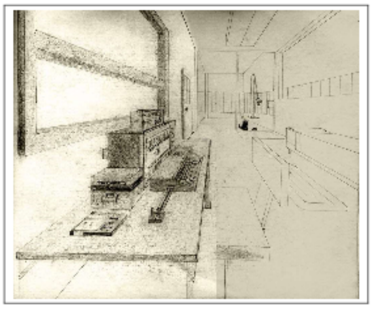


You do pg 2/3/4

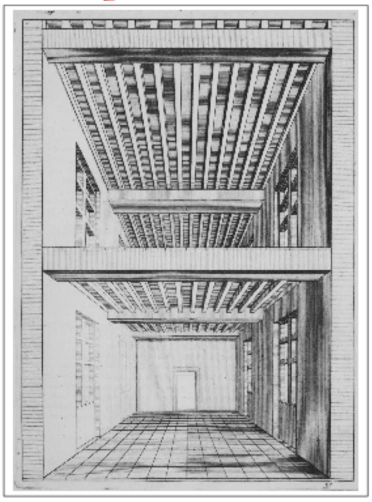
1.1.3 Practice

For each of the following images, determine the vanishing point and horizon line:

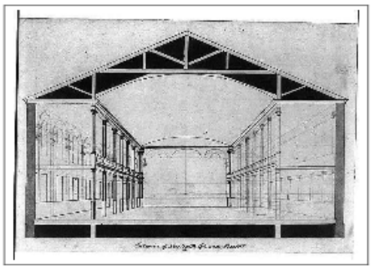
#1



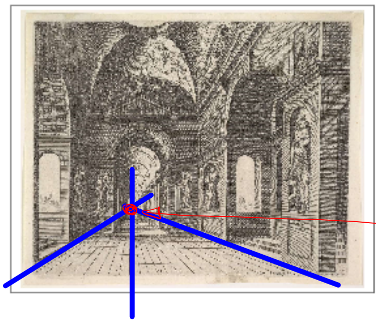
#2



#3



#4

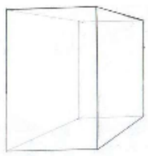


vanishing point which our eyes are drawn to.

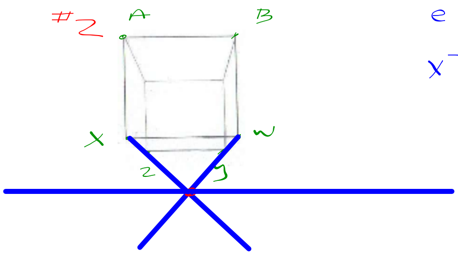
1.1.4 Practice

For each of the following drawings, determine the vanishing point and horizon line:

#1



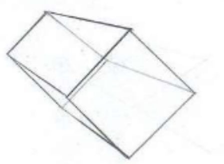
#2



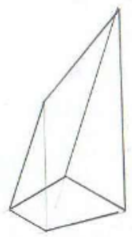
extend $\overline{XZ}$ and $\overline{WY}$

#3

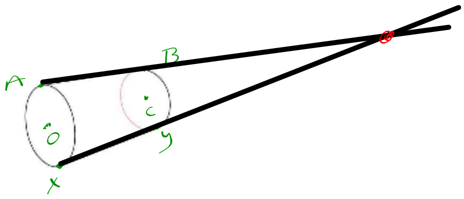
#4



#5



#6

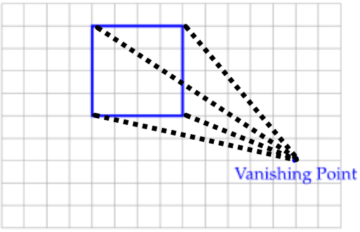


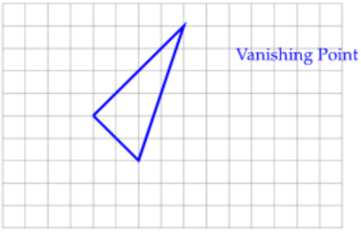
extend $\overline{AB}$ $\overline{XY}$

Drawing the 3D solid on the 2D w̄ central Proj. w̄ one v. point

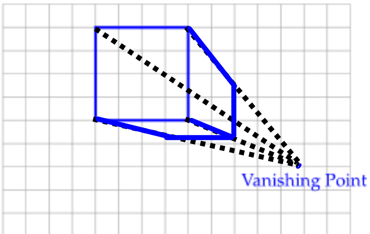
1.1.5 Example

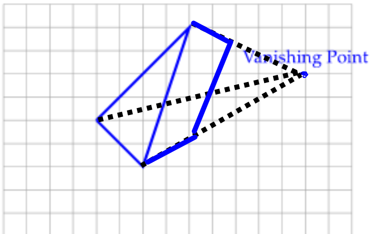
Complete the drawing of the following prisms with one perspective point (the depth of the prism in this case is up to you, there are many possibilities).





Complete the drawing of the following prisms with one perspective point (the depth of the prism in this case is up to you, there are many possibilities).





step i (done)
- draw front face un-distorted.
- pick v. point (where you want eye level)

step ii. from each vertex of face draw dotted v. lines to v. point.

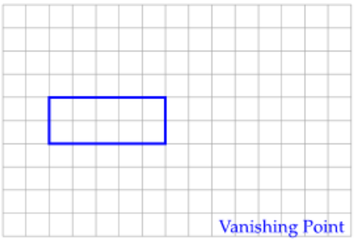
step iii. Draw a (vertical) line parallel to front face limited to correct plane.

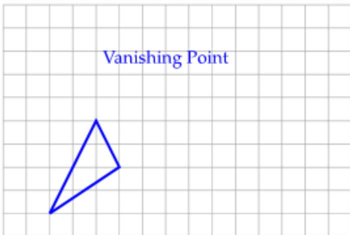
step iv. Connect the vertices

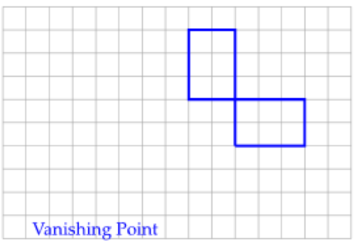
You do ↳

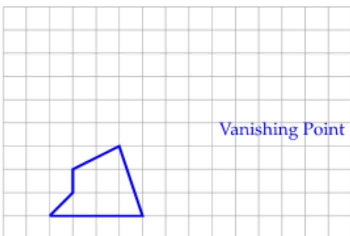
1.1.6 Practice

Complete the drawing of the following prisms with one perspective point (the depth of the prism in this case is up to you, there are many possibilities).





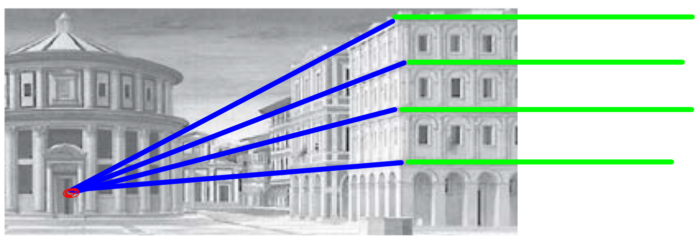




2nd Central Projection

A perspective with 2 vanishing Points

1 v. point!



- straight on front view
- front facade/face is ∥ to drawing

Figure 2.4

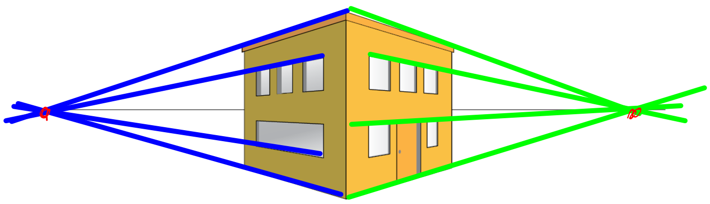


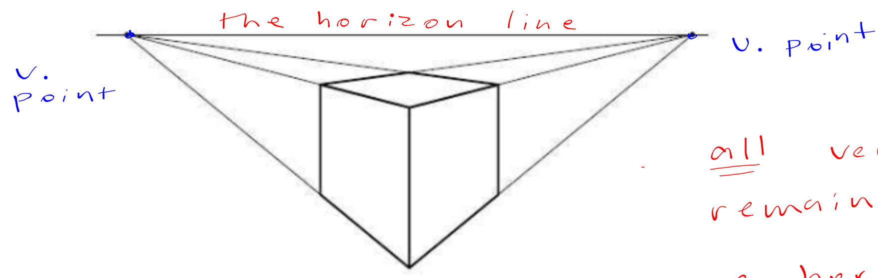
Figure 2.5

2 vanishing points!

What's the difference? (pg 67 recaps the difference)

→ no face of solid is ∥ to drawing.

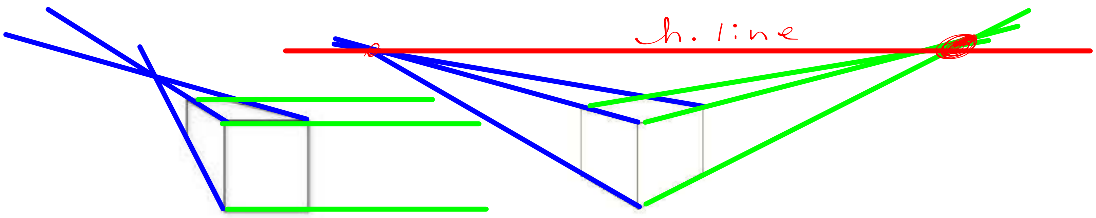
Perspective with Two Vanishing Points This perspective has two vanishing points.



- all vertical line remain parallel
- no horizontal line is parallel

1.2.1 Example

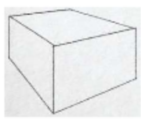
Determine whether the following prisms have one or two vanishing points:



1.2.2 Example

Determine the two vanishing points and horizon for the following object:

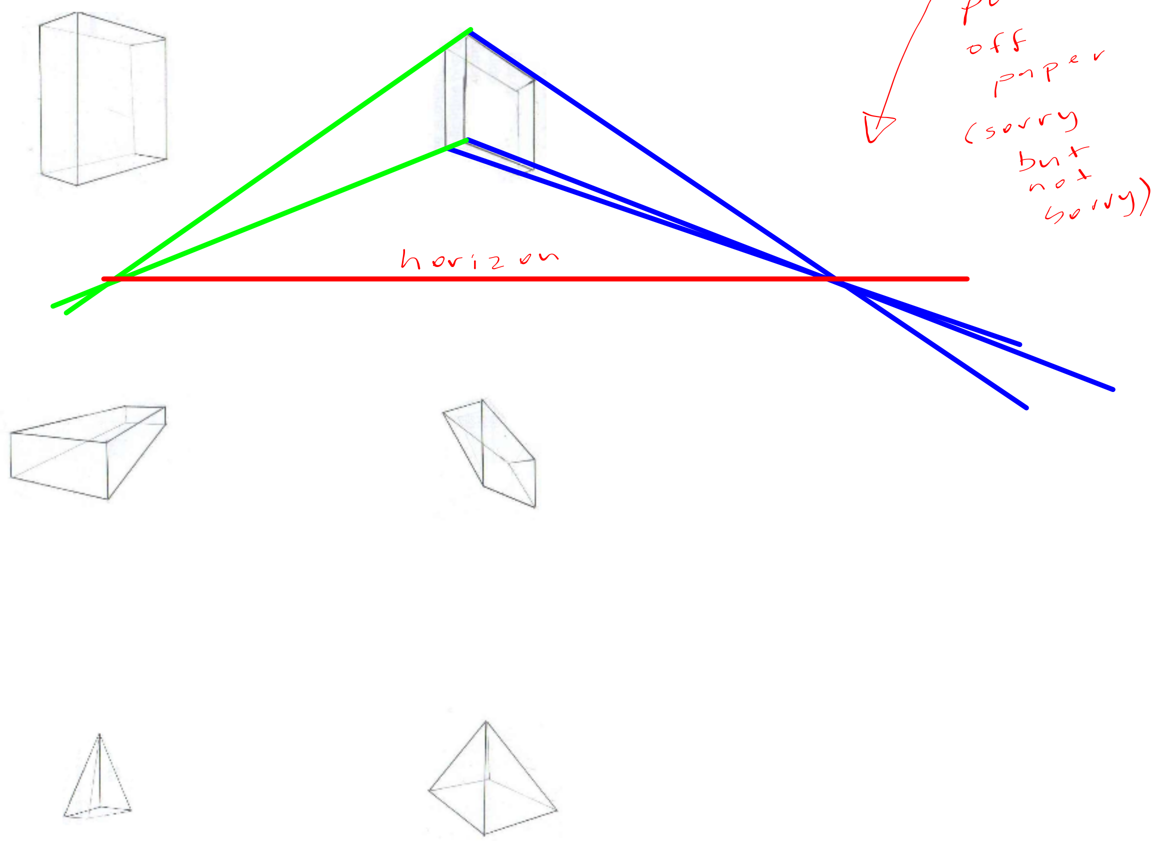
nota: the horizon line must go thru both v. points.



you do and pg 8!

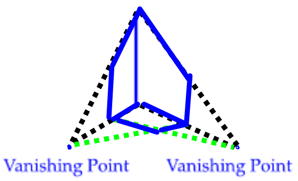
1.2.3 Practice

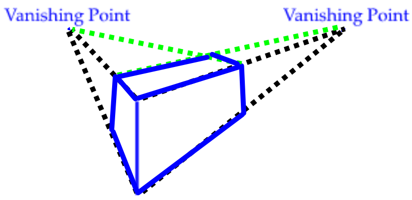
For each of the following drawings, determine the vanishing points and horizon line:



1.2.4 Example

Draw rectangular prisms with two vanishing points (the depth is up to you!):

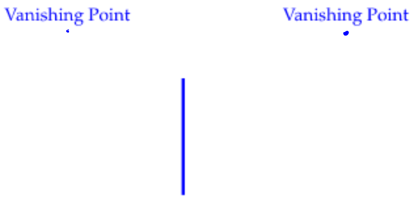




1.2.5 Practice

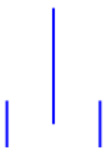
You do the next three!

Draw rectangular prisms with two vanishing points (the depth is up to you!):



1.2.6 Practice

The foll w g is triangul r prism wi h two vanishing p ts. Complete he drawing by firs identifying the vanishing points and horizon line.



step i (done)
· one vertical line, and 2 v. points.
step ii draw dotted lines from end points of line to each v. point.
step iii. draw other || vertical line w/in the dots
step IV. connect the endpoints of new vertical line to v. point.
step V. connect vertices

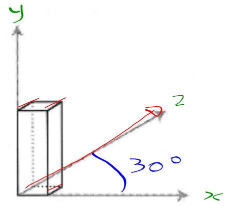
5 projections in total

2 Parallel Projections — 2 types — All vertical AND horizontal line remain ∥ in drawing.

In parallel projections, the parallel dimensions of the object *remain* parallel in their projected form.

1st type:

2.1 Cavalier (Oblique) Projection



Conventions:

z — the oblique line (niether ∥ nor ⊥)

- The depth axis forms a 30° or 45° angle with the horizontal. → good w graph paper
- The receding edges (the depth edges) are reduced in size by approximately 50%. or 60%
- The non-visible edges of the prism are drawn with dotted lines.

2.1.1 Example

Represent a rectangular prism whose real dimensions measure 1cm × 1cm × 2cm (depth) in cavalier (oblique) perspective:

stays the same since in the front face

must be reduced: 2 cm = 1 cm (50%)

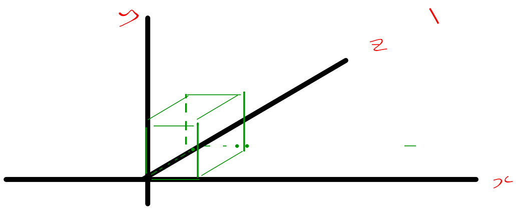
Pre-step: reduce depth measure by 50%

step i. make axis

step ii. draw the front 2D face according to measures (not depth) (from origin)

step iii: draw the depth line segments @ reduced measure ∥ to depth axis.

step iv. connect vertices



2.1.2 Practice

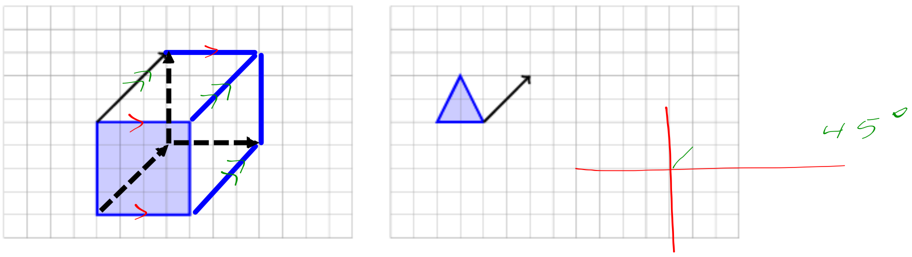
Represent a cube whose real dimensions measure 2cm × 2cm × 2cm in cavalier (oblique) perspective

- You do →
- Bonus: start pg 11

2.1.3 Example: Oblique Perspective as a Translation

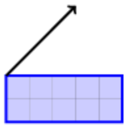
We can also obtain an oblique projection by translating a polygon in a grid. The translation arrow (vector) is usually 45° in a given direction.

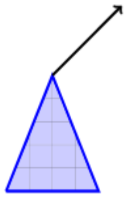
Complete the drawing of the following oblique projections, given the translation arrow:

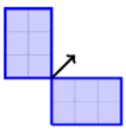


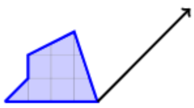
2.1.4 Practice

Complete the drawing of the following oblique projections, given the translation arrow:

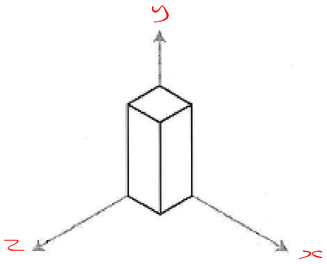






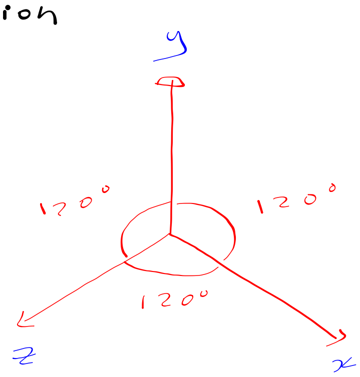


2nd type of Parallel Projection



Conventions:

- The axes meet at 120° angles.
- All the segments that are congruent in reality and that are parallel to one of the axes remain congruent in the representation.



2nd typo

2.2 Isometric (Axonometric) Projection

Isometric projections are parallel projections in which congruent dimensions of the original object (in reality) remain congruent in the representation.

2.2.1 Example

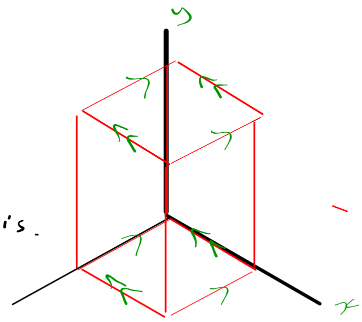
Represent a rectangular prism with a base measuring 2cm × 2cm and a height of 3cm

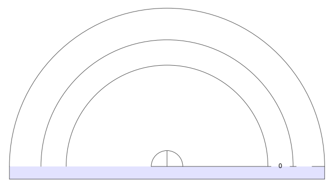
|| to other axis

y axis

→ eye level always above solid

- start w/ base w/ dimensions || to x / z axis.
- then draw heights || to y axis





2.2.2 Practice

Represent a cube with a side measuring 2cm.

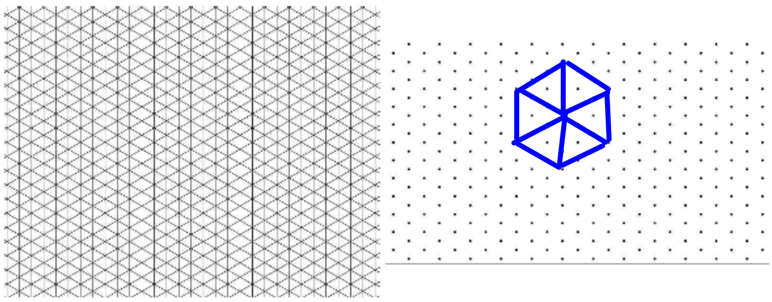
You do

try to draw on cube on pg 13 w/ the isometric graph paper

2.2.3 Example: Using Isometric Grid Paper/Dots

→ 120°

Isometric projections are easier to draw when using isometric grids. There are two types (see below):

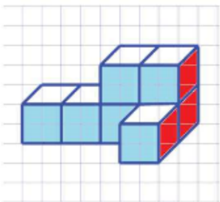


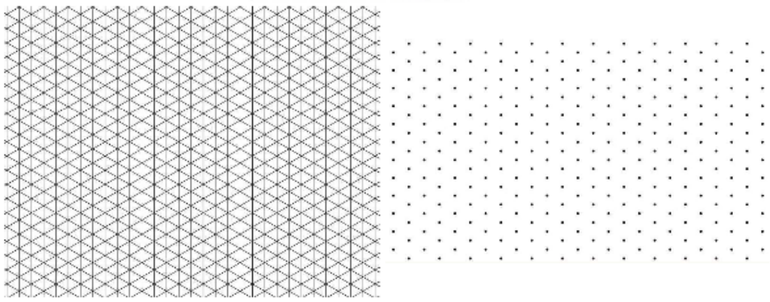
Looks like a different approach when using graph paper. For example, better not to draw the lines behind.

Q: How would you draw a cube using either grid? Draw a cube in each grid above.

2.2.4 Practice

Reproduce the following oblique representation in isometric representation in each grid below:



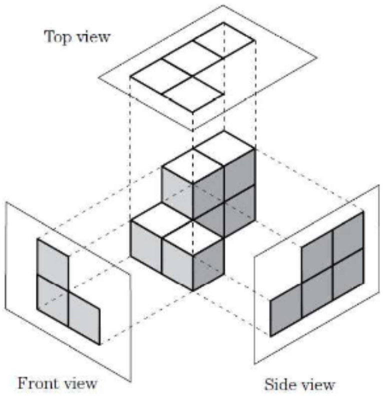


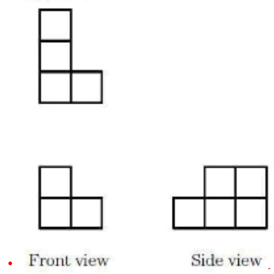
5 different Projections

3 Orthogonal Projection

With Orthogonal projections, we actually project a 3D object onto a 3 2D plane.

Orthogonal Projection:



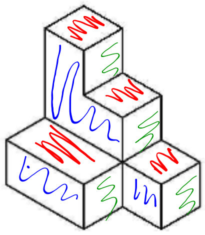


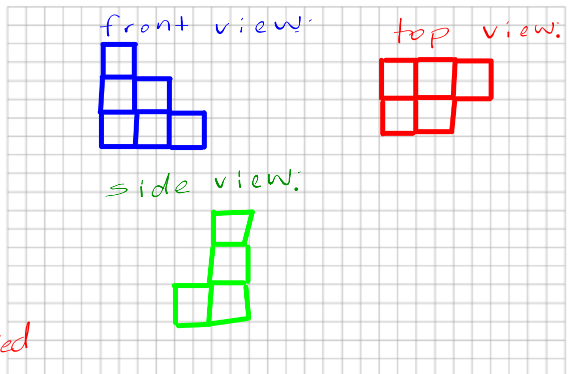
3 flat views

3.1 Example

Represent the top, front and side view of the following object:

Top - red
Front - blue
Side - green



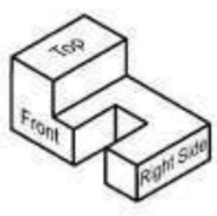


okay to assume their cubes stacked

3.1.1 Practice

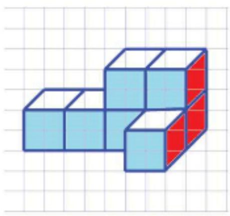
You do (answer key on teams)

Represent the top, front and side view of the following object:



3.1.2 Practice

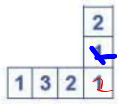
Represent the top, front and side view of the following object:



3.1.3 Example: Drawing from Coded Blue Print

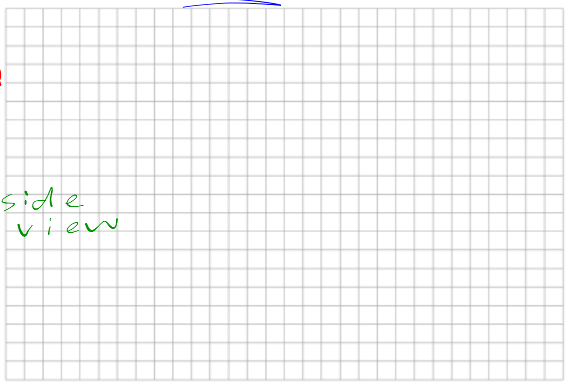
Represent the front and side view of the object with the following coded blue print as well as the entire object in cavalier(oblique) projection and isometric projection:

the # of cubed stacked on each other.

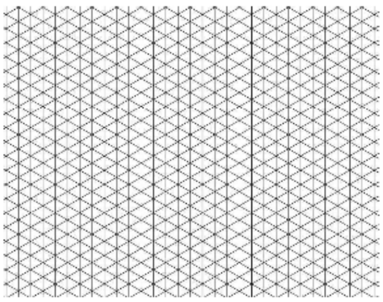


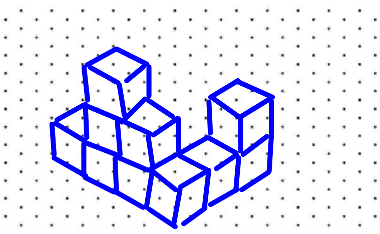
side view

front side



tip: start in middle.





HMWK:

pg 68

pg 69 #2.8 a)-c)

pg 77

pg 78 (not #2.15)

pg 83

pg 85

(answer key for last examples that you'll practice will be uploaded to teams)

3.1.4 Practice

Represent the front and side view of the object with the following coded blue print as well as the entire object in cavalier(oblique) projection and isometric projection:

3	4	3	2
3			
2			
1			

4 Homework

- MHS Workbook p. 375-386
- Geometry Worksheets 4