Lesson 6 : 2 - dimensional polygons

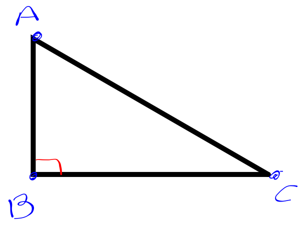
2D

May 30
2023

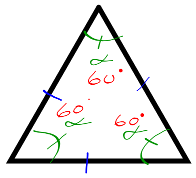
and · Pythagoras Theorem

· Pythagorean Theorem : $c^2 = a^2 + b^2$

Terminology : a right triangle



e.x. equilateral triangle (not a right triangle)



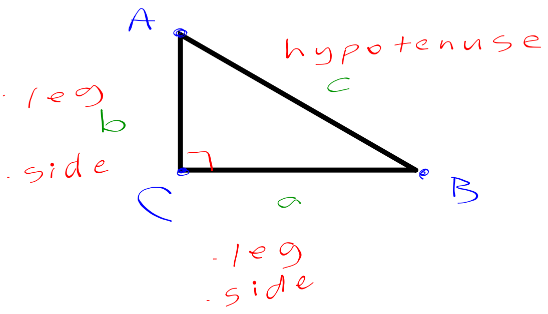
∠ B = 90°
∠ABC = 90°
} a right angle

$\alpha + \alpha + \alpha = 180°$

$\frac{3\alpha}{3} = \frac{180}{3}$

$\alpha = 60°$

The sides of a Right Triangle :
(line segment)



measurements

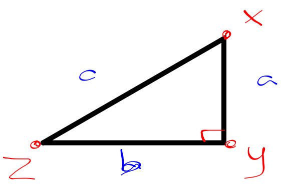
$m\overline{AB} = 2.7$ units

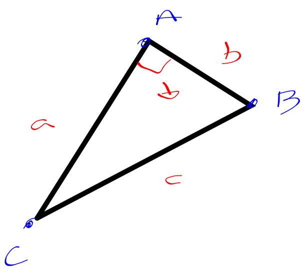
$m\overline{BC} = 4.8$ units

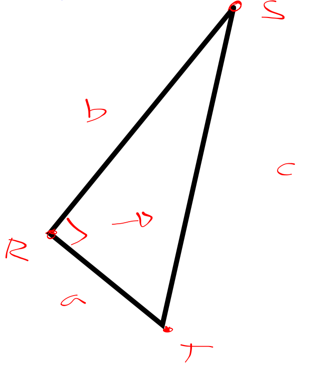
$m\overline{AC} = 5.5$ units

nota bene : · the hypotenuse is the longest side in a right triangle.

· the hypo is the side opposite the 90°.
(side doesn't touch angle)







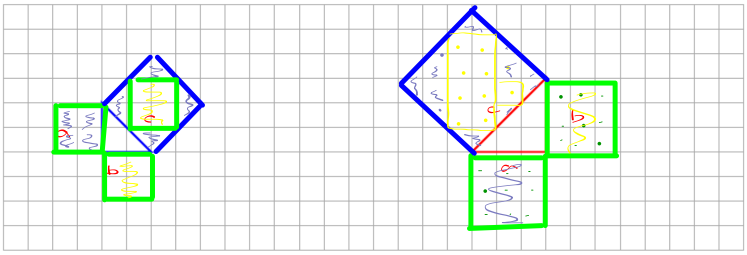
Label the traingles w̄ a, b, c.

the other sides

hypotenuse

Discovering Pythagoras Theorem

Consider the following right-angled triangles in the grid below. Is there are any relationship between the sides and the squares formed by those sides?

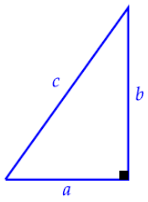


$a^2 + b^2 = c^2$

$2^2 + 2^2 = c^2$

$2 \times 2 + 2 \times 2 =$

The Pythagorean Theorem



For any **right triangle**:

$$a^2 + b^2 = c^2$$

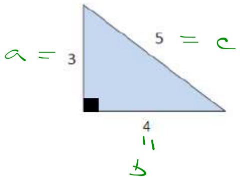
Where c is the *hypotenuse* of the right triangle.

$a^2 + b^2 = c^2$

$a \times a + b \times b = c \times c$

1.1.1 Example

Verify the Pythagorean theorem for the following triangle: (verify it's a right triangle)



$c^2 = a^2 + b^2$

$5^2 = 3^2 + 4^2$

$25 = 9 + 16$

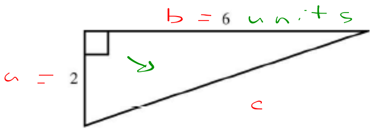
$25 = 25$

✲ $c^2 = a^2 + b^2$ to find hypo (c)

1.2 Applications of the Pythagorean Theorem

1.2.1 Example

Use the Pythagorean theorem to determine the measure of the hypotenuse of the following triangle:



$c^2 = a^2 + b^2$

$c^2 = 2^2 + 6^2$

$c^2 = 4 + 36$

$\sqrt{c^2} = \sqrt{40}$

c = 6.32 units

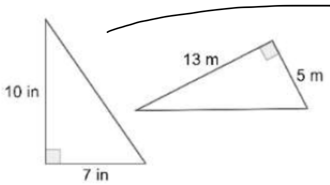
- label triangle
- sub in known values.
- simplify
- solve for c $\bar{w}$ o.o. to both sides.

ANS: 6.32

You do 1.2.2 / 1.2.3 (Bonus: 1.2.4)

1.2.2 Practice

Use the Pythagorean theorem to determine the hypotenuse of the following triangles:



$a^2 + b^2 = c^2$

$7^2 + 10^2 = c^2$

$c = \sqrt{149}$

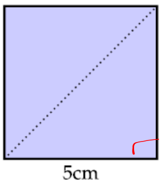
$c = 12.20$ ✗ −1

$c = 12.21$ ✓

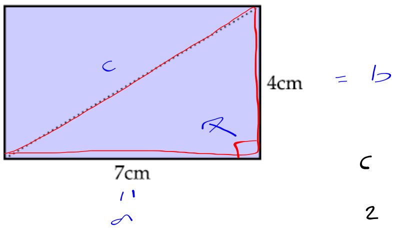
$c = 13.93$

1.2.3 Practice

Use the Pythagorean theorem to determine the measure of the *diagonals* of the following square and rectangle. Round your answer to the nearest hundredth, if necessary:



$c = 7.07\text{cm}$



$c^2 = a^2 + b^2$

$c^2 = 7^2 + 4^2$

$= 49 + 16$

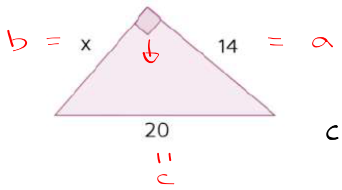
$\sqrt{c^2} = \sqrt{65}$

$c = 8.06\text{cm}$

⊕ $c^2 = a^2 + b^2$ to find sides/legs (a or b)

1.2.4 Example

Use the Pythagorean theorem to find the missing side of the following triangle:



$$c^2 = a^2 + b^2$$

$$20^2 = 14^2 + b^2$$

$$400^{-196} = \cancel{196}^{\cancel{-196}} + b^2$$

$$\sqrt{204} = \sqrt{b^{\cancel{2}}}$$

$$b = 14.28 \text{ units}$$

④ Brackets
③ Exponents
② { Division
multiplication
① { Addition
Substraction

Solving

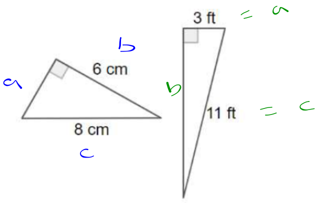
• opposite operation

You do 1.2.5

Bonus 1.2.6

1.2.5 Practice

Use the Pythagorean theorem to find the missing side of the following triangles:

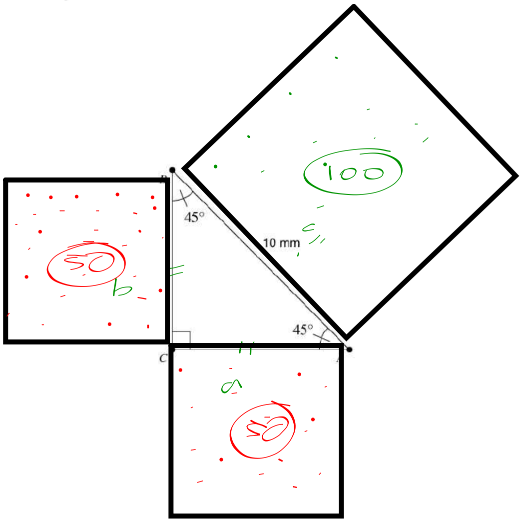


$b = 10.58$ ft

a
$= 5.29$ cm

1.2.6 Example

Use the Pythagorean theorem to find the missing sides of the following *isosceles* triangle. Round your answer to the nearest tenth of a *mm*.



$c^2 = a^2 + b^2$

$10^2 = a^2 + b^2$

100

$\sqrt{b^2} = \sqrt{50}$

$b = 7.07$

$b = 7.1 = a$

$b = 7.1$

since ② $\boxed{a = b}$

sub value of a into ①

$c^2 = a^2 + b^2$

$10^2 = a^2 + b^2$

① $100 = a^2 + b^2$

$100 = b^2 + b^2$

$\frac{100}{2} = \frac{\cancel{2} \cdot b^2}{\cancel{2}}$

$\sqrt{50} = \sqrt{b^2}$

$b = 7.1$ mm

You do 1.2.7

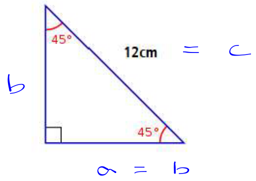
and 1.2.8

(refer to early notes)

$\boxed{\text{Bonus 1.2.9}}$

1.2.7 Practice

Use the Pythagorean theorem to find the missing sides of the following *isosceles* triangle. Round your answer to the nearest tenth of a *cm*.



b = 8.5 cm

1.2.8 Example

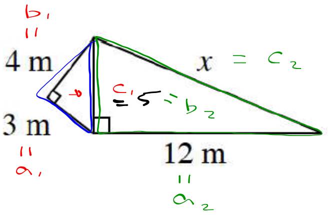
If a triangle has sides measuring $7cm$, $5cm$, and $3cm$, determine whether or not it is a *right* triangle.

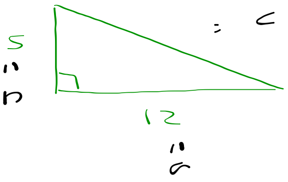
no ... but must justify for full marks ☺

Tip: start w/ triangle you have most info for

1.2.9 Example

Determine the measure of x in the following:





blue small triangle

$c^2 = a^2 + b^2$

$c^2 = 3^2 + 4^2$

$c^2 = 9 + 16$

$\sqrt{c^2} = \sqrt{25}$

$c = 5$

green big triangle

$c^2 = a^2 + b^2$

$c^2 = 12^2 + 5^2$

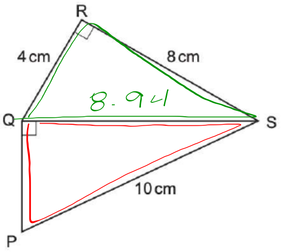
$\sqrt{c^2} = \sqrt{169}$

$c = 13$

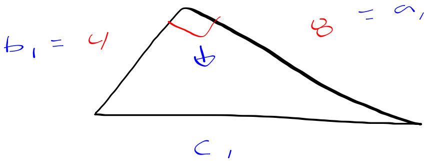
You do:

1.2.10 Practice

Determine the measure of $\overline{QP}$:



green Δ



1st time:

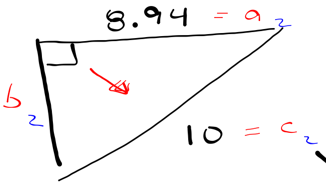
$c^2 = a^2 + b^2$

$c^2 = 8^2 + 4^2$

$c^2 = 80$ preferred answer $= a_2$

$c_1 = 8.94 = a_2$

red 2nd triangle



2nd time

$c^2 = a^2 + b^2$

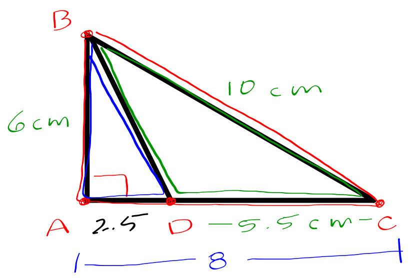
$10^2 = 8.94^2 + b^2$

$100 - 80 = 80 - 80 + b^2$

$\sqrt{40} = \sqrt{b^{\cancel{2}}}$

$b = 4.47$ cm

AND Question 1.2.11



Given:

$m\overline{AB} = 6$ cm

$m\overline{BC} = 10$ cm

$m\overline{DC} = 5.5$ cm

Find

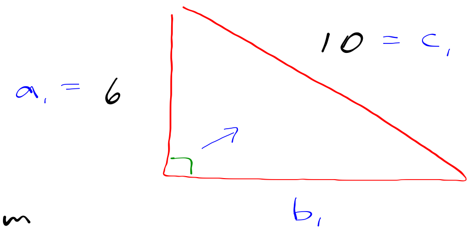
$m\overline{BD}$

$\overline{AD} = \overline{AC} - \overline{DC}$

$= 8 - 5.5$

$\overline{AD} = 2.5$

· 1st biggest triangle

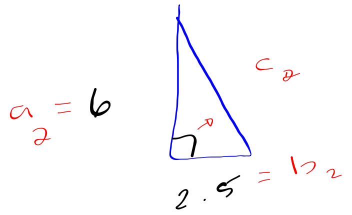


$c^2 = a^2 + b^2$

$\overset{-36}{100} = \overset{-36}{36} + b^2$

$b = 8$ units.

· 2nd smallest triangle



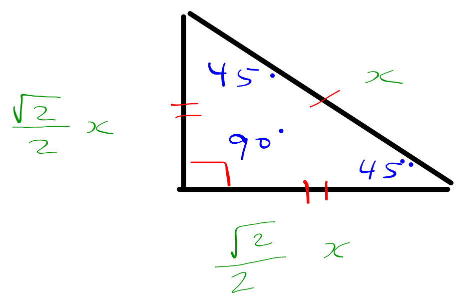
$c^2 = a^2 + b^2$

$c = \sqrt{(6^2 + 2.5^2)}$

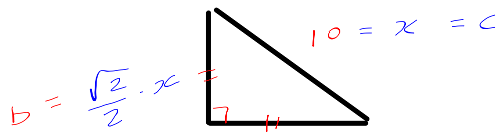
$c = 6.5$ units

2 Special Right Triangles

#1. The Isoceles 45°-45°-90° △



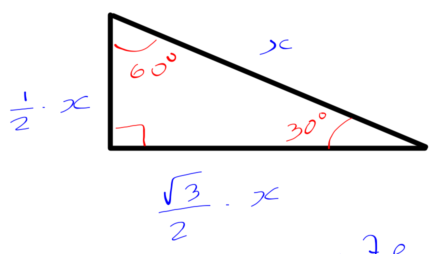
e.x.



$$b = \frac{\sqrt{2}}{2} \times 10$$

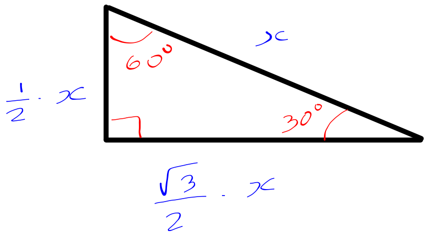
$$b = 7.07$$

#2. The 30°-60°-90° triangle

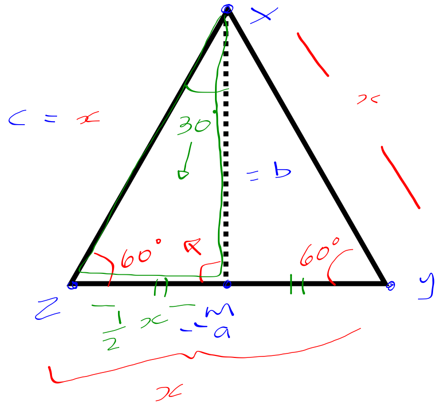


~~✱~~ the side opposite the 30° is half the hypo.

Prove the following ratios :



start $\bar{w}$ equilateral Δ



$\overline{xm}$ - height

* a height in an equilateral or isosceles Δ is also a ⊥ bisect.

#6 $(ab)^n = a^n b^n$

#7 $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$$c^2 = a^2 + b^2$$

$$x^2 = \left(\frac{1}{2}x\right)^2 + b^2$$

$$-\frac{1}{4}x \qquad -\frac{1}{4}x^2$$

$$x^2 = \frac{1}{4}\cancel{x^2} + b^2$$

$$b^2 = \frac{4}{4}\cdot x^2 - \frac{1}{4}x^2$$

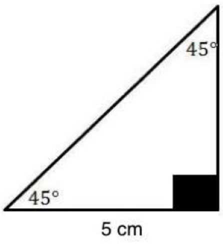
$$\sqrt{b^{\cancel{2}}} = \sqrt{\frac{3}{4}x^2}$$

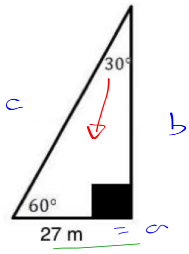
$$b = \sqrt{\frac{3}{4}} \cdot \sqrt{x^{\cancel{2}}}$$

$$b = \frac{\sqrt{3}}{2}x$$

1.3.1 Example

Determine the missing measurements of the following triangles:





$c^2 = a^2 + b^2$ · label

$a = \frac{1}{2} c$

$\frac{27}{1/2} = \frac{\frac{1}{2} \cdot c}{1/2}$

$54_m = c$

$54^2 = 27^2 + b^2$

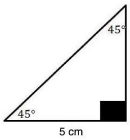
$2916 \underset{-729}{} = \cancel{729} \overset{-729}{} + b^2$

$\sqrt{2187} = \sqrt{b^{\cancel{2}}}$

$b = 46.77$ m

1.3.1 Example

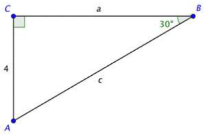
Determine the missing measurements of the following triangles:



c = 7.07 cm

1.3.2 Practice

Determine the missing measures a and c:

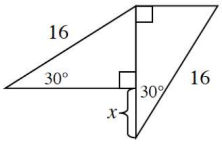


c = 8

a = 6.93

1.3.3 Practice

Determine the missing measure x:



x = 5.86

You do:

- 1st ex 1.31
- 1.3.2
- 1.3.3

HMWK:

pg 121 "use an eq."

pg 125 # 3.12

pg 126

pg 127 #3.16 a)-b) only