Lesson 4: Operations on Polynomials and creating Algebraic Equations task

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Recalls: Fractions and monomials $\times$ $\div$

ex. multiply:

$$\frac{12x^2}{y} \times \frac{6y^2}{2x^2}$$

way #1: detailed way:

$$\frac{(12x^2 \times 6y^2)}{y \times 2x^2}$$

$$\frac{72x^2 \cdot y^2}{2x^2 y}$$

$36y$

way #2: fast way:

$$\frac{12x^2}{y} \times \frac{6y^2}{2x^2}$$

$36y$

Recall:

$$\frac{1}{2} \times \frac{3}{4}$$

$$\frac{1 \times 3}{2 \times 4}$$

$$\frac{3}{8}$$

law #3

$a^m \times a^n = a^{m+n}$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{x^2}{x^2} = x^{2-2} = x^0 = 1$$

law #2

$a^0 = 1$

e.x. Divide

$$\frac{15a^3}{b^2} \div \frac{5ab}{4a^4}$$

Recall: ÷ keep/change/flip

$$\frac{15}{7} \div \frac{5}{4}$$

$$\frac{15}{7} \times \frac{4}{5}$$

$$\frac{12}{7}$$

$$\frac{a^1}{a^4} = a^{1-4} = a^{-3} = \frac{1}{a^3}$$

$$\frac{15a^3}{b^2} \times \frac{4a^3}{5b^1}$$

$$\frac{12a^3 \times a^3}{b^{2+1}}$$

law # 3

$$a^m \times a^n = a^{m+n}$$

$$\frac{12a^6}{b^3}$$

You do

Q1:

$$\left(\frac{-4mn^2}{m^2}\right)\left(\frac{6m^3}{2n}\right)$$

Q2:

$$\frac{\frac{-15a^2}{b^4}}{\frac{5a^5}{b^2}}$$

Q2:

$$\frac{\frac{-15a^2}{b^4}}{\frac{5a^5}{b^2}}$$

← ÷ keep/change/flip

$$\frac{-15a^2}{b^4} \div \frac{5a^5}{b^2}$$

$$\frac{-15a^2}{b^4} \times \frac{b^2}{5a^5}$$

$$\frac{-15a^2 \cdot b^2}{5a^5 \cdot b^4}$$

$$-3\,a^{2-5} \cdot b^{2-4}$$

law #8

$$a^{-n} = \frac{1}{a^{-n}}$$

$$r \cdot a^{-n} = r \cdot \frac{1}{a^n} = \frac{r}{a^n}$$

$$-3a^{-3} \cdot b^{-4}$$

$$-3 \cdot \frac{1}{a^3} \cdot \frac{1}{b^4}$$

$$\frac{-3}{a^3 b^2}$$

Q1:

$$\left(\frac{-4mn^2}{m^2}\right)\left(\frac{6m^3}{2n}\right)$$

1st way: detailed way

$$\frac{-4mn^2 \times 6m^3}{m^2 \times 2n}$$

$$\frac{-24m^4n^2}{2m^2n}$$

$$-12m^2n$$

2nd way: fast way

$$\left(\frac{-4mn^2}{m^2}\right)\left(\frac{6m^3}{2n}\right)$$

$$-12m^2n$$

attention
2 cannot also divide the 4 since the numerator is one term!

Dividing a Polynomial by a Monomial

e.x.

$(3x^2 - 9x^2 + 6x) \div 3x$

$\frac{3x^2 - 9x^2 + 6x}{3x}$ ← there are 3 terms in top

not same thing as:

$3x^2 - 9x^2 + 6x \div 3x$

$3x^2 - 9x^2 + \left(\frac{6x}{3x}\right)$

$3x^2 - 9x^2 + \frac{6x}{3x}$

$\frac{3x^2}{3x} - \frac{9x^2}{3x} + \frac{6x}{3x}$ ← do each division individually

$1 \cdot x - 3x + 2$

$-2x + 2$

e.x.

$(3x^2y^2 - 4xy) \div -xy$

$\dfrac{3x^2y^2 - 4xy}{-xy}$

$\dfrac{3x^2y^2}{-xy} - \dfrac{4xy}{-xy}$

$-3xy - -4$

$-3xy + 4$

Not same thing as:

$3x^2y^2 - 4xy \div -xy$

$3x^2y^2 - \dfrac{4xy}{-xy}$

Recall: an add or subtract sign introduces a new term!

expression	# of terms
$2x - y$	2
$2x(-y)$	1
$3ab - bc$	2
$3ab - (bc)$	2
$-x^2(-2)$	1
$-2 - x^2$	2
$-a(-2b)$	1
$-2b - a$	2

You do: Perform operations

#1. $(10x^7 - 5x^5 - 25x^3) \div 5x^2$ $= 2x^5 - x^3 - 5x$

#2 $\dfrac{15x^6}{y^6} \div \dfrac{5x^2}{10x^4y^3}$ $= \dfrac{30x^8}{y^3}$

#3 $(80x^3y^3 - 40x^7y^4) \div -10xy^2$ $= -8x^2y + 4x^6y^2$

#4 $-\dfrac{\dfrac{8a^3b^4}{a^4}}{\dfrac{6a^5}{2 \cdot b^2}}$ $= \dfrac{-8b^6}{3a^6}$

#5 $6a^2 - 9a^2 - 15a^2b^4 \div -3b^4$ $= 2a^2$

pg 28 #1.22 d) - f)

Adding and Subtracting Polynomial Expressions

Adding:

$(2x^2 + 3x - 1) + (4x^3 + 3x^2 - x + 3)$

Same thing as: $2x^2 + 3x - 1 + 4x^3 + 3x^2 - x + 3$

$2x^2 + 3x - 1 + 4x^3 + 3x^2 - x + 3$

Adding like terms:

$r \cdot a^m + s \cdot a^m = (r+s)a^m$

$4x^3 + 2x^2 + 3x^2 + 3x - 1x - 1 + 3$

$4x^3 + 5x^2 + 2x + 2$

Subtracting

adjacent signs

+ + = +
+ − = −
− − = +

$(x^3 - 2x + 3) - (x^3 + 3x^2 - x + 3)$

Not the same as $x^3 - 2x + 3 - x^3 + 3x^2 - x + 3$

$1 \cdot x^3 - 2x + 3 - 1x^3 - 3x^2 + x - 3$

- distribute the negative sign into each term.
- add like terms

$-2\ \$ + (-4\ \$)$

$0x^3 - x + 0 - 3x^2$

$-3x^2 - x$

You do: Evaluate

#1

$(-x^2 + 4x) - (-x^2 - 5x + 3)$

$-x^2 + 4x + x^2 + 5x - 3$

$9x - 3$

#2

$(-2x^{1/2} + 4) + (x^2 - 4x^{1/2})$

$-6x^{1/2} + x^2 + 4$

$x^2 - 6x^{1/2} + 4$

← not actually polynomial

Multiplying Polynomial w̄ Monomials

let's eat Grandma! v.s. let's eat, Grandma!

e.x.

$-2x(x-3)$ not same thing as: $-2x \cdot x - 3$

$-2x^2 + 6x$

- distribute monomial in, and follow same law of signs

e.x.

$-x^1(-4x^2+2) + 3x^2(x^1 - x^2)$ 2 terms.

$4x^3 - 2x + 3x^3 - 3x^4$

$7x^3 - 2x - 3x^4$

$-3x^4 + 7x^3 - 2x$

You do: Evaluate

#1. $3x^2(x-2) - x^2(2x^2 - 3x + 1)$

$= -2x^4 + 6x^3 - 7x^2$

#3. $y(3x+y) - x(4y-3)$

$= y^2 - xy + 3x$

#2. $x^1y^1(x^2 - y^2) - (2x^2 - 3x + 1) + y(2x^3 - 3xy^2 - 2)$

$x^3y - xy^3 - 2x^2 + 3x - 1 + 2x^3y - 3xy^3 - 2y$

$3x^3y - 4xy^3 - 2x^2 + 3x - 2y - 1$

· Factoring a Polynomial by factoring
·(Decomposing) out a Greatest common Factor
(GCF)
·(Rewriting Polynomials as a product of their Factors/Parts)

Recall : Distributing / Expanding (multiplying)

$2(5x + 8)$

$10x + 16$

$-(7x - 2)$

$-7x + 2$

Now factor out GCF from 2 two terms.

step i : Find the GCF by listing all factors of each terms.
2 integers whose product equals the #.

10: 1, 10; 2, 5; ~~3, 3.3~~

16: 1, 16; 2, 8; 4, 4

∴ GCF = 2

$\frac{10x}{2} + \frac{16}{2}$

$2(5x + 8)$ ✓ ans

step ii . write GCF in front of empty brackets
$2(\;\;)$

step iii : Divide each original term by GCF and put in brackets.

Check:
$2(5x + 8)$
$10x + 16$

step iv : Check your answer by expanding/distributing and ensure you have original expression.

Factor:

$18 - 12y$

6 6

$6(3 - 2y)$

18		12	
1	18	1	12
2	9	2	(6)
3	(6)	3	4

pick greatest CF

check

$6(3 - 2y)$

$18 - 12y$

Factor:

$$\frac{6x^3}{3x^1} - \frac{9x^2}{3x^1} + \frac{3x^1}{3x^1}$$

$$3x(2x^2 - 3x + 1)$$

3 3 1 ← a prime # since only factors are 1 and itself.

note: if there's a prime #, the GCF is the prime # (or 1)

GCF: 3x

check: expanding/distributing

$$3x(2x^2 - 3x + 1)$$

$$6x^3 - 9x^2 + 3x$$

Factor: 4-2

$-2x^7 + 4x^4 - 6x^2y$

$-2x^2(x^5 - 2x^2 + 3y)$

note: if leading term is negative, then GCF is negative.

2 ← prime #

GCF $= -2x^2$

check $-2x^2(x^5 - 2x^2 + 3y)$

$-2x^7 + 4x^4 - 6x^2y$

You do: Factor / Decompose

#1 $3x^2y^3 - 9x^2y^4 + 12xy^3$

$= 3xy^3(x - 3xy + 4)$

#2 $-x^3 + 2x^2 - x$

$= -x(x^2 - 2x + 1)$

#3 $-8x^6 - 4x^5 - 6x^3$

$= -2x^3(4x^3 + 2x^2 + 3)$

#4 $10a^2b^3 - 5a^5b^4 + 25a^3b^7$

$= 5a^2b^3(2 - a^3b + 5ab^4)$

$5a^2b^3(5ab^4 - a^3b + 2)$

BONUS: $2x(3x-2) + 3(3x-2)$

$(3x-2)(2x+3)$

Hmwk:

p 28, p 15 (pg 21) } from textbook

from handouts

all questions except NOT #31-40, #46, #48