p240

Evaluate:

$$(10^1 \times 10^5)^{\frac{1}{2}}$$

$$a^m \times a^n = a^{m+n}$$

$$(10^{1+5})^{\frac{1}{2}}$$

$$(10^6)^{1/2}$$

$$(a^m)^n = a^{m \times n}$$

$$10^{6 \times \frac{1}{2}}$$

$$10^3$$

Lesson 2: May 24, 2023

- Radicals
- Scientific Notation:

↳ a way to rewrite very large/small numbers

Tomorrow May 25th, Thursday Mini-Day
A: 8:30 - 9:30
B: 9:40 - 10:40

$$a \times 10^n$$

$$a \in [0, 10[\qquad n \in \mathbb{R}$$

Decimal Notation — Scientific Notation

ex. 623 000 — 6.23×10^5

6.23 000

0.00478

0.00478

⊘ 4.78×10^{-3} — same value

⊘ 47.8×10^{-4} ↳ ⊘ NOT in scientific notation
4.78×10^{-3} ↳
· since $47.8 \notin [0,10[$
· since 2 digits before decimal

$\boxed{\wedge}$

$\boxed{y^x}$

left right

Practice:

Convert to Decimal Notation

i. 5.89×10^2

$5.89. \times 10^2$

589

ii. 5.89×10^4

5.8900×10^4

58900

left right

iii. 5.89×10^{-2}

005.89×10^{-2}

0.0589

iv. 5.89×10^{-4}

000005.89×10^{-4}

0.000589

nota bene:

- when exponent is positive, the decimal notation is a large #.
- when exponent is negative, the dec. not. is a small #.

$\boxed{\text{minor diff.}}$

You do:

Convert to Decimal Notation

5.89×10^{3} = 5890

5.89×10^{5} = 589 000

5.89×10^{-3} = 0.00589

5.89×10^{-5} = 0.0000589

Convert to Scientific Notation

e.x. 730 000 m^2

7.30 000. m^2

7.3×10^5 m^2

e.x. 0.0000987 mL

0.0000987 mL

9.87×10^{-5} mL

nota bene:

- when exponent is positive, the decimal notation is a large #.
- when exponent is negative, the dec. not. is a small #.

You do:

Convert to Scientific Notation

i. 890 000 000 cm^2

8.9×10^8 cm^2

ii 0.00017 mm

1.7×10^{-4} mm

iii. 987 890 daL

9.8789×10^5 daL

iv. 10 256 000 000 000 km

1.0256×10^{13} km

Performing Operations $\bar{w}$ Scientific Notation

e.x. Evaluate and final answer in scientific notation

B
E
D
M L to R
A
S

$(3.45 \times 10^{3}) \times (9.7 \times 10^{-1})$

$3.45 \times 9.7 \times 10^{3} \times 10^{-1}$

exp law # 3

$33.47 \times 10^{3+(-1)}$

$2 \times 3 \times 4 = 4 \times 3 \times 2$

33.47×10^{2} ✗ ← not in scientific notation, ∴ not done!

33.47×10^{2}

$3.347 \times 10^{1} \times 10^{2}$ exp law # 3

$3.347 \times 10^{1+2}$

3.347×10^{3}

e.x. Evaluate and final in Scientific Notation

$(0.03 \times 10^{3}) \div (0.97 \times 10^{-1})$

$\dfrac{0.03}{0.97} \times \dfrac{10^{3}}{10^{-1}}$

Recall:

$\dfrac{x}{y} \times \dfrac{a}{b} = \dfrac{x \cdot a}{y \cdot b}$

$\dfrac{0.03}{0.97} \times \dfrac{10^{3}}{10^{-1}}$

exp law #4

$0.0309 \times 10^{3-(-1)}$

$0.0309 \times 10^{3+1}$

0.0309×10^{4}

0.0309×10^{4}

$3.09 \times 10^{-2} \times 10^{4}$

law #3

$3.09 \times 10^{-2+4}$

3.09×10^{2}

You do:

Question 2 from handout #1

(at least 1 question and do the rest and question 1 for homework)

Notation of Radicals

root index (a #) → $\sqrt{2}$ ← base/radicand

this reads, the square root of two.

root index → $\sqrt[2]{2}$

nota bene: square rooting is the opposite operation of squaring.

definition: a square root of a # is a # that when squared gives the base/radicand

e.x.

$\sqrt{2}$

$= 1.41$

$(\sqrt{2})^{2}$

2

$\sqrt{3}$

$(\sqrt{3})^{2}$

3

---

ex.

$\sqrt[3]{2}$, this reads the cubed root of 2.

$\boxed{\sqrt[3]{\ }}$ $\boxed{\sqrt[x]{\ }}$

$\boxed{x^{3}}$ $\boxed{y^{x}}$

$= 1.2599$

$\sqrt[3]{2}$

$(\sqrt[3]{2})^{3}$

2

---

ex

$\sqrt[4]{2}$, this reads, the 4th root of 2

$\boxed{\sqrt[x]{\ }}$

$= 1.189$

$\sqrt[4]{2}$

$(\sqrt[4]{2})^{4}$

2

Simplifying Radicals $\bar{w}$ out calculator.

e.x. Rewrite the base/radicands as exponential numbers and simplify

e.x.

$\sqrt{9}$ rewrite as power

$\sqrt{3^2}$

$\sqrt{\cancel{3^2}}$ simplify

3

ex. cubed root
power $\bar{w}$ base 3.

$\sqrt[3]{27}$

$\cancel{\sqrt[3]{3^3}}$

3

$\bar{w}$ base 3

$\sqrt[4]{81}$

$\cancel{\sqrt[4]{3^4}}$

3

$\bar{w}$ base 5

$\sqrt[4]{625}$

$\bar{w}$ base 3

$\sqrt[6]{729}$

$\bar{w}$ base 2

$\sqrt{4}$

You do

Converting Radicals to Exponents
(using laws #10 – 12)

Radicals: Exponential Number:

Why? $\sqrt{a} = a^{\frac{1}{2}}$ (most useful)

$\sqrt[n]{a} = a^{\frac{1}{n}}$ (more generally)

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$ (when base is an exp #)

$\sqrt[3]{x^1} = x^{\frac{1}{3}}$

$\sqrt[4]{x} = x^{\frac{1}{4}}$

$\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[4]{a^8} = a^{\frac{8}{4}} = a^2$

$\sqrt[3]{a^9} = a^{9/3} = a^3$

$\sqrt[2]{x^2} = x^{\frac{2}{2}} = x^1 = x$

$\sqrt{4} = 2$

$\sqrt[2]{2^2} = 2^{\frac{2}{2}} = 2^1 = 2$

$\sqrt{9} = 3$

$\sqrt[2]{3^2} = 3^{\frac{2}{2}} = 3^1 = 3$

$\sqrt{16} = 4$

$\sqrt{4^2} = 4^{2/2} = 4^1 = 4$

$\sqrt[3]{8} = 2$

$\sqrt[3]{2^3} = 2^{3/3} = 2^1 = 2$

You do:

i. $\sqrt{5^4}$

ii. $\sqrt[3]{a^6}$

law #7

iii. $\sqrt[3]{\left(\frac{x}{y}\right)^1} = \left(\frac{x}{y}\right)^{1/3} = \frac{x^{1/3}}{y^{1/3}}$

$\sqrt[n]{a^m} = a^{m/n}$

iv. $\frac{\sqrt[3]{x}}{y} = \frac{x^{1/3}}{y}$

Prove that $\sqrt{a} = a^{\frac{1}{2}}$

$(\sqrt{a})^2 = a$ by definition

$\sqrt{a} \times \sqrt{a} = a$

$3^2 = 3 \times 3$

$a^{\square} \times a^{\square} = a^{1}$

law #3

$\square + \square = 1$

$\frac{\cancel{2}\square}{\cancel{2}} = \frac{1}{2}$

$\square = \frac{1}{2}$

$\therefore \sqrt{a} = a^{1/2}$ $\blacksquare$ Q.E.D

$\boxed{\text{Prove } \sqrt[n]{a} = a^{1/n}}$

Converting Exp # to Radicals

law #12: $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

ex simplify and convert to radicals

i. $a^{\frac{1}{2}} = \sqrt[2]{a^1} = \sqrt{a}$

ii. $b^{1/3} = \sqrt[3]{b^1} = \sqrt[3]{b} \neq \sqrt{b}$

iii. $c^{6/5} = \sqrt[5]{c^6}$

iv. $3 \cdot \boxed{x^{2/3}} = 3 \times \sqrt[3]{x^2} = 3\sqrt[3]{x^2}$ ✓

v. $(3x)^{2/3} = \sqrt[3]{(3x)^2} = \sqrt[3]{3^2 x^2}$
$= \sqrt[3]{9 \cdot x^2}$

vi. $(x^{1/2})^{3/4}$

#5 $(a^m)^n = a^{m \times n}$

$x^{\frac{1}{2} \times \frac{3}{4}} = x^{\frac{3}{8}} = \sqrt[8]{x^3}$

the eighth root of ...

You do:

i. $2x^{2/3}$

ii. $(a^{3/4})^{1/5}$

iii. $x^{\frac{1}{4}} \times x^{2/4} = x^{\frac{1}{4} + \frac{2}{4}} = x^{3/4} = \sqrt[4]{x^3}$

iv. $(2x^{3/5} \times 3x^{-2/5}) = 2 \times 3\, x^{3/5 + (-2/5)}$
$= 6x^{1/5} = 6\sqrt[5]{x}$

Using BEDMAS and laws of exp/rad to simplify expressions

e.x.

$\boxed{\sqrt[4]{49^1}} \times 7^{\frac{1}{2}}$

#12

$49^{\frac{1}{4}} \times 7^{\frac{1}{2}}$

$(7^2)^{\frac{1}{4}} \times 7^{1/2}$

#5

$(a^m)^n = a^{m \times n}$

$7^{2 \times \frac{1}{4}} \times 7^{1/2}$

$7^{\frac{1}{2}} \times 7^{\frac{1}{2}}$

#3

$7^{\frac{1}{2} + \frac{1}{2}}$

$7^1 = 7$

law #3

$a^m \times a^n = a^{m+n}$

Tip:

* convert radicals to exp #.

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

* Rewrite bases as exp #'s w̄ same base.

ex $49 = 7^?$

$49 = 7^2$ sub in

* Follow B E D M A S

You do:

Q1: $3^{1/2} \div \dfrac{1}{\sqrt[3]{729}} = 3^{5/2}$

Q2 $\dfrac{1}{5^3} \times \sqrt[4]{25^{-5}} = \dfrac{1}{5^{11/2}}$

Q3 $\sqrt[3]{\dfrac{27}{8}} \div \left(\dfrac{3}{2}\right)^2 = \dfrac{2}{3}$

handout #2 and #1 for HMWR