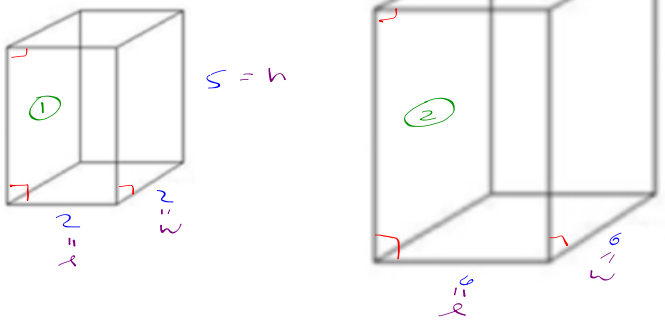
Lesson 14 : Similar Figures

June 15th 2023

- Thursday Cont'd (Focus on 3D solids)
- Friday : Review / Task
- Monday : Practice Pretest
- Tuesday : Real Pretest
- Wednesday : Take-up Pretest
- Thursday : Practice Pretest
- Friday : La Saint-Jean
- Monday : EXAM !

e.x. the 2 prisms are similar.



↳ Recall : definition :

similar figures (2D or 3D) :

→ have congruent (same) angles.

→ their ratios of corresponding sides are all equal to same scale factor (k, aka similarity ratio)

$\frac{15}{5} = 3$, $\frac{6}{2} = 3$, $\frac{6}{2} = 3$ } same k !

∴ the sides are $2x$, the areas $4x$, the volumes are $8x$
the sides are $3x$, the areas are $9x$, the volumes are $27x$
3^2 3^3

" " $4x$, " " $16x$, " " $64x$

$5x$, $25x$ $125x$

$6x$ $36x$ $216x$

$k^1 x$ $k^2 x$ $k^3 x$

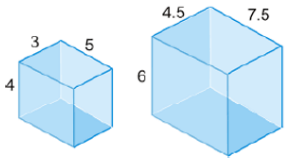
You do.

1.2.2 Example

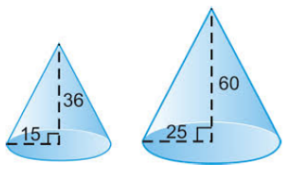
Determine whether the following solids are similar:

↳ how? make proportions/ratios and verify they're all equal:

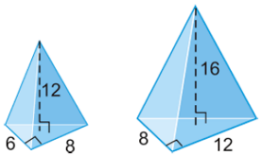
a)



b)

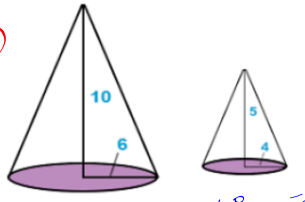


c)



not similar cuz different ratio

d)



similar:

$\frac{3}{4.5} = 0.\dot{6}$

$\frac{5}{7.5} = 0.\dot{6}$

$\frac{4}{6} = 0.\dot{6}$

$\frac{r_B}{r_S}$ $\frac{25}{15} = 1.\dot{6}$

$\frac{h_B}{h_S}$ $\frac{60}{36} = 1.\dot{6}$

} ∴ cones are similar

$\frac{h_B}{h_S} = \frac{10}{5} = 2$

$\frac{r_B}{r_S} = \frac{6}{4} = 1.5$

} different ∴ not similar

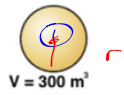
How to find k (1 unknown → 1 equation)

- $k = \frac{\text{corresponding side of figure (1)}}{\text{corresponding side of figure (2)}}$ ex $\frac{\text{hypo (1)}}{\text{hypo (2)}}$
- $k^2 = \frac{\text{area of figure (1)}}{\text{area of figure (2)}}$
- $k^3 = \frac{\text{volume f(1)}}{\text{volume f(2)}}$

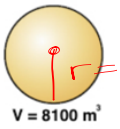
1.2.3 Similarity Ratio (Scale Factor) k

1.2.4 Example

Determine the similarity ratio (scale factor) k for the following *similar* solids:



$r_1 = ?$



$k^3 = \frac{V_{(1)}}{V_{(2)}}$

$k^3 = \frac{300}{8100}$

$\sqrt[3]{k^3} = \sqrt[3]{\frac{300}{8100}}$

$k = \frac{1}{3}$

$k = 0.\dot{3}$

Remember if $k < 1$ all ratios must be: small / big

← isolate/solve for k.

$\sqrt[3]{k^3} = \sqrt{\frac{8100}{300}}$

$k = 3$

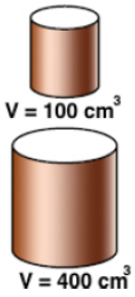
if $k > 1$ ratios big / small

You do 1.2.5 and BONUS 1.2.6.

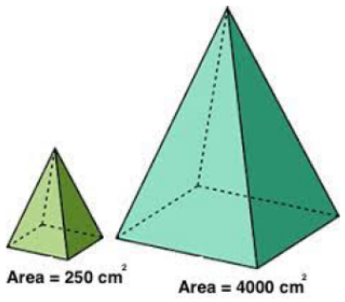
1.2.5 Practice

Determine the similarity ratio (scale factor) k for each of the following *similar* solids:

a)



b)



a)

$k = 1.59$

$k^3 = \frac{400}{100}$

$\sqrt[3]{k^3} = \sqrt[3]{\frac{100}{400}}$

$k = 0.63$

b)

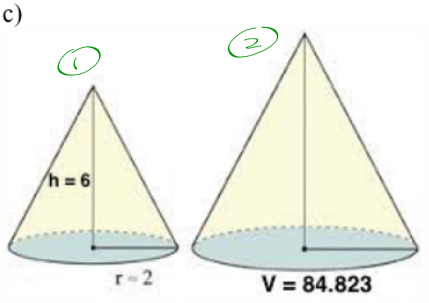
$k = 4$

c) $k = 1.5$

1.2.5 Practice

Determine the similarity ratio (scale factor) k for each of the following *similar* solids:

c)



- WANT: k
- TOOL: $k^3 = \frac{\text{Vol } ①}{\text{Vol } ②}$
- INFO:
  - Vol ② = 84.823
  - Vol ① = ?

WANT: Vol ① 1 HR

TOOL: $V = \frac{A_B \times h}{3}$

$V = \frac{\pi r^2 \times h}{3}$

INFO:
- $V = ?$
- $r = 2$
- $h = 6$

$V = \frac{\pi (2)^2 \times 6}{3}$

$V = 8\pi$

$V_1 = 25.13$

$V_2 = 84.823$

sub

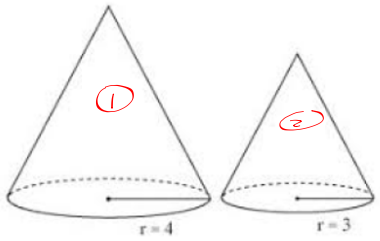
$k^3 = \frac{\text{Vol } ①}{\text{Vol } ②}$

$\sqrt[3]{k^3} = \sqrt[3]{\frac{25.13}{84.823}}$

$k = 0.66664$ or $k = 1.5$

1.2.6 Example

The following Cones are *similar*. If the total area of the first cone is $23m^2$ and its volume is $12m^3$, determine the area and volume of the second cone.



$k = \frac{\text{side } ①}{\text{side } ②}$

$k = \frac{4}{3}$

$A_1 = 23m^2$ $A_2 = ?$

$V_1 = 12\,m^3$ $V_2 = ?$

$k^2 = \frac{A①}{A②}$ TOOL

$k^3 = \frac{V①}{V②}$

$\left(\frac{4}{3}\right)^3 = \frac{12}{V②}$

$\frac{64}{27} = \frac{12}{V②}$

$64\,V② = (12 \times 27)$

$\frac{64}{64}$ $\frac{64}{64}$

$V_2 = 5.06\,m^3$

$\left(\frac{4}{3}\right)^2 = \frac{23}{A_2}$

- sub
- simplify / evaluate.

$\frac{16}{9} = \frac{23}{A_2}$

- solve w/ 0.0
- cross multiply

$16 \cdot A_2 = (23 \cdot 9)$

$\frac{16}{16}$ $\frac{16}{16}$

$A_2 = 12.9375\,m^2$

You do:

1.2.7

1.2.8

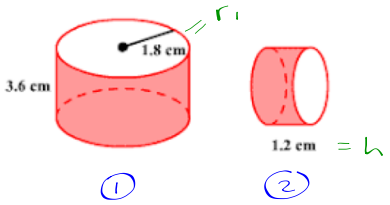
Bonus: 1.3.1

$A_{T②} = 6.786\ cm^2$

$V_② = 1.357\ cm^3$

1.2.7 Practice

The following cylinders are similar. Determine the area and volume of the smaller cylinder.



- Find A_2 2nd figure:

$A_{2_T} = A_\ell + 2A_B$

$A_T = P_B \times h + 2A_B$

$\rightarrow A_T = 2\pi r \times h + 2\pi r^2$

$k^2 = \frac{A①}{A②}$

$r_① = 1.8$

Find r_2

$k = \frac{r①}{r②}$

$\frac{3}{1} = \frac{1.8}{r_2}$

$\frac{3 \cdot r_2}{3} = \frac{1.8}{3}$

$r_2 = 0.6$

$h_2 = 1.2$

Find k:

$k = \frac{h①}{h②}$

$k = \frac{3.6}{1.2}$

$k = 3$

} have all info to find area.

$A_T = 2\pi r \times h + 2\pi r^2$

$A_T = 2\pi(0.6) \times 1.2 + 2\pi(0.6)^2$

$A_T = 6.786\ cm^2$

$r = 0.6$

$h = 1.2$

$V = A_B \times h$

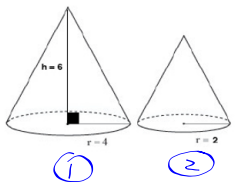
$V = \pi r^2 \times h$

$V = \pi(0.6)^2 \times 1.2$

$V = 1.357\ cm^3$

1.2.8 Practice

The following cones are similar. Determine the area and volume of the smaller cone.



$A_{T(2)} = 35.25\ cm^2$

$V_{(2)} = 12.57\ cm^3$

Answer
Ex 1.3.1
$h = 6\ cm$

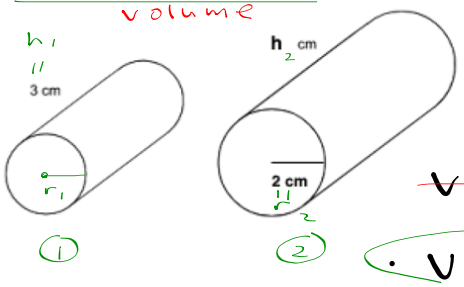
mini-tasks (not so wordy)

1.3 Similarity, Solids and Missing Measurements

$V_1 = \frac{1}{8} \cdot V_2$

1.3.1 Example: The following two solids are similar. The volume of the smaller solid is 8 times smaller than the ~~area~~ of the larger solid. Determine the value of h.

volume



8 times greater:

$V_1 = 8 \times V_2$? ×

$V_2 = 8 \times V_1$ ✓

$V_1 = \frac{1}{8} \times V_2$ ✓

TIPS:
- LABEL
- Translate sentences (directly)
- key word similar:

$k = \frac{\text{side}}{\text{side}}$

$k^2 = \frac{\text{area}}{\text{area}}$

$k^3 = \frac{\text{vol}}{\text{vol}}$

Find k:

$\frac{V_2}{V_1} = \frac{8 \times V_1}{V_1}$

$\frac{V_2}{V_1} = 8$

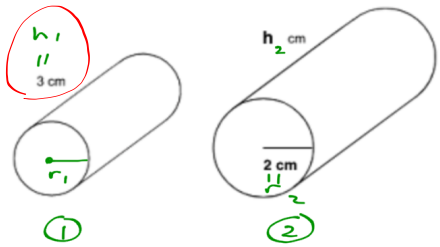
Careful!

$k^3 = \frac{V_2}{V_1}$

$\therefore k^3 = 8$ ← solve w/ o.o.

$\sqrt[3]{k^3} = \sqrt[3]{8}$

$k = 2$



Find h_2

$k = \frac{\text{side } ②}{\text{side } ①}$

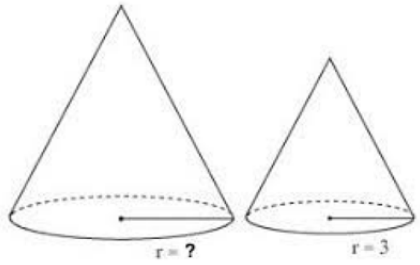
$2 = \frac{h_2}{3}$

$h_2 = 2 \times 3$

$h_2 = 6$ units

1.3.2 Practice

The following two cones are similar. The area of the smaller cone is 9 times smaller than the area of the larger cone. Determine the radius of the larger cone.



HMWK:

pg 194 #4.20 and #4.21 a) and b)

pg 195 #4.22

pg 200 #4.31

BONUS: pg 196 #3.24

} task like!

You do:

1.3.2 – answer $r = 9$

1.3.3 – answer $h = 3.6$ cm

1.3.4 – answer $A = 108$ cm^2

1.3.5 – answer

(task) Yes the snowman will fit since

$h_{sm} < h_{cyl}$

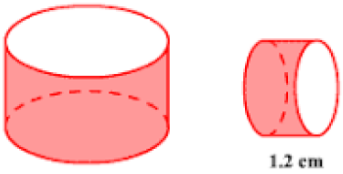
$14.4\text{cm} < 14.48\text{cm}$

and since

$d_{sm} < d_{cylin}$

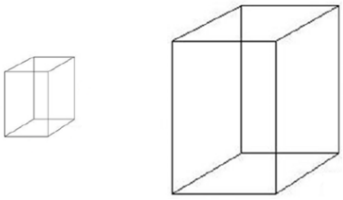
$8.4\text{cm} < 8.8\text{cm}$

1.3.3 Practice: The following cylinders are similar. The volume of the larger cylinder is 27 times greater than the volume of the smaller cylinder. Find the height of the larger cylinder.



1.3.4 Practice

The following two rectangular prisms are similar. The volume of the smaller prism is one eighth times the volume of the larger prism. If the total area of the larger prism is 432 cm^2, what is the total area of the smaller prism?



1.3.5 Practice

You make three-dimensional handmade snowmen that you sell on Etsy. You need to find a cylindrical package to place the snowmen in before mailing them to customers. Below is an image of one of your snowmen and a possible cylindrical package. Your snowman is composed of two similar spheres. The volume of the smaller sphere is $\frac{125}{343}$ times the volume of the larger sphere. If the cylindrical packaging uses 5.22 dm^2 of cardboard, will the snowman fit?