Lesson 13 : Similar Figures

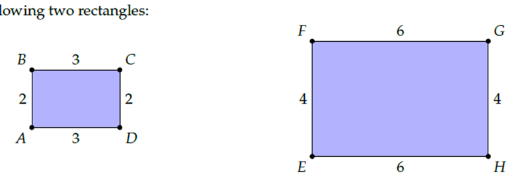
June 13th 2023

Discovery / Recall Activity :

1 Similar Figures

1.1 Activity

Consider the following two rectangles:



* not the same rectangle!
* the rectangles are not congruent!
* BUT they are similar.

Answer the following questions:

construct ratio w corresponding sides

i. • What is the proportion between the sides of the rectangles *ABCD* and *EFGH*?

• ratio
• fraction

$\frac{4}{2} = 2$ $4 : 2$ $2 : 1$

∴ we can say, rectangle EFGH is 2 times larger than ABCD.

ii. • What is the perimeter of rectangle *ABCD*?

$P = 2 + 2 + 3 + 3 = 10$ units

iii. • What is the perimeter of rectangle *EFGH*?

$P = 4 + 4 + 6 + 6 = 20$ units

iv. • What is the proportion between the perimeters of the rectangles? What do you notice?

$\frac{20}{10} = 2$

v. • What is the area of rectangle *ABCD*?

$A = \ell \cdot w = 2 \cdot 3 = 6$ units^2

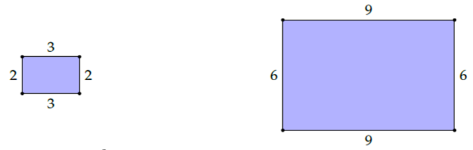
vi. • What is the area of rectangle *EFGH*?

$A = \ell \cdot w = 4 \cdot 6 = 24$ units^2

vii. • What is the proportion between the areas of the rectangles? Now find the proportion between the area of the following two rectangles. Do you notice a pattern?

$\frac{24}{6} = 4$

side lengths:
$\frac{9}{3} = 3$



$A = 2 \cdot 3$
$= 6$ units^2

$A = 6 \cdot 9$
$= 54$ units^2

$\frac{A}{A} = \frac{54}{6} = 9$

(next pg for pattern)

Notice:

- When the length of a similar figure is $2x$, the Area is $4x$.
- When the length of a similar figure is $3x$, the Area is $9x$.
- " " " is $4x$, the Area is $16x$
- " " " is $5x$, the Area is $25x$
- " " " is $6x$, the Area is $36x$
- " " " is $7x$, the Area is $49x$

$\vdots$

- " " " is kx, the Area is k^2x

$\times 2$

Area 1 squre $\xrightarrow{\times 4}$ Area 4 square

$\times 3$

Area 1 square $\xrightarrow{\times 9}$ Area 9 square

- sides:

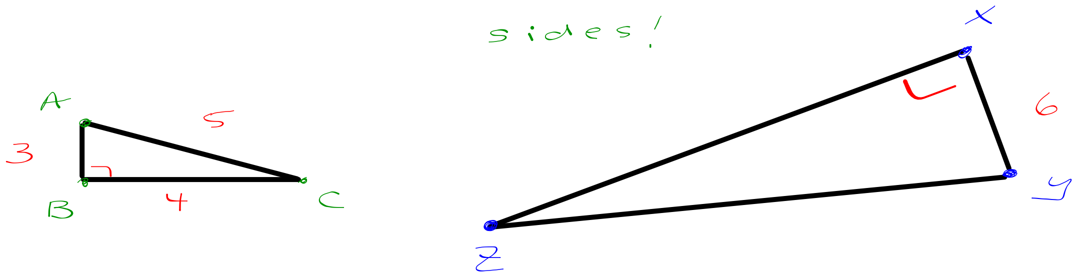
$$\frac{2}{1} = 2$$

- Areas:

$$\frac{2 \times 2}{1 \times 1} = 4$$

Constructing Similar Ratios

→ how? use corresponding sides!



$\dfrac{m\overline{XY}}{m\overline{AB}}$

← smallest side in Big △

← smallest side in Little △

(corresponding sides)

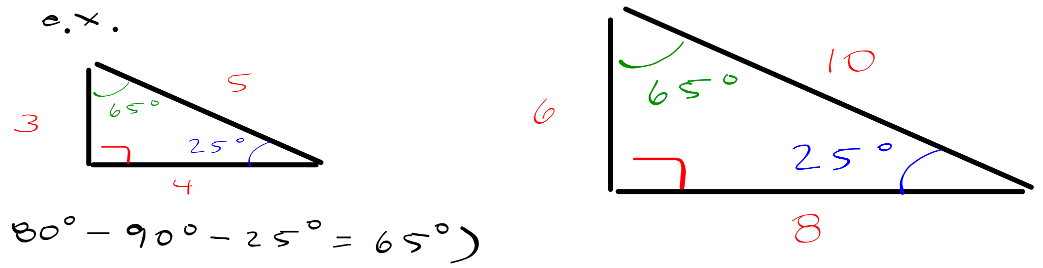
$\dfrac{m\overline{ZY}}{m\overline{AC}}$

← largest side in Big △ (opposite 90°)

← largest side in Little △ (opposite 90°)

Definition of Similar Figures

e.x.



$(180° - 90° - 25° = 65°)$

Two figures (2D or 3D) are similar if:

- their corresponding sides are proportional (not congruent)
- their corresponding angles are congruent.

$3 \neq 6 \quad 4 \neq 8 \quad 5 \neq 10$ (not congruent)

$$\left.\begin{aligned}\frac{3}{6} &= \frac{1}{2} = k\\ \frac{4}{8} &= \frac{1}{2} = k\\ \frac{5}{10} &= \frac{1}{2} = k\end{aligned}\right\}$$

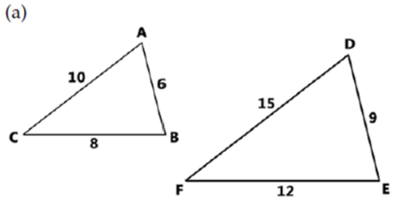
all ratios equal the same k.

, where k is the similarity ratio (scale factor)

1.2.1 Example

Determine whether the following figures are similar. If they are similar, determine the scale factor k:

(a)



→ how? check <u>all</u> ratios are equal to same k.

$$\frac{6}{9} = \frac{2}{3}$$

$$\frac{10}{15} = \frac{2}{3}$$

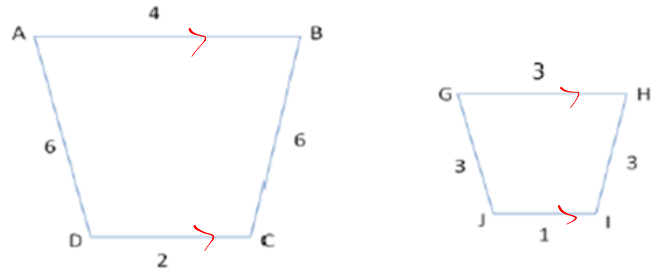
$$\frac{8}{12} = \frac{2}{3}$$

∴ triangles are similar

$\triangle ABC \sim \triangle DEF$

is similar to

(b)



$$\frac{m\overline{AB}}{m\overline{GH}} = \frac{4}{3} = k$$

$$\frac{m\overline{DC}}{m\overline{JI}} = \frac{2}{1} = k$$

not same k

∴ trapezoids are not similar!

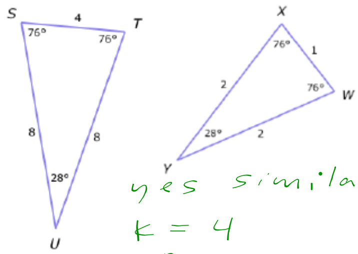
You do page 2 and page 3

☆ take 5 mins

1.2.2 Practice

Determine whether the following figures are similar. If they are similar, determine the scale factor k:

(a)



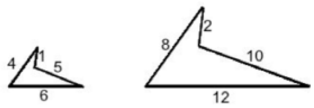
yes similar

$k = 4$

or

$k = \frac{1}{4}$

(b)



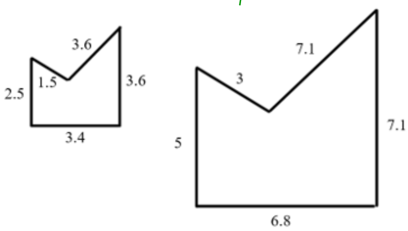
yes similar

$k = 2$

or

$k = \frac{1}{2}$

(c)



no they're not similar

$\frac{5}{2.5} = 2$ $\qquad$ $\frac{7.1}{3.6} = 1.97$

$\frac{6.8}{3.4} = 2$

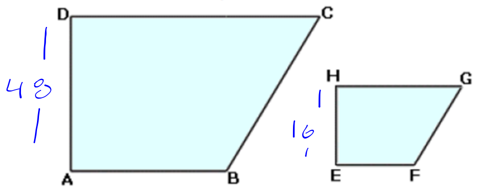
$\therefore$ not the same k

so $\therefore$ not similar

How to find k, similarity ratio

1.3.1 Example

Determine the similarity ratio (scale factor) k between the following two similar figures:



- $m\overline{AD} = 48$ cm
- $m\overline{EH} = 16$ cm

ratio sides

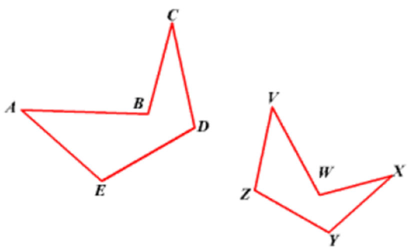
$$k = \frac{48}{16} = 3$$

or

$$k = \frac{16}{48} = \frac{1}{3}$$

1.3.2 Example

Determine the similarity ratio (scale factor) k between the following two similar figures:



- Perimeter of $ABCDE$ is 108 m
- Perimeter of $VWXYZ$ is 81 m

ratio of perimeters

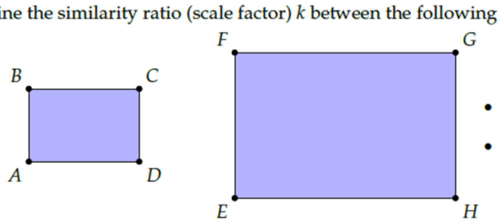
$$k = \frac{108}{81} = \frac{4}{3}$$

or

$$k = \frac{81}{108} = \frac{3}{4}$$

1.3.3 Example

Determine the similarity ratio (scale factor) k between the following two similar figures:



- Area of $ABCD$ is 110 units^2
- Area of $EFGH$ is 990 units^2

$$k = 3$$

$$\frac{\text{Area Big } \square}{\text{Area Small } \square} = k^2$$

$$\frac{990}{110} = k^2$$

$$9 = k^2$$

$$\sqrt{9} = \sqrt{k^2}$$

$$k = 3$$

Put on memory Aid:

3 new equations.

If 2 figures (2D or 3D) are similar:

i. $$k = \frac{\text{corresponding side } ①}{\text{corresponding side } ②}$$

ii. $$k^2 = \frac{\text{area of figure } ①}{\text{area of figure } ②}$$

iii. $$k^3 = \frac{\text{volume of figure } ①}{\text{volume of figure } ②}$$

Determining Unknowns → use equations.

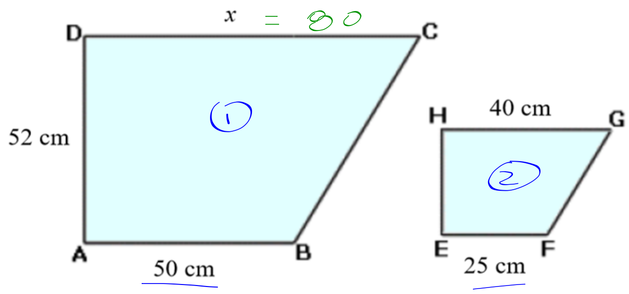
$k = \frac{\text{side}}{\text{side}}$

$k^2 = \frac{\text{area}}{\text{area}}$

$k^3 = \frac{\text{vol}}{\text{vol}}$

1.4.1 Example: Determine the missing measurement(s) given that the two figures are similar.

(a)



WANT: x (side) (1 unk)

TOOL: 1 eq.

$k = \frac{\text{side}}{\text{side}}$

$k = \frac{x}{40}$

label: figure 1, figure 2

INFO: $x = ?$

$40 = 40$ ✓

$k = ?$

or $k = \frac{40}{x}$ → figure 2 then figure 1 $k = \frac{25}{50}$

WANT: k

TOOL: $k = \frac{\text{side}}{\text{side}}$ f 1 f 2

1 $k = \frac{50}{25}$

and all info

find x:

2 $k = \frac{x}{40}$

sub 1 into 2

$k = \frac{50}{25}$

$\frac{50}{25} = \frac{x}{40}$ ← 1 eq / 1 unk → solvable

. solve w/ o.o.

. make it easy by cross multipling

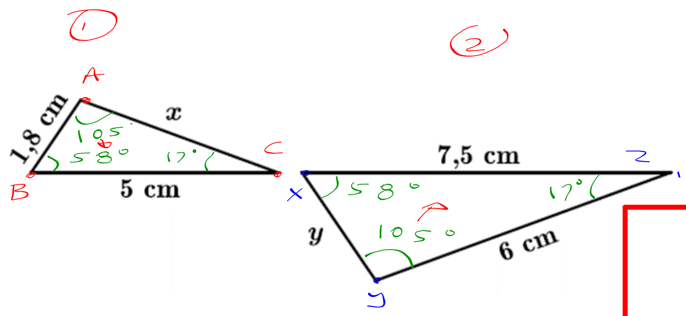
$\frac{(50 \times 40)}{25} = \frac{x \cdot \cancel{25}}{\cancel{25}}$

$x = 80$ → check w/ graph if that makes sense

Find x and y.

b)

tips: label and measure angles.



find k:

$$k = \frac{side}{side}$$

$$k = \frac{7.5}{5}$$ ← figure ② ← figure ①

find x:

$$k = \frac{side}{side}$$

$$k = \frac{7.5}{5}$$ figure ① or figure ②

$$\frac{7.5}{5} = \frac{x}{6}$$

$$\frac{7.5}{5} = \frac{6}{x}$$ figure ② figure ①

$$\frac{\cancel{7.5} \times x}{\cancel{7.5}} = \frac{(6 \times 5)}{7.5}$$

$$x = 4$$

find y:

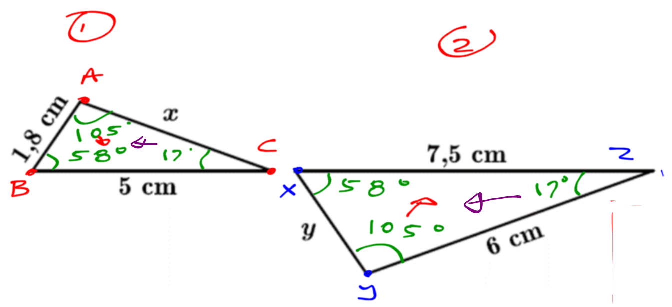
$$k = \frac{side}{side}$$

$$k = \frac{7.5}{5}$$ ← Big △ ← Small △

$$\frac{y}{1.8} = \frac{7.5}{5}$$

$$\frac{\cancel{5} \times y}{\cancel{5}} = \frac{(7.5 \times 1.8)}{5}$$

$$y = 2.7$$

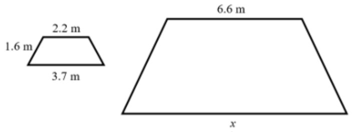


check w graph

you do pg 4 and 5 try the BONUS.

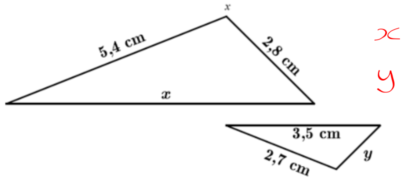
1.4.2 Practice: Determine the missing measurement(s) given that the two figures are similar.

(a)



$x = 11.1$ m

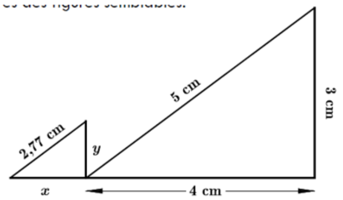
(b)



$x = 7$ cm

$y = 1.4$ cm

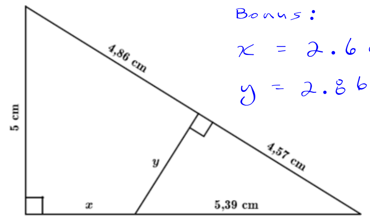
(c)



$x = 2.216$ cm

$y = 1.662$ cm

1.4.3 Bonus Example:

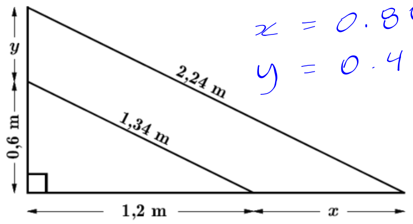


Bonus:

$x = 2.6$ cm

$y = 2.86$ cm

1.4.4 Bonus Practice:



Bonus:

$x = 0.806$

$y = 0.403$

Mini – Tasks (2D)

2.1.1 Example: The two triangles are similar. Line segment XY is 0.3 times line segment AC. Find y.

direct translation

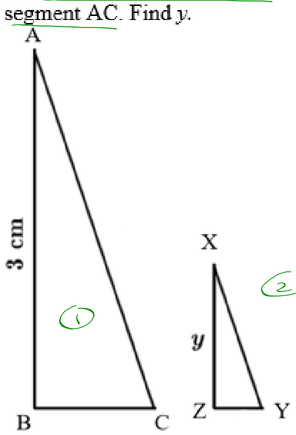


figure 2 →

figure 1 →

$\overline{XY} = 0.3 \times \overline{AC}$

← construct ratio/proprotion

$\dfrac{\overline{XY}}{\overline{AC}} = \dfrac{0.3 \times \cancel{\overline{AC}}}{\cancel{\overline{AC}}}$

$\dfrac{\overline{XY}}{\overline{AC}} = 0.3$

$\therefore k = 0.3$

$\therefore$ find y:

$\dfrac{y}{3} = k$

$3 \times \dfrac{y}{3} = 0.3 \times 3$

$y = 0.9$

TIP:

similar:

#1

- $k = \dfrac{\text{side}}{\text{side}}$
- $k^2 = \dfrac{\text{Area}}{\text{Area}}$
- $k^3 = \dfrac{\text{Volume}}{\text{Volume}}$

#2 Translate sentences into equations

#3. LABEL

nota bene:

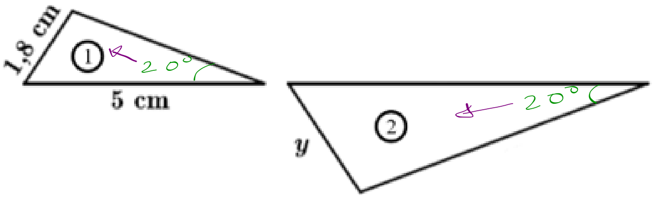
- if $k < 1$, $\dfrac{\text{small figure}}{\text{big figure}}$
- if $k > 1$, $\dfrac{\text{big figure}}{\text{small figure}}$

2.1.2 Example: The two triangles are similar. The area of the large triangle is 4 times greater than the area of the small triangle. Find y.

$A_2 = 4 \times A_1$

$k = \frac{\text{side}}{\text{side}}$

$k^2 = \frac{\text{Area}}{\text{Area}}$



translate:

$A_2 = 4 \times A_1$ ✓

$\frac{A_2}{A_1} = \frac{4 \times \cancel{A_1}}{\cancel{A_1}}$

$\frac{A_2}{A_1} = 4 \therefore k^2 = 4$

ratio of areas!

solve for k

$\sqrt{k^2} = \sqrt{4}$

$k = 2$

Find y

$\frac{\text{side}}{\text{side}} = k$

$\frac{y}{1.8} = k$

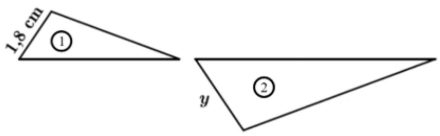
$1.8 \times \frac{y}{\cancel{1.8}} = 2 \times 1.8$

$y = 3.6\text{cm}$

You do

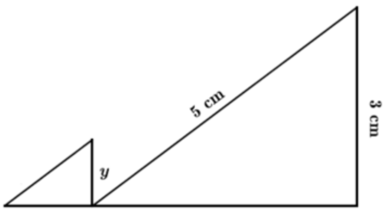
- 2.1.3
- 2.1.4

2.1.3 Practice: The two triangles are similar. The area of the triangle 1 is 0.36 times the area of triangle 2. Find y.



$y = 3$

2.1.4 Practice: The two triangles are similar. The perimeter of the larger triangle is 1.6 times greater than the perimeter of the small triangle. Find y.

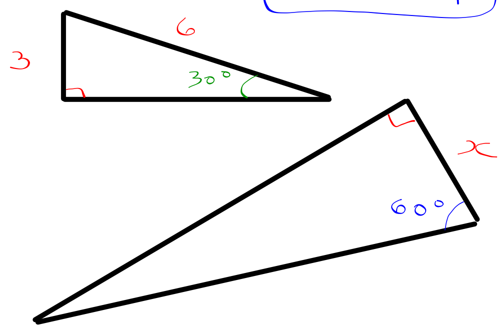


$y = 1.875$

2.1.5. a) Practice

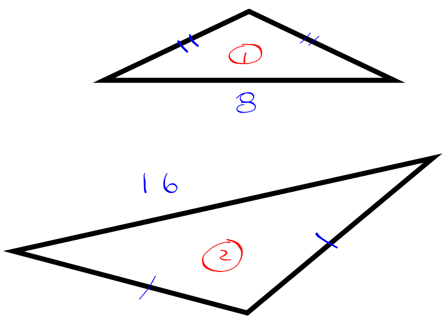
The 2 Δ's are similar.
The area of the larger Δ is 9 times the area of the smaller Δ.
Find x.

$\boxed{x = 9}$



2.1.5b) Practice.
The 2 triangles are similar. What's the area of triangle 2?

Area ① = 12 unit^2



$A_2 = 48$ units^2